S. P. Rigaud
May 31. 1821

A

SYNOPSIS

OF

PRACTICAL MATHEMATICS.

CONTAINING

PLAIN TRIGONOMETRY, MENSURATION OF HEIGHTS, DISTANCES, SURFACES, AND SOLIDS; SURVEYING OF LAND, GAUGING, NAVIGATION, AND GUNNERY.

WITH

TABLES OF THE LOGARITHMS OF NUMBERS, AND OF SINES AND TANGENTS.

FOR THE USE OF SCHOOLS AND MEN OF BUSINESS.

BY ALEXANDER EWING,
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PREFACE.

MATHEMATICS are studied either by gentlemen of birth and fortune, as a necessary part of genteel education, or by those in the middle rank of life, in order to qualify them for the employments or professions which they intend to follow.

The views of these classes are as different as their stations. The gentleman, by studying a system of theory, and such of its applications as are necessary in his other studies, may have all his ends answered. But the soldier, sailor, engineer, surveyor, and man of business, cannot follow his profession rationally, without being expert in most parts of practical mathematics.

That this class may obtain their end in the most effectual manner, it is necessary to throw the several parts to be learned into the shortest form. It is a great encouragement to proceed, when the end of the task is in view; nor is any thing more discouraging to a beginner, than to be told, that the science he is about to learn fills many volumes. The apparent length of the labour sets proficiency at so great a distance, that half a lifetime seems too little to acquire it; and, if the natural desire of knowledge be nipt in the bud by such an idea, it will be difficult afterwards to make any one apply diligently to the study.

But it is not enough to learn the several parts of practical mathematics at school; they must be remembered, or kept in view by some means or other, that the stu-

dent

dent may be always prepared to perform his part with propriety.

The following Synopsis is composed with the design of answering both these ends. As a school-book, it will lessen the labour of teaching, and shorten the time of learning. As a memorandum, the young practitioner will find here many useful rules, delivered in such clear and easy terms, that he cannot mistake their meaning; and even the most learned will find some assistance to his memory.

I have revised and corrected the tables of logarithms, and of logarithmic sines and tangents with attention, and hope they are more correct than any small tables formerly published; and, although printed in the most concise form, will answer every purpose as well as Sherwin's tables.

In section 3, I have endeavoured to show, that most problems in the mensuration of heights and distances, and surveying of land, may be performed accurately enough by a right application of lineal measures, such as the surveying chain and poles; and therefore instruments for measuring angles, are not so very necessary as has been commonly supposed.

The favourable reception which the public has given to the former editions, has encouraged the Author to make this Fourth Edition still more compleat than either of them, by several useful additions interspersed in the different sections; and it is hoped, that the book has now a very good claim to its title, and will be useful to all students, who desire to become expert in Practical Mathematics.

CON-

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Character *of this Book in the* Monthly Review.

" This Work is not offered to the Public as a complete treatise. Its object is to afford easy access to the study of Astronomy; and to enable such Students to solve its Problems, who are acquainted merely with Arithmetic, the Circles of the Sphere and Logarithms: —for such a purpose, the publication is well designed and properly executed."

It may be added, that the Book was intended to assist those who study by themselves, and the Tables will be equally useful to Students in general.

SECT. I.

GEOMETRICAL PROBLEMS.

DEFINITIONS.

1. A Plain angle is the inclination of two right lines to each other, which meeting do not lie in one straight line.

2. An angle is expressed by three letters, of which the letter at the angular point must be in the middle. Thus, the angle at the point B is called the angle ABC, or CBA. *Fig.* 1.

3. Every plain angle is measured by the arch of a circle described about the angular point as a center: Thus, the measure of the angle ABC is the arch *ae.*

4. Angles are equal to each other, when the arches which measure them are equal, the radius being the same in both, that is, when both the arches are described with the same extent of the compasses.

Problem 1. *With a given right line* AB, *and at a given point* A *in it, to make an angle equal to a given plain angle* DEF. *Fig.* 2.

About the center E, with any radius E*n*, describe the arc *mn*; and, about the center A, with the same radius A*f*, describe the arc *fg*; in which, take *fh* equal to *mn*, and through A and *h* draw the straight line AC: The angle BAC is equal to the given angle DEF.

DEFIN. 5. Parallel right lines are every where equidistant from each other, or which, being infinitely produced both ways, would never meet.

PROB. 2. *Through a given point* A, *to draw a right line parallel to a given right line* BC. *Fig.* 3.

Set one foot of the compasses in the point A, and with the other find the nearest distance of BC, by describing the arc *n*; then take any point *e* in the line BC, and about the center *e*, with the same radius, describe the arc *m*; through A and *m* draw the right line DAE, which will be parallel to BC.

DEFIN. 6. When one right line, standing on another, makes the angles on each side of it equal between themselves, these angles are right ones; and the first line is perpendicular to the other.

Note—In some of the following problems, the point where two arches cut each other, is called a section.

PROB. 3. *To bisect a given right line* AB *by another* CD *cutting it at right angles.* *Fig.* 4.

With any radius greater than half of AB, from the points A and B, make the sections C and D; and draw the line CD, which will bisect AB at right angles in E.

COR. 1. The four angles about the point E are all right angles.

2. By this problem a given circle is quartered, and from the middle of any right line another is drawn perpendicular to it.

PROB. 4. *From a point* A *in a given right line* BC, *to draw another line perpendicular to it.* *Fig.* 5.

Take AE equal to AB; and from the points B and E, with

with any radius greater than AB, make the section D; draw DA, which will be perpendicular to BC.

PROB. 5. *To make a right angle; or, from the end A of a given right line AB, to raise a perpendicular. Fig. 6.*

Take the point C nearer to A than to B; about the center C with the radius CA, describe the circle EAD; through the points E and C draw the line ED; and join DA, which will be perpendicular to AB; and BAD is a right angle.

Otherwise, *Fig.* 7.

About the center A, with any radius AE, describe the arc E*nm*; set the same radius from E to *n* and from *n* to *m*; from the points *n* and *m* make the section D; and join AD; which will be the perpendicular required to be drawn; and BAD is a right angle.

PROB. 6. *From a given point A, to let fall a perpendicular upon a given infinite straight line* BC. *Fig.* 8.

About the given point A describe a circle cutting BC in E and F; from the points E and F make the section *n*; and draw the line AD from A towards *n*; and AD is the perpendicular required.

DEFIN. 7. A plain triangle is a figure contained by three straight lines.

PROB. 7. *To make a triangle whose sides shall be equal to three given right lines* AB, BC, *and* CA; *any two of these being greater than the third. Fig.* 9.

Draw the straight line AB of the given length; and from B, with a radius equal to BC, describe an arc; and from A, with a radius equal to AC, describe another

arc cutting the former in C; join AC, CB; and ABC is the triangle required.

Note 1. If the given lines are all equal to one another, the triangle ſhall be equilateral.

2. If two of the given lines be equal, the triangle ſhall be iſoſceles.

3. If the given lines are unequal, the triangle ſhall be ſcalene. The ſame operation ſerves for making every kind.

Defin. 8. A ſquare is a quadrilateral figure whoſe ſides are equal, and its angles all right ones.

9. A rectangle is a quadrilateral figure, whoſe oppoſite ſides are equal, and its angles all right ones. It is ſaid to be contained under two lines, namely, the length and breadth.

Prob. 8. *To make a ſquare on any given right line* AB. *Fig.* 10.

Make the right angle BAD; take AD equal to AB; and from the points D and B, with a radius equal to AB, make the ſection C; join DC and BC, and ABCD is a ſquare.

Prob. 9. *To make a rectangle of two given right lines* AB *and* BC. *Fig.* 11.

Make the right angle ABC; take AB and BC of the given lengths; from A, with a radius equal to BC, deſcribe an arc; and from C, with a radius equal to AB, deſcribe another arc cutting the former in D; join AD and DC; and ABCD is the rectangle required.

Prob.

PROB. 10. *To divide a given right lined angle* ABC *into two equal parts, or to bisect it. Fig.* 12.

About the angular point B, with any radius Bd, describe the arc de; and from the points d and e make the section F; join BF, and the angle ABC is bisected by the right line BF.

N. B. If the angles ABF and CBF are bisected, the given angle will be divided into four equal parts; and each of these being again bisected, it will be divided into eight equal parts; and thus the bisection may be carried as far as you please: But an arch or angle cannot be divided into three, five, or seven equal parts, by Euclid's propositions. This is to be done by trials.

OF PROPORTION.

DEFIN. 10. When we compare any quantity with another of the same kind, in order to estimate the magnitude of the one in respect of the other; the comparison is made, either by considering what part or parts the one is of the other, or how oft the one contains the other.

11. The quantity compared is called the antecedent, and the quantity to which it is compared is called the consequent.

12. The ratio of any two quantities is the number expressing what part or parts the antecedent is of the consequent, or how often the antecedent contains the consequent. Thus, if any quantity A is compared to another B, the ratio is $\frac{A}{B}$. The ratio of 6 to 12 is $\frac{6}{12}$, and of 4 to 8 is $\frac{4}{8}$.

13. When

13. When the ratio between any two quantities is equal to the ratio between any other two, these quantities are proportional.

14. In three quantities, A, B, C; if the ratio of A to B be equal to the ratio of B to C, these three quantities are proportional; expressed thus, As A is to B, so is B to C, or A:B::B:C. Here A and C are called the extreme terms, and B is a mean proportional between A and C. Also C is a third proportional to A and B.

15. In four quantities A, B, C, D; if the ratio of A to B be equal to the ratio of C to D, the four quantities are proportional; or A:B::C:D.

Here A and D are the two extreme terms, and B and C are the means; also D is a fourth proportional to A, B, C.

Examples in Numbers.

1. In the numbers 4, 6, 9, the ratio of 4 to 6 is equal to the ratio of 6 to 9; that is, $\frac{4}{6}=\frac{6}{9}$; therefore 4:6::6:9, and 6 is a mean proportional between 4 and 9; also 9 is a third proportional to 4 and 6.

2. In the numbers, 16, 32, 12, 24, the ratio of 16 to 32 is equal to the ratio of 12 to 24; that is, $\frac{16}{32}=\frac{12}{24}$; therefore 16 : 32 :: 12 : 24. Here the first and third terms, viz. 16 and 12, are antecedents, and the second and fourth terms, 32 and 24, are consequents.

In four proportional numbers, the product of the two extremes is equal to the product of the two means; *Eucl. B. 6 prop.* 16.; thus, if 16 : 32 :: 12 : 24, then $16\times 24=32\times 12=384$.

When four quantities are proportional, several variations may be made in their order and magnitude, which geometers

geometers call *Alternation*, *Inversion*, *Composition*, *Division*, and *Conversion*.

16. Alternate proportion is the comparing of the antecedent with the antecedent, and the consequent with the consequent.

Exam. Suppose four proportional quantities are represented by the proportional numbers 18:6::21:7, these will be alternately proportional, viz. as 18:21::6:7.

17. Inverse proportion is when the consequent is made an antecedent, and the antecedent a consequent; that is, if 18:6::21:7, then, by inversion, as 6:18::7:21.

18. Compounded proportion is when the sum of the antecedent and consequent is compared to the consequent. If 18:6::21:7, then, by composition, as 24:6::28:7.

19. Divided ratio is when the difference between the antecedent and consequent is compared to the consequent. If 18:6::21:7, then, by division, as 12:6::14:7.

20. Converse ratio is when the antecedent is compared to the difference between the antecedent and consequent. If 18:6::21:7, then, by conversion, as 18:12:: 21:14.

Four proportional quantities being given, to vary their order, so that any one of them may stand in the fourth place. This is done by inversion and alternation, as follows:

Exam. Let the four proportionals be 12:3 ::16: 4
Then, by inversion, as 4:16:: 3:12
by alternation, as 16: 4::12: 3
and, by inversion, as 3:12:: 4:16

Hence, if four quantities be proportional, any three of them being given, the fourth may be found.

PROB. 11. *To find a mean proportional between two given right lines* AB *and* BC. *Fig.* 13.

Draw an indefinite line CE, on which take BA and BC of the given lengths; on AC describe the semicircle ADC; and from B draw BD perpendicular to AC; and BD is the mean proportional between AB and BC, or AB:BD::BD:BC.

PROB. 12. *To find a third proportional to two given right lines* AB *and* BC. *Fig.* 14.

Make any angle PAQ; and take AD equal to AB, and AE and DF each equal to BC; join DE; and through F draw FH parallel to DE; and EH is the third proportional required. For AD:DF::AE:EH; that is, AB:BC::BC:EH.

PROB. 13. *To find a fourth proportional to three given right lines* AB, BC, *and* CD. *Fig.* 15.

Make any angle GEF; and take EH equal to AB, HK equal to BC, and EM equal to CD; join HM; and through K draw KL parallel to HM; and ML is the fourth proportional required. For EH : HK :: EM: ML, that is, AB:BC::CD:ML.

PROB. 14. *To divide a given right line* AB *into any number of equal parts, suppose* 10. *Fig.* 16.

Draw AC, making any angle with AB, and through B draw BD parallel to AC; take ten equal parts on AC from A to *m*, without regarding whether A*m* be longer

or

or ſhorter than AB; take the ſame parts from B to *e* on the line BD; join the oppoſite points, as in the figure; and AB will be divided into ten equal parts.

N. B. This operation divides, not only AB, but alſo B*m* or A*e*, into ten equal parts. For *n* 1 is $\frac{1}{10}$, and *n* 2 is $\frac{2}{10}$ of B*m*. By this method, a very ſhort line may be divided into ten, or any other number of equal parts, as may be ſeen on the diagonal ſcale.

PROB. 15. *To make the diagonal ſcale of any length. Fig.* 17.

Make the right angle BAC; take ten equal parts on AC, and through each diviſion draw lines parallel to AB: Having determined the length of A*o*, divide it into ten equal parts, and take the ſame parts from C to *m*; then, through the ninth diviſion, counting from *o* towards A, draw the line 9C; and through the other diviſions, draw lines parallel to it, and draw *om* parallel to AC; theſe lines will divide each of the ten parts between *o* and A into ten equal parts.

Take the line *o*A in your compaſſes, and carry it along the line AB, as often as you pleaſe; and through theſe points, draw lines parallel to AC or *om*; and the ſcale is made.

When the great parts A*o*, *o* 1, &c. are accounted hundreds, the ſmall diviſions between *o* and A are tens, and the units are reckoned on the parallels, between A*o* and C*m*; but, when the great parts are accounted tens, the ſmall are units; and their parts on the parallels are decimals of an unit.

To take any number from the ſcale.

Take the hundreds between *o* and B, the tens between *o* and A, and carry back both feet of the compaſſes to the parallel whereon the units are marked.

A ſcale of equal parts, ſuch as fig. 18. ſerves for taking numbers below an hundred, eſpecially when a large plan is to be made. Its conſtruction and uſe is evident by inſpection.

The lengths of the ſides of right lined figures may be taken from theſe ſcales, to a ſufficient degree of accuracy for any purpoſe; but, in order to make or meaſure right lined angles, we muſt have recourſe to the circle.

Every arc of a circle is meaſured by certain right lines, called Chords, Sines, Tangents, and Secants; and, for nautical purpoſes, a Rhumb line. *Vide* Trigonometry.

PROB. 16. *To conſtruct the lines of chords, rhumbs, ſines, tangents, and ſecants. Fig.* 19.

1. To make the line of Chords.

Having deſcribed the ſemicircle ADB, and drawn the right line DB, divide the quadrant DB into nine equal parts; and, about the center B, deſcribe arcs through the ſeveral diviſions, cutting the right line DB in 10, 20, 30, &c. and the line of chords is made for every ten degrees.

In the ſame manner, if each of theſe parts of the quadrant be divided into ten equal parts, and arcs deſcribed through the ſeveral points of diviſion, the line of chords will be made.

Note.

Note. The chord of ſixty degrees is equal to the radius of the circle.

2. To make the line of Rhumbs.

Draw the right line DA, and divide the quadrant DA into eight equal parts; and about the center D deſcribe arcs cutting the right line DA in the points 1, 2, 3, &c. and DA will be a line of rhumbs, or points of the mariner's compaſs.

N. B. This is alſo a line of chords, each diviſion being 11°. 15', and is uſed to deſcribe the angle of a ſhip's courſe.

3. To make the line of Sines.

From each point of diviſion in the quadrant BD, let fall perpendiculars on the radius CB; and theſe diviſions being numbered 10, 20, &c. from C to B, will be a line of ſines. The ſine of 90° is equal to the radius of the circle.

4. To make the line of Tangents.

Through D draw DE parallel to CB; then a ruler laid to the center C, and each diviſion in the quadrant DB will cut the line DE in the points 10, 20, &c.; and DE will be the line of tangents. The tangent of 45° is equal to the radius of the circle.

5. To make the line of Secants.

Produce CB to F; ſet one foot of the compaſſes in the center C, and with the other deſcribe arcs through every diviſion in the tangent line, cutting the line CF in the points 20, 30, &c.; and CF will be a line of ſecants.

The ſecant of every arc is greater than the radius of the circle; and therefore the ſecant line begins at the ſine of 90°, and goes on without the circle.

The line of ſemitangents is made from the line of tangents; the ſemitangent of 10° being the whole tangent of 5°, &c.

The diviſions on the lines above deſcribed, being transferred to right lines drawn on a ſcale of wood, or other matter, are of conſiderable uſe in practical mathematics.

The ſcale of equal parts, and the line of chords, ſerve for making and meaſuring right lined figures; and the lines of ſines, tangents, and ſecants, are uſeful in ſpherical projections.

Prob. 17. *To make an angle of a propoſed number of degrees, ſuppoſe 55° by the line of chords. Fig.* 20.

Draw the right line AB; about the center A, with a radius equal to the chord of 60°, deſcribe the arc CD; then take the given number of degrees (55°) from the ſame line of chords, and ſet it from C to *m*, and through A and *m* draw the line AE; and EAB is the angle required.

Prob. 18. *To meaſure a right lined angle by the line of chords.*

With a radius equal to the chord of 60°, deſcribe an arch about the angular point; take the arch in your compaſſes, and apply it to the line of chords; and thus the quantity of the angle will be known.

When

When an obtuse angle is to be made or measured, its measure being greater than 90°, must be taken from the scale at twice: Thus, suppose an angle of 160° was to be made; having described an arc with the chord of 60°, take 90° and 70°, or 80° and 80° from the line of chords, and set them one after another on that arc.

DEFIN. 21. A regular polygon, is a figure whose sides and angles are all equal, and is denominated by the number of its sides. One of five sides is called a Pentagon, of six sides, a Hexagon, &c.

22. The angle at the center of a regular polygon is contained between two right lines drawn from its center to the extremities of any of its sides, and is found by dividing 360° by the number of sides.

23. The angle at the circumference of a regular polygon is contained between any two adjacent sides of the polygon, and is found by subtracting the angle at the center from 180°.

A TABLE, shewing the Names, number of Sides, Angles at the Center, and at the Circumference of regular Polygons, from three to twelve Sides, inclusive.

Names.	*Sides.*	*Ang. at the Center.*		*Ang at the Circumf.*	
Trigon	3	120°	00′	60°	00′
Square	4	90	00	90	00
Pentagon	5	72	00	108	00
Hexagon	6	60	00	120	00
Heptagon	7	51	$25\frac{5}{7}$	128	$34\frac{2}{7}$
Octagon	8	45	00	135	00
Nonagon	9	40	00	140	00
Decagon	10	36	00	144	00
Endecagon	11	32	$43\frac{7}{11}$	147	$16\frac{4}{11}$
Dodecagon	12	30	00	150	00

PROB.

PROB. 19. *To inscribe a regular polygon in a circle by the line of chords.*

Make an angle at the center of the circle equal to the angle at the center of the polygon required to be drawn; this will give one side of the polygon, which being applied round the circle, the lines joining the several points in the circumference will be the sides of the polygon.

Exam. Suppose a pentagon were to be inscribed in a given circle ABD. *Fig.* 21.

At the center, make the angle ACB of 72°, and join AB; then applying AB round the circumference, will give the points D, E, F; join BD, DE, EF, FA, and ABDEF is the pentagon required.

In the same manner, any other regular polygon may be inscribed in the circle.

N. B. The radius of the circle is equal to the side of a hexagon inscribed in it.

PROB. 20. *To make any regular polygon on a given right line* AB. *Fig.* 21.

At the points A and B, make the angles CAB, ABC, each equal to half of the angle at the circumference of the polygon; and from the point C with the radius CA or CB, describe a circle; and AB properly applied in the circumference will give the sides of the polygon required.

Exam. of a pentagon. Make the angles CAB and ABC each equal to 54°, the half of 108°; and from the center C, with the radius CA, describe a circle, in which apply the right lines BD, DE, EF, and FA, each equal to AB; and ABDEF is a pentagon described upon the given line AB.

PROB.

PROB. 21. *To draw a tangent to the circle from any point* A, *which is not within the circle. Fig.* 22.

1. If the given point A is in the circumference, draw a line from the center C to A, and through A draw BAD at right angles to AC; and BD is the tangent required.

2. If the point A is without the circumference, join CA, and on it describe the semicircle AEC, cutting the arch AE in E; then, through A and E, draw AD, which will touch the circle in the point E.

PROB. 22. *To describe a circle about a given triangle* ABC; *or, to describe a circle through any three given points* A, B, C, *which do not lie in the same right line. Fig.* 23.

Bisect the sides of the triangle, or the distances of the points AB and AC, at right angles, by the right lines DF and EF; and from the point F, where they meet, with a radius equal to FA, FB, or FC, describe the circle ABC.

PROB. 23. *To describe an oval figure representing an ellipse, whose longest diameter* AB *is given Fig.* 24.

Divide AB into three equal parts in the points C and D; and about the centers C and D, with the radius CD, describe two circles, intersecting each other in the points E and F; through the points C and D, draw the lines EN, EM, FO, and FP; and about the centers F and E, with the radius EN, describe the arches NM and PO; and AHBG is the oval figure required.

SECT. II.

PLAIN TRIGONOMETRY.

DEFINITIONS.

1. A Plain triangle hath ſix parts, *viz.* three ſides and three angles; any three of theſe being given, except the three angles, the reſt may be found; and the method of performing this in every poſſible caſe is called *Trigonometry*.

2. The ſides of plain triangles are eſtimated by meaſures of length, ſuch as, inches, feet, yards, miles, &c.

3. The angles of plain triangles, are meaſured by circular arcs deſcribed about the angular points; thus the meaſure of the angle BCD is the arc BD; and of DCH the arc DH. *Plate* 2. *Fig.* 25.

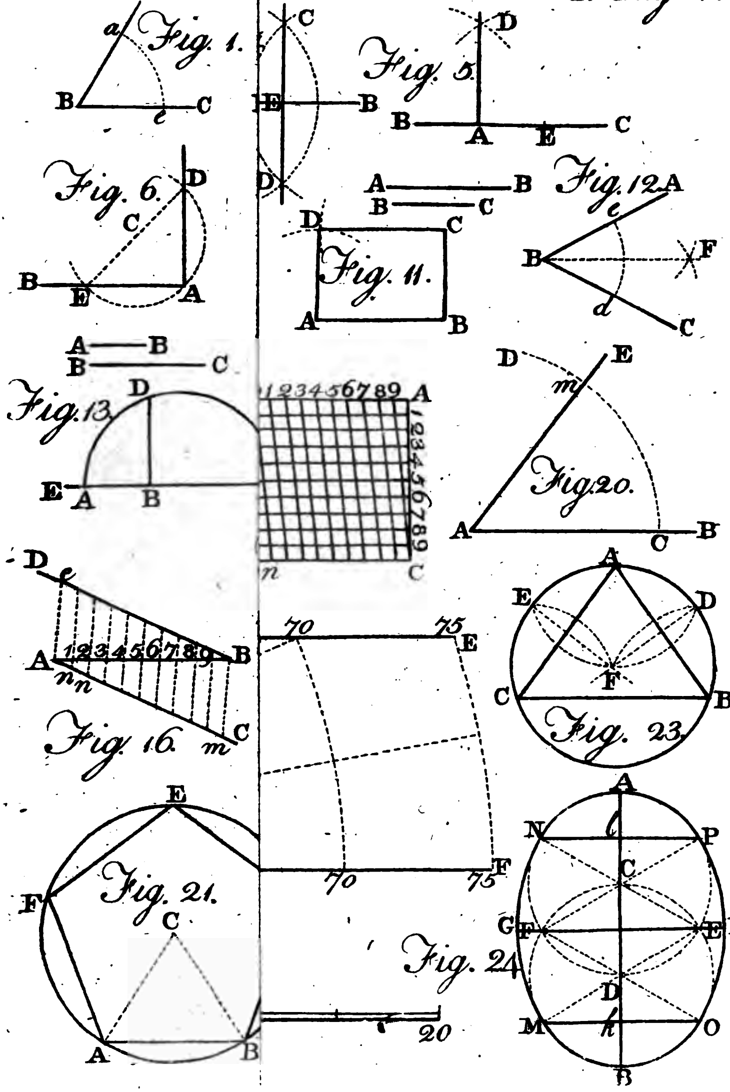
4. The circumference of every circle conſiſts of 360 equal parts, called *Degrees*; each degree of 60 minutes, and each minute of 60 ſeconds, &c. marked thus, 1°, $1'$ $1''$, ſignifying, one degree, one minute, one ſecond.

5. An angle is ſaid to be of the ſame number of degrees, minutes, and ſeconds, with the arc that meaſures it.

6. The whole circumference of the circle is the meaſure of four right angles; the meaſure of a right angle is a quadrant of the circle, or 90°; of two right angles is 180°; and the meaſure of an obtuſe angle is greater than 90°, and of an acute angle, is leſs.

7. The

PLATE 1.st Pag. 10



7. The complement of an arc or angle, is its difference from a quadrant, or 90°; thus the complement of the arc BD, or the angle BCD, is the arc DH, or the angle DCH; and the complement of the arc BK, or the angle BCK, is the arc HK, or the angle HCK.

8. The ſupplement of an arc or angle, is what it wants of a ſemicircle, or 180°; thus the arc DHA, or the angle DCA, is the ſupplement of the arc DB, or of the angle DCB.

N. B. This is ſometimes called the complement of an arc to a ſemicircle.

9. The diameter of a circle, is a right line drawn through the center, and terminated by the circumference, as AB.

10. The radius, is any right line drawn from the center, to the circumference, as CA, CB, or CD, &c.

11. The chord of an arc, is a right line joining its extremities; thus, DEO is the chord of the arc DBO, or of DHAO.

N. B. Every chord line, except the diameter, belongs to two arcs, the one leſs, and the other greater than a ſemicircle.

12. The ſine of an arc or angle, is a right line drawn from one end of the arc, perpendicular to the radius drawn to the other end of it; or, it is half the chord of double the arc; thus, DE is the ſine of the arc DB, or DHA; and DF is the ſine of DH or DBV.

N. B. As every chord line, except the diameter, belongs to two arcs; ſo every ſine, except the radius, belongs alſo to two arcs; the one leſs than a quadrant,

 and

and the other its supplement; because every sine is half the chord of double the arc.

13. The tangent of an arc or angle, is a right line touching one end of the arc, and produced until it meets with the right line drawn from the center through the other end of the arc; thus, BG is the tangent of the arc BD, and HI of HD.

14. The secant of an arc or angle, is a right line drawn from the center through one end of the arc, and produced until it meets with the tangent drawn from the other end of the arc; thus CG, is the secant of the arc BD, and CI of DH.

15. The versed sine of an arc or angle, is that part of the diameter intercepted between the arc and its sine; thus, BE is the versed sine of the arc BD, and AE is the versed sine of the arc DHA.

16. The co-sine, co-tangent, co-secant, and co-versed sine of an arc or angle, is the sine, tangent, secant, versed sine of the complement of that arc or angle.

Thus DE, BG, CG, EB, being respectively the sine, tangent, secant, and versed sine of the arc DB, which is the complement of DH; therefore DE is called the co-sine, BG the co-tangent, CG the co-secant, and EB the co-versed sine of the arc DH, or of the angle HCD.

17. The chords and versed sines of arcs increase continually from the least quantity to 180°, or a semicircle; but the sines, tangents, and secants increase only to 90°.

18. The sine of a right angle, or 90°, is equal to the radius; but the tangent and secant of 90° are infinite; because the tangent BG is parallel to the radius CH.

19. When

19. When the ſine, tangent, or ſecant of an obtuſe angle is ſought in the tables, that of its ſupplement muſt be taken; thus for the ſine, tangent, and ſecant of 120° take thoſe of 60°.

20. Since the meaſure of a right-lined angle is the arc of a circle deſcribed about the angular point with any radius, we can have as many different arcs expreſſing the quantity of an angle as we pleaſe to aſſume different *radii*; each of theſe arcs hath a ſine, tangent, ſecant, and verſed ſine; and therefore the ſine, tangent, &c. of the ſame angle may be expreſſed by innumerable different right lines, all in proportion to each other as the *radii* aſſumed; for, let EAF be any right-lined acute angle, *Fig.* 26. produce the right lines EA and AF at pleaſure; about the center A deſcribe the ſeveral arcs, EB, HC, and LD, and draw the ſines EF, HI, and L*r*, and the tangents BG, CK, and DM; and then, by ſimilar triangles, as

$$EF : HI :: AE : AH$$
$$GB : KC :: AB : AC$$
$$AG : AK :: AB : AC$$

All trigonometrical calculations depend on this principle; as alſo, the conſtruction of triangles, and the method of meaſuring the parts required by ſcale and compaſſes.

For, a radius being taken at pleaſure, and the lengths of the ſines, verſed ſines, tangents, and ſecants of all arcs from 1′ to 90°, calculated in proportion to it, and ranged in order in tables, or marked on a ſcale, affords a canon for calculating or meaſuring the angles and ſides of any triangle; one ſide, and other two of its parts, being given.

21. The three angles of any plain triangle are equal to two right angles, or 180°.

Cor. 1. If one angle of a plain triangle be given, the sum of the other two angles may be found, by subtracting the given angle from 180°.

2. If two angles of a plain triangle are given, the third may be found, by subtracting the sum of the two given angles from 180°.

3. The two acute angles of any right angled plain triangle, are equal to one right angle, or 90°.

4. If one of the acute angles of a right angled plain triangle be given, the other is found by subtracting the given angle from 90°.

22. In any right angled plain triangle ABC, *Fig.* 27. the line AC opposite to the right angle, is called the Hypothenuse; and AB and BC, which contain the right angle, are called Sides, or Legs.

23. Similar triangles are those which have their angles equal, each to its correspondent angle.

24. The sides about the equal angles of similar triangles are proportional. *Eucl. B.* 6. *prop.* 4.

Theorem. If from any point in the side of a plain triangle, a right line be drawn parallel to one of the other sides, it will cut off a triangle similar to the whole. *Fig.* A. *plate* 2.

Let ABC be a triangle, and from any point E in the side AB draw EF parallel to BC; I say, the triangle AEF is similar to the triangle ABC.

For, since the line AB falls upon the two parallels EF and BC, the angle AEF is equal to the angle ABC; and, for the same reason, the angle AFE is equal to the angle

angle ACB; alſo the angle at A is common to the two triangles AEF, ABC; and therefore the triangle AEF is ſimilar to the triangle ABC. In like manner, if FD be drawn parallel to AB, it may be demonſtrated that the triangle FDC is ſimilar to the triangle ABC. Hence, and from the 4th prop. of Euclid's 6th book, we have the following proportions, viz.

AE : EF :: AB : BC	CD : DF :: CB : BA
EF : FA :: BC : CA	CF : FD :: CA : AB
FA : AE :: CA : AB	DC : CF :: BC : CA

25. By Radius, in Trigonometry, is underſtood, the radius of the tables of ſines and tangents, commonly called the Trigonometrical Canon; and is generally ſuppoſed to conſiſt of 100000000 equal parts.

CONSTRUCTION *of the* TABLES *of* SINES *and* TANGENTS, *&c.*

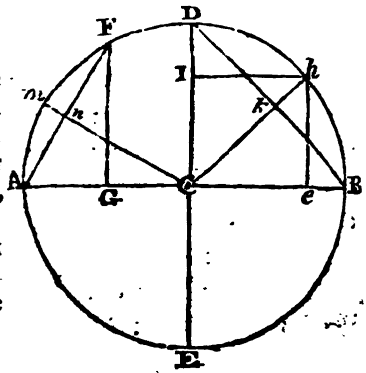
1. The diameter of a circle being given, to find the chord of 90°.

The diameter AB being given, the radius CB or CD will be known; and the chord of 90°, or a quadrant, viz. $BD=\sqrt{CB^2+CD^2}$ by the 47th Euclid, B. 1.

Exam. Let the diameter AB=2, then the radius CB or CD=1, and $BD=\sqrt{CB^2+CD^2}=\sqrt{2}=1.414213562$. Now, becauſe the ſine of an arch is half the chord of double the arch, we have the ſine of 45°, viz. B*k* or D*k*=,70710678 1, which is half the chord of 90°.

The

The co-ſine of 45° is equal to its ſine: and becauſe the difference between the co-ſine and the radius is the verſed ſine of any arch leſs than 90°, therefore, from the radius = 1 ſubtract .707106781 the remainder kh = .292893219 is the verſed ſine of 45°.



Set the radius from A to F; join FA, and from F draw FG perpendicular to CA; alſo biſect the arch AF in m; join Cm cutting the chord AF in n: then AF = 1 is the chord of 60°, and FG is its ſine: Am is an arch of 30°, An is its ſine and Cn is its co-ſine. Now A$n = \frac{1}{2}$, therefore the ſine of 30° is .5000000, and its co-ſine is Cn.

2. The radius of the circle and the ſine of any arch being given, to find its co-ſine.

Exam. The radius CA = 1; Am an arch of 30°, its ſine An = .5000000: it is required to find Cn, the co-ſine of Am?

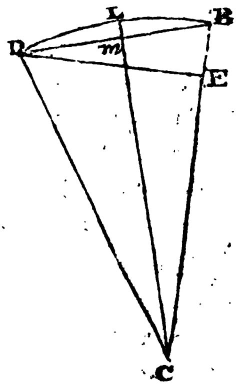
By the 47th prop. of Euclid B. 1. $CA^2 = Cn^2 + nA^2$, and therefore $Cn = \sqrt{CA^2 - An^2}$.

From the ſquare of CA = 1.
Subtract the ſquare of An = .25

There remains the ſquare of Cn = .75 and $\sqrt{75}$ = .8660254, the co-ſine of 30°, or the ſine of 60°.

3. The

3. The radius of the circle, and the ſine of an arch being given, to find the ſine of half that arch.



Let DB be an arch, and its ſine DE given, to find Bm, the ſine of half the arch DB.

The ſine DE being given, the co-ſine CE will be known. The co-ſine being ſubtracted from the radius CB $=1$, the remainder EB is the verſed ſine of the arch DB. Join DB; then in the right angled triangle DEB, the ſides DE and EB being known, we have $DB=\sqrt{\overline{DE}^2+\overline{EB}^2}$, and half of the chord DB, viz. Bm, is the ſine of half of the arch DB.

Exam. Let DB be an arch of $30°$; then its ſine DE=.5000000 and its verſed ſine EB=.1339746; Required the ſine of $15°$?

Anſ. .25881904

$$DE^2 = .25000000000000$$
$$EB^2 = .01794919344516$$
$$DB^2 = .26794919344516$$

The ſquare root of .26794919344516=.51763809, and the half of .51763809 is .25881904 the ſine of $15°$,

4. The ſine Bm of an arch BL being given, to find ED the ſine of double that arch.

The ſine Bm being given, the co-ſine Cm may be found; the triangles CBm, DBE are equiangular; for the

the angles at *m* and E are right, and the angle EBD is common; therefore the sides are proportional, viz.

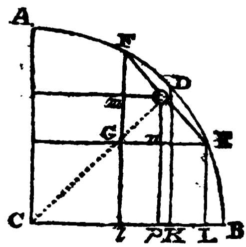
As CB : C*m* :: BD : DE, that is,

As the radius
Is to the co-sine of the given arch BL;
So is twice the sine of the given arch $=2Bm$ or DB,
To DE the sine of double the arch BL.

Exam. Let BL be an arch of 15°; its sine $= .25881904$, it is required to find the sine of 30°, the double of 15°?

The co-sine of 15° is .965925826, and twice the sine of 15° is .51763809. then,

As Rad. $= 1 : .965925826 :: .51763809 : .499999989$, or .5000000 nearly $=$ the sine of 30°, which was to be found.



5. The sines of two arches BD and DF being given, to find FI the sine of their sum, and EL the sine of their difference.

Describe the quadrant ADB, and take BD, and DF of the given quantities: then BF is the sum of the two arches BD, DF, and BE their difference. Draw the radii CD, CB: take DE=DF and join FE: also draw FI, DK and EL perpendiculars to CB: then CD being radius, FO is the sine of FD, and CO its co-sine. Through O draw OP parallel to DK, and through E and O draw EG and O*m* parallel to CB. The triangles, CDK, COP and FO*m* are equiangular, because of the parallel lines, Therefore, as CD : DK :: CO : OP $=$ I*m*
and as CD : CK :: FO : F*m* $=$ *m*G because FO=OE.

Now

Now $OP+mF=FI$, the sine of BF the sum of the arches BD, DF; and $OP-mF=EL$, the sine of BE the difference of the arches BD, DF.

Coroll. Because the difference of the arches BE, BD, BF, are equal to one another, the arch BD is an arithmetical mean between the arches BE and BF.

Exam. Suppose the mean arch $BD=45°$; and $DF=15°$; it is required to find the sine of their sum BF, and of their difference BE?

The radius $CD=1$; the sine of 45° $DK=.70710678$, the co-sine of 15°, $CO=.96592582$, and the sine of 15°, $FO=.2588190$.

1 as $CD : DK :: CO : OP$ that is,

as $1 : .70710678 :: .96592582 : .683012696=OP$

2 as $CD : CK :: FO : Fm$. In this exam. $DK=CK$ that is, as $1 : .70710678 :: .258819 : .183012669=$ Fm or mG.

$$OP= .683012696$$
$$Fm=\pm .183012669$$

Sum is the sine of $60°=.866025365=FI$

Diff. is the sine of $30°=.500000027=EL$ true to the 8th place of decimals.

6. In the same figure, the radius is to double the co-sine of the mean arch BD, as the sine of DF the difference of the arches, is to the difference of the sines of the extreme arches BF and BE.

For $CD : CK :: FO : Fm$, and by doubling the consequents $CD : 2CK :: Fm : 2\,Fm$ or FG.

Exam. Let BD the mean arch $=20°$, the common difference 10°, which is also the least extreme; required the sine of 30°, the greater extreme?

D The

The mean arch is 20°; its co-fine = .93969262, which being doubled is 1.87938524 = 2CK, and the fine of the common difference 10° is .17364817 = FO.

Then as CD : 2CK :: FO : 2Fm, that is,

as 1 : 1.87938524 :: .17364817 : .32635180 = FG

Now, .32635180 being added to the fine of 10° = .17364817, the fum, .4999999 or .5000000 nearly, is the fine of 30° the greater extreme: or the fame number being fubtracted from the fine of 30°, the remainder is the fine of 10°, the leaft extreme.

Coroll. 1. Hence, if the fines of all arches from the beginning of the quadrant to any point, differing from one another by a certain interval, fuch as 1°, are given, the fines of all other arches from that point to its double may be found: for example, fuppofe the fine of every arch confifting of whole degrees up to 15° is known; then by the laft article, the fine of any arch from 15° to 30° may be found: For the radius is to the double co-fine of 15°, as the fine of 1° is to the difference of the fines of 14° and 16°, and the radius is to the double co-fine of 15°, as the fine of 2° is to the difference between the fines of 13° and 17°; and as the fine of 3° is to the difference between the fines of 12° and 18°: and thefe differences being added to the fines of the lefs, will give the fines of the greater arches.

2. If the mean arch be 30°, as radius is to the double co-fine of 30°, fo is the fine of 1° to the difference of the fines of 29° and 31°; and fo is the fine of 2°, to the difference of the fines of 28° and 32°: as alfo, fo is the fine of 3° to the difference of the fines of 27° and 33°: But in this cafe double the co-fine of 30° is the fquare root of 3 = 1.7320508, therefore, if the fine of any arch below

below 30°, be multiplied by the square root of 3, the product will be the sine of an arch as much above 30°, as the given arch was below it. If, for example, the sine of 20° be multiplied by the square root of 3, the product will be the sine of 40°.

3. If the sines of all arches below 60°, are known, those above 60° may be found by addition: for since the radius is to double the co-sine of 60°, as the sine of 1° is to the difference between the sines of 59° and 61°; and the radius being 1, twice the co-sine of 60°, or twice the sine of 30° is also 1; and since the first and second terms of the proportion are equal, the third and fourth terms will also be equal; therefore the sine of 1° is the difference between the sines of 59° and 61°. For the same reason the sine of 2° is the difference between the sines of 58° and 62°, and the sine of 3° is the difference between the sines of 57° and 63°; therefore add the sines of 1°, 2°, 3°, 4° severally to the sines of 59°, 58°, 57° and 56°; the sums shall be the sines of 61°, 62°, 63° and 64, and so on to 90°.

Exam. Having the sines of 45° and 15° given, to find the sine of 75°.

To the sine of 45° = .70710678
Add the sine of 15° = .25881904

The sum is the sine of 75° = .96592582

7. To find the sine and co-sine of 1 minute, the radius being 1. In very small arches, such as 1 minute, the length of the arch and of its sine are very nearly equal; therefore, the length of the arch may be taken for its sine.

If the diameter of a circle be 2, or the radius 1, the circumference will be 6.2831853071; divide this by the

number of minutes in $360^\circ = 21600'$, and the quotient is the length of the arch of $1' = .000290888208$, which may be taken for the sine of $1'$, true to the 12th place of decimals.

The square of the sine of any arch being subtracted from the square of the radius, the remainder is the square of the co-sine of that arch.

In this example, the square of $.0002908882$ is $.0000000846159448992 4$, which being subtracted from 1, the square of the radius, there remains $.9999999153 84$, &c. and by extracting the square root, the co-sine of $1'$, or the sine of $89^\circ\ 59'$ comes out $.9999999577$.

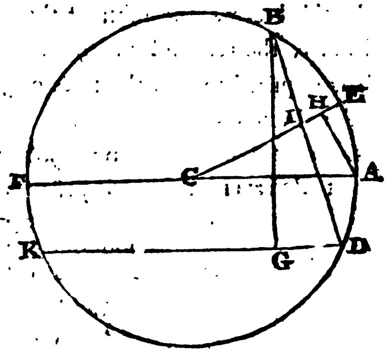
The sine and co-sine of $1'$ being found, the sine of $2'$ may be found by Article 4th, For the radius is to twice the co-sine of any arch, as its sine is to the sine of double that arch. That is,

As $1 : 1.9999999154 :: .0002908882 : .0005817753 =$ Sine of $2'$.

8. Twice the rectangle under the sine of half the sum, and the co-sine of half the difference of two arches, is equal to the rectangle under the radius and the sum of their sines.

Let ABFK be a circle, its center C and diameter FA: let AB, AD be two arches. Draw the straight line DK parallel to AF, and BG perpendicular to DK: join BD and bisect it in I; through I, draw the radius CE and through A, draw AH parallel to BD; then BG is the sine of BAD the sum of the two arches, and BI is the sine of BE half their sum; also CH is the co-sine of AE the half of their difference.

The



The triangles BGD, CHA are equiangular, for the angles at G and H are right, and the angles GDB, CAH are equal, by reaſon of the parallel lines BD and AH; therefore the ſides are proportional, viz. as BD : BG :: CA : CH or as 2BI : BG :: CA : CH, then $2BI \times CH = BG \times CA$. But $2BI \times CH = BI \times 2CH$, therefore $BI \times 2CH = BG \times CA$. Q.E.D.

From this propoſition the following practical rule is derived:

9. The ſine and co-ſine of 1 minute being given, to find the ſines of all arches from 1′ to 90°.

As the radius = 1, is to the double co-ſine of 1 minute, ſo is the ſine of 1′, to the ſum of the ſines of 2′ and 0′; and ſo is the ſine of 2′ to the ſum of the ſines of 3′ and 1′; and ſo is the ſine of 3′ to the ſum of the ſines of 4′ and 2′; and ſo is the ſine of 4′, to the ſum of the ſines of 5′ and 3′; and ſo is the ſine of 5′ to the ſum of the ſines of 6′ and 4′, &c. and ſo on from minute to minute until the ſine of every arch in the quadrant is found.

In the application of this rule; becauſe the firſt term of the proportion is 1: multiply the double co-ſine of 1 minute

minute by the third term, and subtract the next less sine from the product, the remainder is the sine sought.

Exam. It is required to find the sines of 3′ and 4′?

For the sine of 3′

double the cos. of 1′ =	1.9999999154
sine of 2′ reversed =	3677185000.
	99999995
	15999999
	199999
	139999
	13999
	1199
	59
product =	.00116355250
Subtr. the sine of 1′ =	.0002908882
remains the si. of 3′ =	.0008726643

For the sine of 4′

	1.9999999154
Sine of 3′ rever.	3466278000
	159999993
	13999999
	399999
	119999
	11999
	799
	59
	.00174532847
Subtr. S. of 2′	.0005817763
Sine of 4′ =	.0011635521

It

It may be observed that the double co-sine of 1′ being 1.9999999154 is very nearly = 2 and therefore when the sine of any arch is required true to 7 places, multiply the sine of the arch next less by 1′ than that whose sine is sought by 2 and subtract the sine of the arch next less by 1′ than that whose sine was multiplied, from the product, the remainder is the sine required.

Exam. It is required to find the sines of 3′, 4′, 5′ and 6′, &c. the sine and co-sine of 1 minute being known?

S of 2′ =	.0005817763	S. 3′ = .0008726644
Multiply by	2	2
product	.0011635526	.0017453288
Subtr. S. of 1′ =	.0002908882	S. 2′ = .0005817763
S. of 3′ =	.0008726644	S. of 4′ = .0011635525

S. 4′ =	.0011635525	S. 5′ = .0014544406
Multiply by	2	2
product	.0023271050	.0029088812
Subtr. S. 3′ =	.0008726644	S. 4′ = .0011635525
S. of 5′ =	.0014544406	S. of 6′ .0017453287

2. The sine of 22° 29′ being .3824147 and the sine of 22° 30′ = .38268343: required the sine of 22° 31′?

Sine of 22° 30′ =	.38268343
Multiply by	2
	.76536686
Subtract the S. of 22° 29′	.3824147
The S. of 22° 31′ =	.3829521

Tri-

Thus may the ſine of any arch be calculated, or the truth of any ſine in the table examined, if the ſines of two arches differing by one minute next below that arch be known; than which, nothing more eaſy can be deſired.

10. The radius and the length of an arch being given to find its ſine.

The radius being 1; call the length of the given arch a; and the ſine of that arch is $a - \frac{a^3}{6} + \frac{a^5}{120} - \frac{a^7}{5040} + \frac{a^9}{362880} - \frac{a^{11}}{39916800}$ &c.

The firſt term of this ſeries gives the ſine of any arch leſs than 1 degree, true to 7 places; and the two firſt terms gives the ſine of any arch below 5 degrees true to the ſame number of places; but in greater arches more terms of the ſeries muſt be applied.

Exam. It is required to calculate the ſine of 10 degrees?

Arch of 1′ =	.0002908882
Multiply by	600′=10°
a =	.1745329200
Same reverſed	29235471.
	174532920
	122173044
	6981316
	872664
	52360
	3490
	1570
	35
a^2 =	.0304617400

$$
\begin{array}{lr}
a^2 = & .030461740 \\
a \text{ reversed} & 29235471 \\
\hline
 & 304617400 \\
 & 213232180 \\
 & 12184696 \\
 & 1523087 \\
 & 91385 \\
 & 6092 \\
 & 2741 \\
 & 61 \\
\hline
a^3 = & .005316657642 \\
\hline
\frac{a^3}{6} = & .00088609607
\end{array}
$$

$$
\begin{array}{lr}
a^3 = & .005316657642 \\
a^2 \text{ reversed} & 47164030 \\
\hline
 & 159497292 \\
 & 2126630 \\
 & 318994 \\
 & 5316 \\
 & 3711 \\
 & 212 \\
\hline
a^5 = & .000161952155 \\
\hline
\frac{a^5}{120} = & 000001349601
\end{array}
$$

$$
\begin{array}{lr}
a = & .174532920 \\
\frac{a^3}{6} = & -.000886096 \\
\hline
 & .173646824 \\
\frac{a^5}{120} = & +\ 000001349 \\
\hline
\text{The Sine of } 10° = & .173648173
\end{array}
$$

10. The fame things being given, the co-fine of an arch is $1-\frac{a^2}{2}+\frac{a^4}{24}-\frac{a^6}{720}+\frac{a^8}{40320}-\frac{a^{10}}{3628800}$ &c.

Exam. It is required to calculate the co-fine of 10 degrees, or the fine of 80 degrees?

$a^2 = .030461740$
reverfed 47164030.

9138522
121847
18277
304
213
12

$a^4 = .0009279175$

$\frac{a^4}{24} = .0000386634$

1. 000000000
$\frac{a^2}{2} = -$ 015230870

.984769130
$\frac{a^4}{24} = +$ 000038663

Co-fine of 10° = .984807793; or the S. of 80°

Note 1. Thefe feriefes converge fwiftly in arches near the beginning and end of the quadrant, but very flowly in thofe near the middle; and therefore the fines of the firft and laft 10° being calculated, the fines of the intermediate arches may be found more eafily by the former rules.

2. The fame feries will ferve for finding the fine of any arch however expreffed; for example, fuppofe it were required to calculate the fine of one fixteenth part of the quadrant, or 5° 37′ 30″ = 337.5 minutes.

Length of 1′ = .0002908882
Multiply by 337.5′

14544410
20362174
8726646
8726646

$a = .09817476750$
$a^2 = .00963828497$
$a^3 = .00094623638$
$a^5 = .00000912008$

$a =$

$$a = .0981747675$$
$$\frac{a^3}{6} = -\ .0001592060$$
$$.0980155615$$
$$\frac{a^5}{120} = +\ .0000000760$$
$$\text{Sine of } 5^\circ\ 37'\tfrac{1}{2} = .0980156375$$

The table of sines being made, the tangents, secants, and versed sines may be calculated by the following theorems:

1. The co-sine of any arch is to its sine as the radius is to the tangent of that arch. *Plate* 2. *Fig*. 25.

Let DB be an arch of a circle, DE its sine, CE its co-sine, and CB the radius. The two triangles CED, CBG are equiangular, because the angles at E and B are right, and the angle at C common to both; therefore their sides are proportional, viz. as CE : ED :: CB : BG, the tangent of the arch BD.

Exam. Let BD be an arch of 30°, its sine DE = .5000000; its co-sine = .8660254; required the tangent of 30°?

Cos.	*Sine*	*R.*	*Tang.*
As .8660254 :	.5000000 ::	1 :	.57735027 = BG

.8660254) .50000000 (.57735027 = the tangent of 30°.

2. The sine of 15° = .258819045 and its co-sine = .9659258; required the tangent of 15°?

As .9659258 : .258819045 :: 1 : 26794917

.9659258) .258819045 (.26794917 = the tangent of 15°.

2. The sine of any arch is to its co-sine as the radius is to the co-tangent of that arch.

In the same figure; the triangles CDE, CIH are equiangular, for the angles at E and H are right, and the angle DCE is equal to the angle at I because CB is parallel to HI; therefore

 As

As DE : EC :: CH : HI = the co-tangent of the arch DB

Exam. Required the co-tangent of 15°; its sine being .2588190 and co-sine .9659258?

As .2588190 : .9659258 :: 1 : .37320513 = the co-tangent of 15°.

3. The radius is a mean proportional between the co-sine of any arch and its secant, or as Cos. : R :: R : Sec.

For the triangles CED : CBG are similar, as has been shewn, therefore as EC : CD :: BC or CD : CG the secant of the arch BD.

Exam. Let BD be an arch of 30°, its co-sine is .8660254; required the secant of 30°?

Cos. *R.* *R.* *Sec.*

As .8660254 : 1 :: 1 : 1.1547005 = CG.

.8660254) 1 0.00000000 (1.1547005 = the secant of 30°.

4. The versed sine of any arch less than 90° is the difference between its co-sine and the radius: thus the difference between the radius CB, and the co-sine CE is BE, the versed sine of the arch BD; and the versed sine of any arch greater than 90° is the sum of the radius and the co-sine of that arch.

Let AHD be an arch greater than 90°, its sine is DE and co-sine DF or CE. Now it is manifest that the versed sine of the arch AHD is AE, which is the sum of its co-sine EC and the radius AC. Hence, the co-sine of any arch being given, its versed sine may be found.

Exam. 1. The co-sine of 20° is .93969262: Required the versed sine?

From the radius	1.00000000
Subtract	.93969262
The versed sine of 20° =	.06030738

2. The

21 The co-sine of 140° is .76604443: What is the versed sine?

To the radius 1.00000000
Add the co-sine 0.76604443

The versed sine of 140° = 1.76604443

These sines, tangents, secants and versed sines, here treated of, are called natural sines, tangents, &c. and the logarithms of these numbers are called Logarithmic sines, tangents, &c. Tables of either kind are called the trigonometrical Canon; for by them all the Cases of Trigonometry may be solved, as will appear farther on.

Plain triangles are either right angled, or oblique angled; and each kind hath several cases.

RIGHT ANGLED TRIANGLES.

CASE I. *Fig.* 27.

In the right angled plain triangle ABC, suppose the two sides AB and BC were given, and it were required to find the angles at A and C.

If the given sides are equal between themselves, each of the acute angles is half a right angle, or 45°; but, if not, the angles may be found by this proportion:

As one of the given sides
Is to the other,
So is the radius
To the tangent of the angle adjacent to the first side.

For, produce AB and AC to D and E, so that AD may be equal to the radius. About the center A, with the *radii* AB and AD, describe the arcs B*e* and DF, whose tangents are BC and DE; each of these arcs is the measure of the angle at A, and the triangles ABC and

ADE

ADE are similar: Now, since AB, BC, and AD are known; DE, or the tangent of the angle at A may be found thus, as AB:BC::AD:DE. That is,

As AB:BC::Rad:Tan. A. Hence,

As any side of a right angled plain triangle
Is to the other side containing the right angle,
So is the radius
To the tangent of the angle adjacent to the first side.

In the same manner it appears, that CB:BA::Rad.: Tan. C; and when either angle is found, subtract it from 90°, and the remainder is the other angle.

CASE II. *Fig.* 28.

In the right angled plain triangle ABC, suppose the hypothenuse AC and the side AB were given, and it were required to find the angles at A and C.

As the hypothenuse AC
Is to the side AB,
So is the radius
To the sine of the angle at C.

Make CE equal to the radius; and about the center C describe the two arcs *An*, *Em*, whose sines are AB, ED; either of which is the sine of the angle C, according to the radius assumed: And because the triangles ABC, ECD are similar, and CA, AB, and CE, are known; ED, or the sine of the angle at C, may be found thus,

As CA:AB::CE:ED; that is,

As CA:AB::Rad:Sine C.

In the same manner it may be demonstrated, that AC:CB::Rad.:Sine A. Therefore,

As

As the hypothenuſe
Is to any ſide,
So is the radius
To the ſine of the angle oppoſite to that ſide.

Hence we have rules for finding any part of a right angled plain triangle, the hypothenuſe, or a ſide, and any other two of its ſix parts, being given.

CASE III. *Fig.* 27.

In the right angled triangle ABC, ſuppoſe the ſide AB, and the angles at A and C were given, to find the ſide BC, and the hypothenuſe AC.

1. To find the ſide BC.

Since by Caſe 1. AB : BC :: Rad. : Tan. A,
Therefore Rad. : Tan. A :: AB : BC.

2. To find the hypothenuſe AC.

Since by Caſe 2. AC : AB :: Rad. : Sine C.
Therefore Sine C. : Rad. :: AB : AC.

CASE IV. *Fig.* 28.

In the right angled triangle ABC, ſuppoſe the hypothenuſe AC, and the angles at A and C were given, and it were required to find the ſides AB and BC.

1. To find AB.

Since by Caſe 2. AC : AB :: Rad. : Sine C;
Therefore Rad. : Sine C :: AC : AB.

2. To find BC.

By the ſame Caſe, Rad. : Sine A :: AC : CB.

EXAM-

EXAMPLES *in right angled Triangles.*

CASE 1. *Fig.* 27. In the right angled triangle ABC, there is given the side AB=429, and BC=316, to find the angles at A and C.

CONSTRUCTION.

Make the right angle ABC; take the numbers expressing the lengths of AB and BC from a scale of equal parts, and set them from B to A and C; then join AC, and the triangle is described.

Measure the angle at A by a line of chords, or protractor, and subtract it from 90°, the remainder is the angle at C.

Making AB Radius, we have.

As AB	429	2.6324573
Is to BC	316	2.4996871
So is the radius (tang. 45°).		10.0000000
To tangent A 36° 22'.		9.8672298

Whose complement 53° 38' is the angle C.

EXAM. 2. *Fig.* 29. Suppose there are three cities, E, D, and C, so situate, that E is 542 miles directly west of D, and C 318 miles north of D; it is required to find the bearing of C from E? That is, in the right angled triangle CDE, there is given DE=542, and DC=318; to find the angles at E and C.

ANSW. E=30° 24' C=59° 36'.

3. When the sun shines, suppose a tower of 147 feet high, projects a shadow of 219 feet 6 inches on the horizontal plain, What is the altitude of the sun at that time? ANSW. 33° 48'.

CASE 2. *Fig.* 28. In the right angled triangle ABC, there is given the side AB=286, and the hypothenuse AC=463; to find the angles at A and C.

CON-

CONSTRUCTION.

Make the right angle ABC; take the number expressing the length of AB from a scale of equal parts, and set it from B to A; take also the number expressing the length of AC from the same scale, and setting one foot of the compasses in A, with the other cross the line BC in C; draw the straight line AC, and the triangle is described.

Measure the angle ACB by a line of chords, and subtract it from 90°, the remainder is the angle CAB.

Making AC radius, we have

As AC 463	2.6655810
Is to AB 286	2.4563660
So is the radius (sine 90°)	10.0000000
To sine C 38° 8′	9.7907850

Whose complement 51° 52′ is the angle at A.

N. B. When the proportion lies in tangents, the radius is the tangent of 45°; and when in sines, radius is the sine of 90°.

EXAM. 2. *Fig.* 30. Suppose a ship sails between south and west from E to C 500 miles, and thereby makes 300 miles of southing=DE; what was the angle of course CED?

Here, in the right angled triangle CDE, there is given CE=500, and ED=300, to find the angles at C and E.

ANSW. C=36° 52′, and the course is S. 53° 8′ W.

3. Suppose the hypothenuse of a right angled triangle is 273 miles, and one of its sides is 194 miles; required the two acute angles? ANSW. 45° 17′, and 44° 43′.

CASE 3. *Fig.* 27. In the right angled triangle ABC, there is given the side AB=476, and the angle at A=47° 9′; to find the side BC and the hypothenuse AC.

ANSW. BC=513.1 AC=699.9.

CONSTRUCTION.

Make the angle at A of the given quantity by a line of chords or protractor; take the given length of AB from a scale, and set it from A to B; from B raise the perpendicular BC and the triangle is described.

Measure BC and AC by taking them severally in the compasses, and applying them to the scale from which AB was taken.

1. Making AB radius.

As Rad. tan. tan=45°	10.0000000
So tan. A=47° 9′	10.0326241
So is AB= 476	2.6776070
To BC= 513.1	2.7102311

2. Making AC radius.

As Sine C=42° 51′	9.8325609
To radius=S. 90°	10.0000000
So is AB=476	2.6776070
To AC= 699.9	2.8450461

EXAM. 2. *Fig.* 30. Suppose a ship sails S. W. by S. until she hath made 319 miles of southing; how many miles hath she sailed, and how far is she west from the meridian of the place sailed from?

Here, in the right angled triangle CDE, there is given ED=319, and the angle at E=33° 45′; to find EC and CD.

ANSW. EC 383.6 m. and CD 213.1.

3. When the sun's altitude is 30° 20′, suppose the shadow of a tree on the horizontal plain is 69 feet 3 inches; what is the height of the tree; and, what is the distance from the top of the tree to the extremity of its shadow?

ANSW. The height of the tree is 40.5 feet, the other 80.23 feet.

CASE

CASE 4. *Fig.* 28. In the right angled triangle ABC, there is given the hypothenuse AC=357; and the angle at C=42° 13′; to find the sides AB and BC.

ANSW. AB=239.8, BC=264.3.

CONSTRUCTION.

Make the angle CAB of the given quantity; take the number expressing the length of AC from a scale of equal parts, and set it from A to C; from C let fall the perpendicular CB, and the triangle is described.

Measure the sides AB and BC on the scale from which AC was taken,

Making AC radius.

1.	As radius = S. 90°	10.0000000
	To Sine C = 42° 13′	9.8273279
	So is CA = 357	2.5526682
	To AB = 239.8	2.3799961
2.	As radius = S. 90°	10.0000000
	To Sine A = 47° 47′	9.8695891
	So is AC = 357	2.5526682
	To CB = 264.3	2.4222573

EXAM. 2. *Fig.* 29. Suppose the slope side of a hill CE is 3625 links of a surveying chain, and the angle of ascent at E is 19° 30′; what is the length of the horizontal level ED, and the perpendicular height of the hill CD? ANSW. ED=3417 CD=1210.

When the two sides of a right angled plain triangle are given to find the hypothenuse; or, when the hypothenuse and one of the sides are given to find the other side; the problem may be solved by the *47th prop. of Euclid's*, 1 *B.* as follows:

1. When the two sides are given to find the hypothenuse.

Square each of the sides, add them; and extract the square root of the sum; the root is the length of the hypothenuse.

EXAM. *Fig.* 27. Let AB=429 and BC=316; required the length of the hypothenuse AC?

ANSW. 532.82.

To the square of AB = 184041
Add the square of BC = 99856

The sum is the square of AC = 283897
And the square root of 283897 is 532.82 = AC.

2. The hypothenuse and one side being given to find the other side.

Subtract the square of the given side from the square of the hypothenuse, and extract the square root of the remainder; the root is the length of the side required.

EXAM. *Fig.* 28. Let the hypothenuse AC=463, and the side AB=286; required the side BC?

ANSW. 364.1

From the square of AC = 214369
Subtract the square of AB = 81796

There remains the square of BC = 132573
And the square root of 132573 is 364.1 = BC.

Or, because the difference of the squares of any two straight lines is equal to the rectangle contained under their sum and difference; the last problem may be solved thus: multiply the sum of the hypothenuse and given side by their difference, the square root of the product is the length of the side required.

In

In the laſt Example, AC+AB=749, and AC−AB=177, then 749×177=132573=the ſquare of BC, as before, and BC=364.1.

OBLIQUE ANGLED TRIANGLES.

When one ſide of an oblique angled triangle, and all the angles are given, to find the other two ſides; or, when two ſides with an angle oppoſite to one of them are given, to find the other two angles and the third ſide; the ſolution depends on the following theorem.

Any ſide of a plain triangle is to any other ſide, as the ſine of the angle oppoſite to the firſt ſide, is to the ſine of the angle oppoſite to the other. *Fig.* 31. Let ABC be an oblique angled plain triangle, let fall the perpendicular AD; then, by Caſe 2. of right angled triangles,

As BA : AD : : Rad. : Sine B,
And as CA : AD : : Rad. : Sine C;

Therefore BA × Sine B = CA × Sine C.
Conſequently, as BA : AC : : Sine C . Sine B.

In the ſame manner, it might be demonſtrated, that any other two ſides are to each other as the ſines of their oppoſite angles; and, on the contrary, the ſine of an angle is to the ſine of any other angle, as the ſide oppoſite to the firſt angle is to the ſide oppoſite to the other.

CASE

CASE I. *Fig.* 31. In the oblique angled triangle ABC, there is given the side AC=256, the angle at B=40° 34', and the angle at C=56°; to find the sides AB and BC.

CONSTRUCTION.

Make the angle at C of the given quantity by a line of chords; take the number expressing the length of AC from a scale of equal parts, and set it from C to A; then at the point A, make the angle CAB of the given quantity, and the triangle is described. Measure AB and BC on the scale from which AC was taken.

Subtract the sum of the angles B and C from 180°, the remainder is the angle at A; and then,

As Sine B : Sine C :: CA : AB=326.34,

As Sine B : Sine A :: AC : CB=391.06.

N. B. When two of the angles are equal, the sides opposite to them being also equal, the case will be solved by one proportion.

Exam. In the triangle ABC, suppose the side AC=256 and each of the angles at B and C=67° 39'; required the side BC?

Angle B =	67° 39'	From	180°
—— C =	67 39	Subtr.	135 18
Sum	135 18	Angle A =	44 42

As Sine B = 67° 39'	9.9656153
To Sine A = 44 42	9.8471991
So is AC = 256	2.4082400
To BC = 194.9	12.2554391
	2.2898238

EXAM. 2. *Fig.* 32. Suppose D and E are two stations from whence the object C can be seen, and the angles at D and

D and E measured with an instrument; and the distance of the stations DE measured with a chain, and it were required to find the distance of the object C from each station.

Let DE=479 feet, the angle at D=36° 50′, and the angle at E=47° 9′; required the distances DC and CE?

ANSW. DC=353.1 CE=288.7.

3. The distance of two mountains lying east and west of each other is 29 miles: There is a third mountain to the northward of these, so situated, that the angle contained by a right line drawn to it from the eastern mountain, and the line joining the two first mountains is 52° 12′; and the angle contained by a right line drawn to it from the western mountain, and the line joining the two first mountains is 68° 28′: Required the distance of the third mountain from each of the other two?

ANSW. 31.36m. and 26.64 miles.

CASE 2. *Fig.* 31. In the oblique angled triangle ABC, there is given the side AB=324, and BC=468, with the angle at A=80° 30′; to find the angles at B and C; and the side AC.

CONSTRUCTION.

Make the angle BAC of the given quantity; take AB from the scale, and set it from A to B; then take BC from the same scale, and setting one foot of the compasses in the point B, with the other cross the line AC in C and join BC, and the triangle is described.

Measure the angles at B and C by the line of chords; and measure AC on the same scale from which AB and BC were taken.

To find the angle at C.

As BC : BA : : Sine A : Sine C=40° 3′; which add to the

the angle at A, and subtract the sum from the 180°, the remainder is the angle at B.

To find the side AC

As Sine C : Sine B : : BA : AC=395.45.

N. B. In this case, if the angle at A be right or obtuse, the angle at C being acute, will be found out by the proportion; but if the angle at A be acute, the angle at C may be either acute or obtuse, and therefore the species of this angle cannot be known by the fourth proportional; but must be determined before the solution is begun.

Exam. 2. *Fig.* 32. In a triangular field CDE, suppose the angle at C was measured with an instrument, and found to be 70° 50'; and the sides CE and ED being measured with a chain, CE was 490 links, and ED 532 links; required the angles at D and E, with the length of the side DC?

Answ. D=60° 27', DC=423.2.

3. Let two sides of an oblique angled plain triangle be 500 and 300, and let the angle opposite to the greatest side be 70° 30'; required the other two angles and the third side? Answ. The angles are 34° 26' and 75° 4' and the side 512.6.

Theorem, *Fig.* 33. The half difference of any two quantities added to their half sum, produceth the greater quantity; and their half difference being taken from their half sum leaves the lesser quantity.

Let AB and BC represent two quantities, whose half sum is AF or FC, and half difference FB. It is manifest, that FB, the half difference, added to AF the half sum, is equal to AB, the greater quantity; and FB, the half difference, being taken from FC, the half sum, leaves BC the lesser quantity.

Case

CASE 3. *Fig.* 34. Two ſides of an oblique angled triangle AB and BC, with the angle ABC contained between them, being given, to find the other angles at A and C, and the third ſide AC.

If the two given ſides are equal between themſelves, ſubtract the given angle from 180°, and half of the remainder is either of the other angles; and find the third ſide by Caſe 1.

If the given ſides are unequal, the angles may be found by this Theorem.

In any plain triangle ABC,

As the ſum of any two ſides BC+BA.
Is to their difference BC—BA
So is the tangent of half the ſum of
the angles oppoſite to thoſe ſides $\frac{A+C}{2}$
To the tangent of half their difference $\frac{A-C}{2}$

Take BE and BF, each equal to AB; join FA and AE; and through C draw CD parallel to AE: Then F, A, E, are points in the circumference of a circle whoſe center is B, and FAE, FDC are right angles; FC is the ſum of the ſides AB and BC, and EC their difference.

The angle BAE is equal to the angle AEB, and their ſum is equal to the ſum of the two angles BAC and ACB; becauſe the angle ABF is equal to either of theſe ſums; therefore, the angle AEB, or its equal FCD, is the half ſum of the angles BAC, ACB; and the angle EAC, or its equal ACD, is their half difference; be-

caufe, this added to BAE, their half fum, is equal to BAC, the greater of the two.

Making CD the radius, FD is the tangent of the angle FCD, the half fum, and DA is the tangent of ACD, the half difference of the angles BAC and ACB; the triangles AFE, DFC are fimilar; therefore,

As FC : CE : : FD : DA; that is,

As FC, the fum of AB and BC,

Is to EC their difference,

So is FD,* the tangent of half the fum of the angles BAC and ACB,

To DA, the tangent of half their difference.

The half difference added to the half fum, gives the greater angle, and fubtracted gives the leffer.

EXAM. 1. In the oblique angled triangle ABC, there is given the fide AB=317, the fide BC=429, with the contained angle at B=48° 40′; required the angles at A and C, and the fide AC?

CONSTRUCTION.

Draw the ftraight line BC, and at the point B make the angle ABC of the given quantity; take the numbers expreffing the lengths of BA and BC from a fcale of equal parts, and fet them from B to A and C, then join AC and the triangle is defcribed.

Meafure the angles at A and C by the line of chords, and meafure AC on the fcale from which AB and BC were taken.

429		180° 00
317		48 40
746=BC+BA	2)	131° 20′=Sum of the ang. A and C.
112=BC—BA		65° 40′=their half fum.

As

As $BC+BA = 746$
Is to $BC-BA = 112$
So is tan. $\frac{A+C}{2} = 65° 40'$
To tan. $\frac{A-C}{2} = 18° 22'$

To the half sum $65° 40'$
add the half diff. $18\ \ 22$
Greater ang. $A=84\ \ 02$

From the half sum $65° 40'$
Subt. the half diff. $18\ \ 22$
The lesser ang. $C=47° 18'$

Having found the angles, the side AC is found by Case 1. thus, as Sine C : Sine B :: BA : AC=323.8.

Exam. 2. *Fig.* 32. Suppose CDE a triangular field; the side DE being measured, is 564 links of a surveying chain, and EC=368 links; also the angle at E being measured with an instrument, is 54° 20′; what are the angles at C and D, and what is the length of the side CD?

Answ. C=85° 6′, D=40° 34′, CD=459.8.

3. Suppose the distance of the earth from the sun is 81000000 miles, and the distance of the moon from the earth is 240000 miles; and that the angle contained by two right lines drawn from the center of the earth to the centers of the sun and moon is 89° 6′; what is the distance of the moon from the sun?

Answ. 80938000 miles.

Case 4. When the three sides AB, BC, CA, of any oblique angled plain triangle ABC are given; to find the angles at A, B, and C.

If any two of the given sides are equal, the triangle being isosceles, its angles may be found thus. *Fig.* 35.

Suppose AB is equal to AC, let fall the perpendicular AE, and the point E will fall in the middle of the side

 BC;

BC; therefore, by Case 2. of right angled triangles, as AB:BE::Rad.:Sine BAE, whose complement is the angle at B or C; and its double is the angle BAC.

When the sides are unequal, *Fig.* 36, let fall the perpendicular AE, upon the base, or longest side BC; and, about the center A, with a radius equal to the shortest side AC, describe the circle FDG. Produce BA to G; then BG is the sum of the sides BA and AC, and BF their difference; also BE and EC are the segments of the base, and BD their difference. The rectangle under CB and BD is equal to the rectangle under GB and BF, by the property of the circle; *Euclid, B.* 3, *corrol. to prop.* 36. Therefore, as CB:BG::BF:BD; that is,

As the base BC
Is to (BG) the sum of the other two sides BA+AC,
So is (BF) the difference of these sides BA—AC
To (BD) the difference of the segments of the base BE—EC.

Now, take the half of this difference, and also the half of BC, their sum is BE the greater segment, and their difference is EC the lesser segment. Then, in the right angled triangle ABE, there is given AB and BE; to find the angles, which is done by Case 2. of right angled triangles, viz.

As AB:BE::Rad.:Sine BAE, whose complement is the angle at B. For the same reason, in the right angled triangle AEC, as AC:CE::Rad.:Sine EAC, whose complement is the angle at C. And BAE+EAC=the angle BAC.

EXAM. In the oblique angled triangle ABC, there are given the three sides, viz. AB=264, BC=384, and AC=196; to find all the angles at A, B, and C.

CON-

CONSTRUCTION.

Draw the straight line BC; take the number expressing its length from a scale, and set it from B to C; then take the length of AB from the same scale, and setting one foot of the compasses in the point B, with the other describe an arch, and take the length of AC from the scale, then setting one foot of the compasses in the point C, with the other cross the former arch in A; draw the straight lines AB and AC, and the triangle is described.

Measure the angles at A, B, and C, by a line of chords, or with a protractor. Let fall the perpendicular AE.

264	264
196	196
460=BA+AC	68=BA—AC.

(1.) As BC	384	2)384	2)81.4
Is to BA+AC	460	192	40.7
So is BA—AC	68	40.7 add and subtract	
To BE—EC	81.4	BE=232.7	
		EC=151.3	

(2.) As AB : BE :: Rad. : Sine BAE=61° 49′ whose complement 28° 11′ is the angle at B.

(3.) As AC : CE :: Rad. : Sine EAC=50° 31′, whose complement 39° 29′=the angle at C, and BAC=112° 20′.

EXAM. 2. *Fig.* 37. Let D and E represent two ports in the same latitude, whose distance is 480 miles; and suppose a ship from D sails 300 miles between north and east, and another sails from E 240 miles between north and

and west, and then meets with the first ship at C; required the course of each ship?

Answ. The course of the first ship is N. 65° 49′ E. and of the other N. 59° 13′ W.

A Table of Proportions, for solving the several Cases of right angled Triangles. *Fig.* 27 and 28.

Cases.	*Given.*	*Sought.*	*Proportions.*	*Rad.*
1	AB, BC	A & C	AB : BC : : Rad. Tan. A whose complement is C	AB
2	AC, AB	A & C	CA : AB : : Rad. : Sine C whose complement is A.	AC
3	AB & A	BC AC	Rad. : Tan. A : : AB : BC Sine C : Rad. : : AB : AC	AB AC
4	AC & C	AB BC	Rad. : Sine C : : CA : AB Rad. : Sine A : : CA : CB	AC

The solution of the 1st Case admits of no variation. In the 2d Case, if AB be made radius, the proportion may be, as AB : AC : : Rad. : Secant A, whose comp. is C. In the 3d Case, if AC be made radius, the solution may be as Sine C : Sine A : : AB : BC; and, if AB be made radius, as Rad. : Secant A : : AB : AC. The solution of the 4th Case may be, as Sec. A : R : : AC : AB; and, as Sec. C : R : : AC : CB, each side being radius.

These proportions are true, but unnecessary; for, since every case may be solved by a right application of sines or tangents, there is no reason for introducing secants.

A Table

A TABLE of PROPORTIONS for solving the several Cases of Oblique angled Triangles. *Fig.* 31. 34. and 36.

Cases	*Given.*	*Sought.*	*Proportions.*
1	AC and B, C.	AB BC	Sine B : Sine C : : CA : AB. Sine B : Sine A : : AC : CB.
2	AB, BC, and A.	C, B, and AC.	CB : BA : : Sine A : Sine C. Sine C : Sine B : BA : AC.
3	AB, BC, and B.	A, C, and AC.	BC+BA : BC—BA : : Tan. $\frac{A+C}{2}$: Tan. $\frac{A-C}{2}$ Add and subtract the half sum and half diff. and you have the angles A and C, and then, Sine C : Sine B : : BA : AC.
4	AB, BC, and CA.	B C and A.	BC : BA + AC : : BA—AC : BE—EC. Hence, BE and EC will be known. AB : BE : : Rad. : Sine BAE, whose complement is B. AC : CE : : Rad. : Sine CAE, whose complement is C. And the sum of BAE and CAE is the angle BAC.

The

The 4th case may be solved by the following Theorem:

In any plain triangle, the rectangle contained by half the sum of the sides, and the excess of that half sum above the side opposite to the angle sought, is to the rectangle contained by the differences between the half sum of the sides, and the two sides containing the angle sought; as the square of the radius is to the square of the tangent of half the angle sought.

EXAMPLE 1. of Case 4, resumed. *Fig.* 36.

1. To find the angle BAC.

AB=264	422	422
BC=384	264	196
CA=196	158	226
2)844 sum		
422 half		
Subt. 384		
38		

As 422 × 38	2.6253125 1.5797836
	4.2050961
Is to 158×226	2.1986571 2.3541084
So is squ^r Rad.	20.0000000
	24.5527655 4.2050961
To squ^r tan $\frac{A}{2}$	2)20.3476694
56° 10′ 20″ 56 10 20	10.1738347
112° 20′ 40″=BAC	

2. To

2. To find the angle at C.

As 422×158	2.6253125
	2.1986571
	4.8239696
Is to 38×226	1.5797836
	2.3541084
So is squ^r rad.	20.0000000
	23.9338920
	4.8239696
To squ^r tan. $\frac{C}{2}$	2)19.1099224
$19° \; 44' \; 34''$	9.5549612
$19 \; 44 \; 34$	
ACB $= 39 \; 29 \; 8$	

3. To find the angle at B.

As 422×226	2.6253125
	2.3541084
	4.9794209
Is to 38×158	1.5797836
	2.1986571
So is squ^r rad.	20.0000000
	23.7784407
	4.9794209
To squ^r tan. $\frac{B}{2}$	2)18.7990198
$14° \; 5' \; 6''$	9.3995099
$14 \; 5 \; 6$	
ABC $= 28 \; 10 \; 12$	

Proof.

$$\begin{array}{rrrr} A = & 112^\circ & 20' & 40'' \\ B = & 28 & 10 & 12 \\ C = & 39 & 29 & 8 \\ \hline & 180^\circ & 0 & 0 \end{array}$$

We have made, as uſual, eight Caſes in Plain Trigonometry, *viz.* four in right angled triangles, and four in oblique; but theſe may be reduced to three: for,

1. Since the ſides of any plain triangle are in proportion to each other as the ſines of the angles oppoſite to theſe ſides; by this we can ſolve the 2d, 3d, and 4th Caſes of right angled triangles, as alſo the 1ſt and 2d Caſes of oblique angled triangles.

2. Since the ſum of any two ſides of a plain triangle is to their difference, as the tangent of half the ſum of the oppoſite angles is to the tangent of half their difference; hence we can ſolve the 1ſt Caſe of right angled triangles, and the third Caſe of oblique angled triangles.

3. When the three ſides of a plain triangle are given to find the angles; if the triangle be right angled, the acute angles may be found by Theorem 1ſt above; and if it be oblique angled, the angles may be found by the Theorems, p. 37.

Some authors comprehend the whole of Plain Trigonometry, in three Caſes as follows:

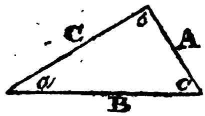
CASE I.

The three ſides of any plain triangle being given, to find the angles.

Let A, B, C, repreſent the three ſides, and a, b, c, the oppoſite angles. Put s = the ſine, s′ = the co-ſine, t = the tangent, and t′ = the co-tangent of any angle.

Alſo

Alſo put $P = \frac{1}{2}\,\overline{A+B+C}$, L = the logarithm of any number and L′ = the arithmetical complement of a logarithm. Then



the $s\,\frac{1}{2}\,a = \sqrt{\frac{\overline{P-C}\times\overline{P-B}}{B\times C}}$ or by Logarithms

the $s\,\frac{1}{2}\,a = \frac{1}{2}\,\overline{L\,P-C+L\,P-B+L'B+L'C}$

the $s\,\frac{1}{2}\,b = \sqrt{\frac{\overline{P-A}\times\overline{P-C}}{A\times C}}$ and by Logarithms

$s\,\frac{1}{2}\,b = \frac{1}{2}\,\overline{L\,P-A+L\,P-C+L'A+L'C}$

The $s\,\frac{1}{2}\,c = \sqrt{\frac{\overline{P-A}\times\overline{P-B}}{A\times B}}$ and by Logarithms

$s\,\frac{1}{2}\,c = \frac{1}{2}\,\overline{L\,P-A+L\,P-B+L'A+L'B}$

Exam. Let the ſide A = 49, B = 96 and C = 66: it is required to find the angles at *a*, *b*, and *c*?

1. For the angle *a*.

$$\begin{array}{rl}
49 & \\
96 & \\
66 & \\
\hline
2)211 & \\
\hline
P = 105.5 &
\end{array}
\qquad
\begin{array}{rr}
P = & 105.5 \\
C = & -\ 66. \\
\hline
P-C = & 39.5
\end{array}
\qquad
\begin{array}{rr}
 & 105.5 \\
B = & -\ 96. \\
\hline
P-B = & 9.5
\end{array}$$

L P—C	= 39.5	1.5965971
L P—B	= 9.5	0.9777236
L′ B	= 96	8.0177288
L′ C	= 66	8.1804561
		2)18.7725056
$s\,\frac{1}{2}\,a$	= 14° 5′ 6″	9.3862528

or the angle *a* = 28° 10′ 12″

2. For

2. For the angle *b*.

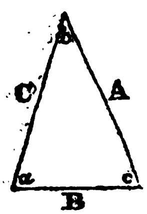
P = 105.5	L.P—A = 56.5	1.7520484
A = —49.	L.P—C = 39.5	1.5965971
	L′ C = .66.	8.1804561
P—A = 56.5	L′ A = 49	8.3098039
		2)19.8389055
	$s\frac{1}{2}b =$ 56° 10′ 20″	9.9194527

or the angle $b =$ 112° 20′ 40″

3. For the angle *c*.

L P—B = 9.5	0.9777236
L P—A = 56.5	1.7520484
L′ A = 49.	8.3098039
L′ B = 96.	8.0177288
	2)19.0573047
$s\frac{1}{2}c =$ 19° 44′ 33″	9.5286523

or the angle $c =$ 39° 29′ 6″



In an isosceles triangle, when the side A is equal to the side C, the angle *b* contained between the equal sides may be found by this Theorem:

$$s\frac{1}{2}b = \frac{B}{A}\times\frac{r}{2} = \frac{B\,r}{2\,A} = \frac{B}{2\,A}$$

because the radius r=1; and by Logarithms $s\frac{1}{2}b =$ L B + L′ 2 A.

Exam. Let A = 80 C = 80 and B = 60; required the angles *b*, *a*, and *c*?

L.B = 60	1.7781513
L′ 2A = 160	7.7958800
$s\frac{1}{2}b =$ 22° 1′ 27″	9.5740313

the angle $b =$ 44° 2′ 54″

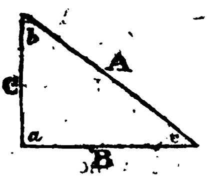
from 180°
Subtr. $b =$ 44 2′ 54″
2)135 57 6
67° 58′ 33″ = *a* or *c*

When

When the three ſides of a right angled plain triangle are given, to find the angles, the Theorem becomes more ſimple. For example, ſuppoſe the angle a is a right angle, then

$$s\,c = \frac{C}{A}, \text{ and } s\,b = \frac{B}{A}; \text{ or by logarithms.}$$

$s.\ c = L\,C + L'A$ and $s.\ b = L\,B + L'A$



Exam. Let the hypothenuſe $A = 50$. the ſide $B = 40$ and the ſide $C = 30$; required the angles at c and b?

For the angle c.

	$L\,C = 30$	1.4771213
	$L'\,A = 50$	8.3010300
angle c =	$36^\circ\ 52'\ 11''$	9.7781513

For the angle b.

	$L\,B = 40$	1.6020600
	$L'\,A = 50$	8.3010300
angle b =	$53^\circ\ 7'\ 48''$	9.9030900

CASE 2. When one ſide and all the angle of any plain triangle are given; or when two ſides and an angle oppoſite to one of them are given, to find the other parts of the triangle.

Becauſe in any plain triangle the ſides are to each other as the ſines of their oppoſite angles; the ratio between any ſide, and the ſine of its oppoſite angle is equal to the ratio between any other ſide, and the ſine of its oppoſite angle, and therefore $\frac{A}{s.a} = \frac{B}{s.b} = \frac{C}{s.c}$. From theſe equations any of their terms may be found. For example, when the ſide A and the angles are given, the ſide $B = \frac{sb}{sa} \times A$; and the ſide $C = \frac{sc}{sa} \times A$. or by Logarithms.

Logarithms. The ſide B = L A + L s b + L′a, and the ſide C = L A + L s c + L′s a.

Exam. Let the ſide A = 32, the angle $a = 17° 44'$, the angle $b = 96° 12'$ and the angle $c = 66° 4'$: requir-ed the ſides B and C?

For the ſide B

L A	= 32	1.5051500
L s b	= 96° 12	9.99745.23
L′ s a	= 17 44	0.5162883
ſide B	= 104.4	2.0188906

For the ſide C

L A	= 32	1.5051500
L s c	= 66° 4′	9.9609548
L′ s a	= 17 44	0.5162883
ſide C	= 96.	1.9823931

And if the ſides B and C and the angle b were given, to find the angles at c and a with the ſide A?

$s\,c = \frac{C}{B} \times s.b$, or by logarithms sc = LC + L s.b + L′B

The ſide $A = \frac{sc}{sa} \times C$. or by logarithms, A = LC + L sa + L′sc.

Exam. Let the ſide B = 43, the ſide C = 32, and the angle $b = 84° 2'$; required the angles c and a, with the ſide A?

For the angle c.

L C	= 32.	1.5051500
L s. b	= 84° 2′	9.9976408
L′ B	= 43	8.3665315
		9.8693223

$s.\,c = 47° 44' 40''$

hence the angle $a = 48° 13' 20''$

For

For the side A.

L C = 32	1.5051500
L $s.a$ = 48° 13′ 20″	9.8725842
L′ sc = 47 44 40	0.1306785
	1.5084127

Side A = 32.24.

CASE 3. When two sides of a plain triangle, and the angle contained between them are given, to find the other two angles and the third side.

Suppose c=the given angle; A and B the two given sides. Put $S=90°-\frac{1}{2}c$, d = the $\frac{1}{2}$ difference of the angles a and b, then tangent $d=\frac{A-B}{A+B}\times$ tan. S. having found d, $S+d=a$, and $S-d=b$.

The side $C=\sqrt{\frac{4AB\times s.\frac{1}{2}c}{rr}+\overline{A-B}^2}$ See the Fig. on p. 59.

Exam. Suppose the side A = 141, B = 92, and the contained angle c = 54° 20′ and it were required to find the angles at a and b, and the side c.

For the angles a and b.

90°	141
$\frac{1}{2}c$ = 27 10	92
S = 62 50	A—B=49
	A+B=233

By logarithms, tan. d = L A—B + L tan. S + L′ A+B.

L A—B = 49	1.6901961
L tan. S = 62° 50′	10.2897176
L′ tan. A+B=233	7.6326441
tan. d = 22° 17′	9.6125578

S. =

$$\begin{array}{rrr} S. = & 62^\circ & 50' \\ d = & 22 & 17 \\ \hline \text{angle } a = & 85 & 7 \\ \hline \text{angle } b = & 40 & 33 \end{array}$$

For the side C. by logs. L 2 A + L 2 B + 2 L s $\frac{1}{2}c$ = Q; and 2 L A − B = L R; lastly, L C = $\frac{1}{2}$ L Q + R.

L 2 A =	282	2.4502491
L 2 B =	184	2.2648178
L s $\frac{1}{2}$ c =	27° 10	9.6595173
Ditto	27 10	9.6595173
Q =	10816.86	4.0341015
L A − B =	49	1.6901961
		2
R =	2401	3.3803922
Q =	10816.68	

$\sqrt{13217.68} = 114.968$

The side C = 114.968

By this Theorem the third side C is found directly from the data, without finding the angles a and b.

In this Case; when the given angle c is a right angle, the other two angles and the hypothenuse may be found thus; the $t.\ a = \frac{A}{B} \times r$. the $t.\ b = \frac{A}{B} \times r$, wherein the radius r being $= 1$ may be left out. The hypothenuse $c = \sqrt{A^2 + B^2}$

And by logarithms, L a = L A + L B. $t.\ b$ = L B + L A and C = $\frac{1}{2}$ L $\overline{A^2 + B^2}$

Exam. In a right angled triangle, let the side A = 213, the side B = 199, containing the right angle c: required the angles a and b, and the hypothenuse C?

For

For the angles a and b.

L A = 213	2.3283796
L′ B = 199	7.7011469
angle $a = 46^\circ\ 57'$	10.0295265
$b = 43^\circ\ 3'$	

For the hypothenuse C.

$A^2 = 45369$	
$B^2 = 39601$	
84970	4.9292656
	$\frac{1}{2}$ 2.4646328
C = 291.49	

TRIGONOMETRICAL PROBLEMS.

PROB. I. One side of a right angled triangle, and the sum of the other side and hypothenuse being given; to find the angles, the hypothenuse, and the other side severally. *Fig.* 38.

EXAM. In the right angled triangle ABC, suppose the side AB=70, and the sum of BC and AC=200; it is required to find the angles ACB, CAB, with the hypothenuse AC, and the side BC?

ANSW. ACB=38° 36′, CAB=51° 24′, AC=112.32, BC=87.68.

CONSTRUCTION. Make the right angle ABD; make AB=70, and BD=200; then find the point C equally distant from A and D, and join AD and AC.

SOLUTION. In the right angled triangle ABD, we have AB and BD given; to find the angle BAD; from which subtract its complement, viz. the angle D=DAC, and there will remain the angle CAB, whose comple-

ment is the angle ACB; and then, in the right angled triangle ABC, we have all the angles with the side AB; to find AC or CB; either of which being found, subtract it from 200, and the remainder is the other side.

PROB. 2. One side of an oblique angled triangle, the angle opposite to that side, and the sum of the other two sides, being given; to find the other two angles and sides severally. *Fig.* 39.

EXAM. In the oblique angled triangle ABC, suppose the side BC=532, and the angle BAC=110° 30′; also the sum of the sides BA and AC=637; it is required to find the angles ACB and ABC, with the sides BA and AC?

ANSW. ACB=45° 5′, ABC=24° 25′, AC=234.7, AB=402.3.

CONSTRUCTION. Make the angle at D of 55° 15′ equal to half BAC; take 637 from the scale, and set it from D to B; then take 532 from the same scale, and setting one foot of the compasses in B, with the other cross the line DC in C; and join BC; find the point A equally distant from D and C; and join AC. Now, since AD is equal to AC, the angle ACD is equal to the angle ADC, and each of them half of the angle BAC.

SOLUTION. In the triangle BCD, we have DB, BC, and the angle at D; to find the angle BCD.

Subtract the angle ACD from BCD, the remainder is the angle ACB; and, by taking the sum of BAC and ACB from 180°, we have the angle ABC. Then, in the triangle ABC, we have all the angles, and the side BC; to find either AB or AC.

PROB. 3. One side of an oblique angled triangle, the angle opposite to that side, and the difference between the

the other two sides, being given; to find the other angles and sides severally. *Fig.* 40.

Exam. In the oblique angled triangle ABC, let the side BC=250, and the angle BAC=96° 50′, also the difference between the sides AB and AC, viz. BD=106; required the angles ACB and ABC, with the sides AB and AC?

Answ. ACB=57° 55′, ABC=25° 15′, AB=213.4, AC=107.4.

Construction. Subtract the angle BAC from 180° and divide the remainder by 2, the quotient is the angle ADC; which being subtracted from 180°, the remainder is the angle BDC.

Make the angle BDC. Take 106 from the scale, and set it from D to B; then take 250 from the scale; and setting one foot in B, with the other cross the line DC in C, and join BC; produce BD, in which find the point A equally distant from D and C, and join AC.

Solution. In the triangle BDC we have BD, BC, and the angle BDC; to find the angle BCD.

Having found the angle BCD, add it to ACD; the sum is the angle ACB.

Subtract the sum of the angles BAC and ACB from 180°, the remainder is the angle ABC.

Then, in the triangle ABC, we have all the angles, and the side BC; to find AC or AB; either of which being found, the other will be known by applying the difference BD to the side found.

Description of Gunter's Scale.

The Logarithms were invented by Lord Napier, and published at Edinburgh in the year 1614; and, about

ten years after, Mr. Edmund Gunter, Professor of Astronomy in Gresham College, London, applied the logarithms to lines on a two foot scale, whereby all questions in trigonometry, and such parts of practical mathematics as depend on that science, may be readily solved without calculation, by applying a pair of compasses to the divisions on the scale.

Gunter's Scale contains,

1. A line marked S. R. (sine rhumbs) on which are the logarithm sines of the degrees to each point and quarter point of the mariner's compass.

2. A line marked T. R. (tangent rhumbs) being the logarithm tangents of the said points and quarters.

3. A line marked Numb. (numbers) on which the logarithms of numbers are laid down.

4. A line marked Sin. containing the logarithm sines.

5. A line of logarithm versed sines marked V. S.

6. A line of logarithm tangents marked Tan.

7. A meridional line marked Mer.

8. A line of equal parts marked E. P.

The two first and two last lines are used only in navigation; the rest in trigonometry.

The figures on the several lines have always the same values, except on the line of numbers; there, the figures represent either the series of numbers from 1 to 100, or from 1 to 1000, and consequently have different values, according to the series they represent.

Because the logarithms of proportional numbers have equal differences, these logarithms on a scale will be at equal distances; therefore, if four numbers are proportional, the distance between the first and second terms will be equal to the distance between the third and fourth

fourth on the scale. Hence, the rule for working any proportion on Gunter's Scale is, Set one foot of the compasses to the first term of the proportion, and extend the other to the second; then, keeping that extent in the compasses, set one foot to the third term, and the other will fall upon the fourth term or answer.

SECT. III.

The Application of PLAIN TRIGONOMETRY *to the* MENSURATION *of* HEIGHTS *and* DISTANCES.

TO perform problems of this kind, the first thing to be done is, to measure the length of any straight line on the ground; and, for this purpose, some measure of a determinate length must be chosen, such as a yard, fathom, pole, the surveying chain, or some part of it.

Measures should be adapted to the quantities to be measured. Small distances may be measured by a foot rule, or yard, or by a tape divided into inches, and rolled up in a box; but large distances require larger measures, such as long poles of some known length, or the surveying chain.

The Scots surveying chain is a measure of $74\frac{2}{5}$ English feet, (commonly made only 74 feet), consisting of 100 links, each link being 8.928 inches. The English surveying chain is 66 feet in length, consisting of 100 links, each link being 7.92 inches.

To measure an accessible distance on the ground, as from A *to* B. *Fig.* 56.

Apply one end of your measure to A, directing the other towards B, and at the extremity E put a mark, or

or pin; then carry the measure into the position EG, placing a mark at G; and again carry the measure into the position GK, and so on until you come to B. The number of pins will show the number of measures from A to B; and hence the length of the line AB will be known.

When a large distance, such as the side of a field, is to be measured with the surveying chain, ten pins, or arrows of wood, or iron, must be made use of to number the chains; thus, he who leads the chain takes the ten arrows at setting out, and, at the end of the first chain sticks one in the ground; and, when they have advanced forward another chain, he sticks another in the ground, and the man at the other end of the chain takes up the first arrow, and so on; the man who leads the chain putting down one at the end of every chain, and the man who follows the chain taking them up as he advances, until he have all the ten; and then they must be carried to the man who leads the chain: And thus the number of chains in any distance will be known.

Besides the surveying chain, a pole of wood ten links long, the lengths of the links being marked on it, and one of them divided into ten equal parts, is necessary to measure a few links and parts of a link accurately; and thus the distance will be known in links, and 10ths of a link, or 1000 parts of the whole chain.

Although the whole chain is commonly made use of, it is difficult to measure a line accurately with it, on account of its great length. A half chain, or one of 50 links, can be held more straight, and managed more easily, and consequently less liable to errors than the whole chain.

The

The common method of meaſuring an acceſſible diſtance is abundantly ſimple, and ſeems eaſy enough to execute; and yet, even in this ſimple operation, it is very hard to avoid errors entirely.

The ſurveying chain is very well contrived for the purpoſe of meaſuring land; and, when applied with care, the area of any field may be found; but, in meaſuring diſtances from whence calculations are to be made, in order to find out larger diſtances, greater exactneſs is neceſſary than can be expected from the ſurveying chain: For it requires a great deal of care to keep the chain always truly ſtraight; and even ſuppoſing this done, we cannot be ſure that the chain does not lengthen by conſtant ſtretching; and, if this be true, ſmall errors may ariſe from different degrees of tenſion; nor is it poſſible to avoid theſe errors with any flexible meaſuring line; and, therefore, when the true length of any ſtraight line on the ground is required, we muſt chuſe a meaſure which will neither contract nor lengthen in the application, ſuch as a pole of wood, or a rod of iron of ſome known length.

When a diſtance is to be meaſured by a pole, the operation is performed in the ſame manner as by the chain. A ſmall wire, or ſome ſuch mark, ſhould be ſtuck in the ground at the end of the pole, that, when it is moved forward, the hindermoſt end in every ſucceſſive poſition may lie at the ſame point where the foremoſt end lay in the laſt poſition; and, for this purpoſe, a notch ſhould be made in the end of the pole to receive the pin; and then the pole being carried forward, and its end laid cloſe to the pin in every advance, the diſtance may be meaſured very accurately.

Otherwiſe,

Otherwiſe, take two ſtraight poles of any convenient length, ſuch as 10 feet, having their ends well ſmoothed, and take ſupporters for them, ſo contrived, that they may be eaſily raiſed or depreſſed with ſcrews, or by ſome other means. Then, having placed the poles upon their ſupporters in a line at one extremity of the diſtance to be meaſured, and made them level by a plummet; lift the hindermoſt pole with its ſupporter, and carry it forward to the extremity of the other; place it there in a level-line, adjuſting it as before; then take up the hindermoſt of the two poles as they now lie, and carry it forward with its ſupporter, and ſo on, always bringing the poles to a true level, and taking care that their ends touch each other exactly. By this method, a diſtance may be meaſured with great accuracy.

The length of any acceſſible right line may be found by a careful application of ſome known meaſure; but, in many problems, the lengths of right lines are required, which are either wholly, or partly, inacceſſible, and ſometimes inviſible, ſuch as the diameters and perpendicular heights of mountains. In order to ſolve ſuch problems, we muſt not only be able to meaſure ſtraight lines, but to find out their poſitions, or to meaſure the quantity of any right lined angle accurately. This may be done by the Theodolite, Graphometer, Quadrant, &c. But the ſame thing may be performed as well by meaſuring certain right lines.

There are two kinds of Angles to be meaſured, namely, Horizontal and Vertical.

A horizontal angle is contained between two right lines which lie in the plane of the horizon, ſuch as the two

two ſides of a field, or any two imaginary ſtraight lines drawn from a given point in the horizontal plane. For example, ſuppoſe any point on the ſurface of the earth is taken, and from that point one ſtraight line runs north, and another north-eaſt; the angle contained by theſe lines is called a Horizontal Angle.

A vertical angle is contained by two ſtraight lines drawn in a plane perpendicular to the plane of the horizon. For example, ſuppoſe a ſtraight line is drawn from any point on the ſurface of the earth in the horizontal level, and, from the ſame point, another ſtraight line is drawn to the top of a high building, a mountain, a ſtar, or to any object which is above the plane of the horizon, the angle contained by theſe two ſtraight lines, or its complement, is called a Vertical Angle.

To meaſure a Horizontal Angle on the Ground, as ABC, *by any lineal meaſure.*

This may be done by taking a radius at pleaſure in any of the containing ſides, and meaſuring either the chord, ſine, tangent, or ſecant of the angle.

1ſt, By meaſuring the chord; *Fig.* 41. Meaſure any diſtance, ſuch as a chain from B to A, and there ſtick an arrow in the ground. Meaſure the ſame diſtance from B to C, and there alſo place an arrow. Then meaſure the diſtance from A to C, and the three ſides of the iſoſceles triangle ABC are known. Let fall the perpendicular B*e*, and the point *e* will be in the middle of AC; therefore A*e* is known; and, as BA : A*e* : : Rad. : Sine AB*e*; which doubled, is the angle ABC.

This method is general, and will anſwer equally well whatever the length of AB or BC is; but, if AB and BC

are each one chain, or any meaſure conſiſting of 100, or 1000 equal parts, the angle ABC may be found from a table of ſines, without calculation; thus, Meaſure AC in 10th parts of a link, or 1000th parts of the chain; then AB being 1000, and the radius of a table of ſines alſo 1000, A*e* being a number of the ſame parts, will be the natural ſine of the angle AB*e*; and therefore, this number being found in a table of natural ſines, the quantity of the angle AB*e* will be known. If you have not a table of natural ſines, take the logarithm of A*e*, and find it among the logarithm ſines, neglecting the indices of both, and you have the angle ABE, which being doubled, is the angle ABC.

If there be no miſtake in meaſuring the right lines, and if AC lies in, or near, the true level, the angle will be found perfectly true. An error in AC will affect the angle more than an equal error in AB or BC; but even here, if the error does not exceed $\frac{1}{10}$ of a link, or $\frac{1}{1000}$ part of the radius, the angle will not be above two minutes wrong, (unleſs the angle be large), which is a degree of exactneſs far exceeding that of the common inſtruments.

EXAM. 1. The radius being one chain, ſuppoſe the length of the chord AC is $67\frac{3}{10}$ links, or .673 1000th parts of the chain; the half of this is .3365, which is the natural ſine of 19° 40′; and therefore the angle ABC is 39° 20′.

2. Let the chord AC be $127\frac{7}{10}$ links, or 1277 of ſuch parts as the radius is 1000; the half of this number is .6385, which is the natural ſine of 39° 41′; and therefore the angle ABC is 79° 22′.

The radius being one chain, an error of $\frac{1}{10}$ of a link, or $\frac{8}{10}$ of an inch in the length of AC, would cauſe an

error

error of three minutes in an angle of about 80°; but such an error could scarcely happen, if due care be taken in measuring AC.

2*dly*, To find out the quantity of a horizontal angle by measuring its tangent; *Fig.* 29. Let CED represent an angle on the ground. Measure any distance, such as one chain from E to D in one of the sides containing the angle for a radius; then from the point D mark out a perpendicular to ED, and place a pin, or arrow, in C, at its extremity, in the other side containing the angle. Measure CD, and reduce it to 1000th parts of ED; it is the natural tangent of the angle CED; and hence the angle will be known from the tables as before.

Exam. Suppose the radius ED is one chain, or 1000 equal parts of any measure, and that the tangent DC is found to be $43\frac{8}{10}$ links, or .438 parts of ED; this number is the natural tangent of 23° 39′; and so much is the angle CED.

3*dly*, The quantity of a horizontal angle may be found by measuring its secant; for, (in the same figure,) having measured the radius ED, and found the point C, where the perpendicular drawn from D meets with the other side; EC is the natural secant of the angle CED; and EC being measured in 1000th parts of the radius, and found in a table of secants, the angle CED will be known. But it is to be observed, that unless the angle be above 45°, there is greater uncertainty in measuring the secant than the tangent; and I would never recommend this method, except in large angles, and when the tangent could not be measured on account of some obstacle in the ground.

Upon the whole, the best method of finding the quantity of any horizontal angle, is to measure either its chord or its tangent. Acute angles can be measured more accurately by their chords than obtuse angles; and, therefore, when the angle is large, I would divide it into two parts, equal or unequal; and, by measuring the chord of each part separately, the quantity of the whole will be known.

There is less chance of error in measuring the tangent of an angle than in measuring its chord, especially if the angle be between 30° and 80°, on account of the rapid increase of the tangents; for, if the angle be about 60°, and the radius one chain, an error of one inch in the tangent will make the angle only one minute wrong; and so great an error could scarcely be committed by a careful person.

Acute angles only can be measured directly by their tangents; but the method will apply to obtuse angles, by dividing them into parts, and measuring the tangent of each part separately; and, in this case, the radius should be measured in a straight line, dividing the angle into two parts, as near equal as can be guessed by the eye; and, on the extremity of this radius, set off a perpendicular to it towards each side containing the angle, these will be the tangents of the parts; and being measured, the quantity of the whole angle will be known. This should be done in any angle which is about or above 80°.

At

At a given point on the ground, to mark out a ſtraight line perpendicular to another given ſtraight line paſſing through that point.

Take a ſmooth board of any form, and thereon draw two right lines cutting each other at right angles, and place two ſights on each line a few inches diſtant from each other. Contrive ſome means to fix the board horizontally on the top of a ſtaff. Then, having fixed the ſtaff in the ground at the given point, with the board on the top of it; turn the whole, until through one pair of ſights you ſee along the given right line, and then the other pair of ſights will be in the direction of the perpendicular required. In that direction, place a pole at any convenient diſtance, and the right line between your ſtaff and the pole is perpendicular to the given right line.

Note. Sights are ſmall pieces of wood, or braſs, having ſmall holes, or ſlits, in them, to view diſtant objects through. They are fixed perpendicular to the plane of any inſtrument, and ſerve to direct the view in a right line. Such as have croſs hairs in them are reckoned the beſt.

A DESCRIPTION *of Two* POLES *for Meaſuring* VERTICAL ANGLES.

Provide two ſtraight poles of wood, one about 10 or 12 feet long, divided into feet, inches, and 10th parts of an inch. Provide alſo a ſtool about 12 or 15 inches high, having three feet, the whole of a conſiderable weight to ſtand firm, and having a hole in the middle of it to receive the end of the pole eaſily; and let the end of the pole that goes into the ſtool be put through the middle

middle of a board of hard wood, which board muſt be firmly fixed to the pole, and ſo near the end of it, that, when the pole is put into the ſtool, the board may be only two or three inches diſtant from the ſurface of the ſtool, and parallel to it. Make three or four ſcrews paſs through the board and the ſtool, to fix the pole firmly to the ſtool, and alſo to ſet it truly perpendicular to the plane of the horizon. In order to aſcertain this, a plummet may be hung from ſome point of the pole, or it may be adjuſted by a ſeparate plummet. Alſo, let there be two pieces of wood or braſs put upon the pole to ſlide up or down, each having a fiducial edge to ſhow the true point where they reſt on the pole.

Make alſo another pole about four or five feet long, having a ſharp end to ſtick in the ground, or elſe a three-footed ſtool to fix it in like the other; but there is no need of ſcrews here; if the pole ſtand firm any how, the end will be anſwered. At, or near the top of this pole, fix a ſmall ſpirit-level with hair ſights, and to turn round an axis, which axis muſt be perpendicular to the pole. By this means, the ſights will be eaſily directed towards the top of any high object, and alſo turned into the level line. And thus the two poles are prepared for uſe.

To Meaſure any Vertical Angle, not exceeding 64°, by the two Poles above deſcribed.

Let AEB be the vertical angle to be meaſured; A being the top of a high object, and FB the level ground. *Fig.* 43.

At any convenient point C, place the longeſt pole CD, and adjuſt it truly perpendicular to the plane of the hori-

zon.

zon. Meaſure ſome convenient diſtance from C to F, and there place the ſhorteſt pole FE with the ſpirit-level. Turn the level on its axis until A, the top of the high object, is ſeen through the ſights; then move the ſliding mark D on the long pole, until the fiducial edge reſt on the long pole preciſely in a right line between E and A, which will be known by ſeeing A and D through the ſights. Then turn the ſights into the level line, and move the other ſliding piece until its fiducial edge reſt at G, where the level line meets the long pole.

Hence GD, the diſtance between the ſliding pieces on the long pole, will be known; and EG, the diſtance between the poles, is known. Therefore,

As the diſtance between the poles EG
Is to the ſpace between the ſliding pieces GD,
So is radius
To the tangent of the angle AEB; which was required.

If the poles, level, and ſliding marks be made, adjuſted, and applied as above, there is no danger of any error in the angle.

The poles may be placed at any diſtance; but, if the ground will admit, it will be beſt to place them at ſome determined diſtance, ſuch as 10 or 20 feet, or, rather at 100 or 1000 equal parts of ſome known meaſure, as 100 inches, 100 half feet, or one ſurveying chain of 100 links; and then the diſtance of the poles being the ſame number with the radius of the tables, if GD be reduced to 1000th parts of EG, and found in the table of tangents, the quantity of the angle will be known without calculation.

EXAM.

Exam. 1. Let the distance of the poles EG be 10 feet, and the distance between the level point G and the sliding piece D on the long pole be $17\frac{3}{4}$ inches; it is required to find the quantity of the angle AEB?

Here $17\frac{3}{4}$ inches is 1.4791 feet; and, because the radius is 10 feet, 1.4791 feet in 100000th parts of the radius is .14791, which, in the tables, is the natural tangent of 8° 25′, the angle sought.

2. Let the distance of the poles be 20 feet, and GD on the longest pole 7 feet 9.75 inches; required the angle AEB?

7 feet 9.75 inches is 7.8125 feet; to make this 1000000 parts of the radius, divide 20 feet and 7.8125 feet both by 20, and GD is .390625, which is the natural tangent of 21° 20′, the quantity of the angle AEB.

The long pole may have only one sliding piece upon it; and, when that is the case, having placed the poles at a proper distance, turn the sights into the level line, and move the slider until the fiducial edge rests at G in the level, and note down the height of it; then turn the sights towards the high object, and move the sliding piece until you see the fiducial edge of it, and the top of the high object in a right line, and note down the height; then the difference of the heights of G and D will be known, and consequently the angle may be found as before.

The longest pole may have a sliding piece in it, to be extended upwards when there is occasion; and, when that is necessary, the slider may be moved up only so far, that the top of the pole and of the high object may appear through the sights in a straight line; and then the sliding

sliding mark will be of no other use but to find the point G on the long pole in the level-line with E, the top of the shortest pole.

The long pole may be divided only into feet and inches; for the odd parts being taken with a pair of compasses, and applied to a scale, the distance between the sliding marks will be known in 100dth parts of an inch; and this will be sufficiently exact for any purpose.

When the angle to be measured is small, as most of those are which serve to find the altitude and horizontal level of rising grounds, hills, mountains, &c. the distance of the poles may be 20 feet or more; but, when a large angle is to be measured, as in taking the altitude of the sun, or any star, the distance of the poles must be less.

When the angle is less than 45°, the distance of the poles may be 10 feet at least; but, if the angle be above 45°, either the distance of the poles must be less than 10 feet, or the longest pole must be lengthened by a sliding piece, or by fixing another pole on the top of the former. The distance of the poles should never be much less than 10 feet, nor much greater than 20 feet.

If the angle be under 12°, and the distance of the poles 10 feet, the 100dth part of an inch on the long pole is the tangent of about 20 seconds; and, if the distance of the poles be 20 feet, the 100dth part of an inch is the tangent of about 8 seconds; but, if the angle be 45°, or above, the 100dth part of an inch is the tangent of 10, or 4 seconds, according as the distance of the poles is 10 or 20 feet.

To measure a verticle angle by the Poles on sloping ground; that is, when the ground slopes from the longest towards the shortest Pole. Fig. 1 and 2. Plate 5.

If the top of the shortest pole be in a level line with the under part of the longest, measure from C to D, and from the square of CD subtract the square of the shortest pole ED; the square root of the remainder is the level distance of the poles EC.

If the level line meets the longest pole in some point *n*, subtract C*n* from ED, and from the square of CD subtract the square of the remainder D*o*, and the square root of the last remainder is *o*C, the level distance of the poles. By repeating this operation, the level distance between any two stations may be found, although measured on sloping ground. The rest of the operation is the same as before.

The instruments (now in use) for measuring angles, consist of a circle, or some part of it, divided into degrees; an index, moveable about the center, and sights of some kind or other for viewing distant objects, with a contrivance to fix it on a fulcrum, or foot.

To measure a horizontal angle ABC, *Fig. 41. with any circular instrument.*

Place poles at A and C. Set the instrument on its foot, so that its center be at the angular point B, and truly level. Set the index to the beginning of the degrees; then turn about the instrument until through the sights you see the pole at A; fix the instrument in this position, and turn the index about, until through the sights you see the pole at C; and the number of degrees

passed

passed over by the index is the measure of the angle ABC.

To measure any vertical angle with a circular instrument.

Set the plane, or face of the instrument, truly perpendicular to the plane of the horizon, and the index truly level; then turn the index until through the sights you see the top of the high object; and the number of degrees passed over by the index is the measure of the vertical angle.

Heights and *Distances*, which are the subject of the following problems, are either accessible or inaccessible. Terrestrial objects, such as trees, towers, or other high buildings, hills, mountains, &c. are accessible in certain situations, and inaccessible in others. Celestial objects, such as the sun, moon, and stars, are always inaccessible, on account of their distance.

Having shown how any angle, either horizontal or vertical, may be measured, I shall, in what follows, only point out the angles to be measured in the field, and leave the practitioner to his own judgment with respect to the means and manner of performance.

PROB. I. *To measure the height of an accessible object, such as a growing tree, a tower, or other high wall* AB. *Fig.* 42.

From B, the root of the height, measure any convenient distance to E, and there measure the angle AEB; then, in the right angled triangle ABE, the side EB and the angles are known; to find the side AB.

Exam. Let the distance EB=366 links, and the angle AEB=37° 30′; required AB?

Answer. 280.84.

There are several other methods of measuring an accessible height, which may be applied with advantage on some occasions; such as,

1*st*, By its shadow. When the sun shines, fix a pole in the ground perpendicular to the plane of the horizon; measure the shadow of the pole, and of the object whose height is required, and also the height of the pole; and then,

As the length of the shadow of the pole,
Is to the length of the shadow of the high object;
So is the height of the pole,
To the height of the object.

Exam. Let the length of the shadow of the pole be 47 inches, and its height $39\frac{1}{2}$ inches, and the shadow of a tree 79 feet 6 inches; what is the height of the tree? *Ans.* 66 8 feet.

N B. This is the shortest and easiest method of measuring the heights of growing trees, or other accessible objects, when great exactness is not required.

2*dly*, By a mirror, *Fig.* 44. Place a plain mirror at E, and move back to D, chusing your station so, that your eye being at F, you may see the top of the high object A, at some particular point of the mirror E.

Then, because the angle of incidence AEB is equal to the angle of reflection FED, and the angles at D and B are right ones; the triangles FDE, ABE, are similar.

Measure

Measure BE, ED, and DF; and then, as ED : DF :: EB : BA.

Note. In order to have a steady view, it will be proper to use a pole DF, and apply your eye to F.

EXAM. Let EB = 248 feet, ED = 8 feet, and the height of the eye DF = 5 feet; required AB? *Ans.* 155.

It may sometimes happen, that a man would wish to measure a height and a distance when he hath no instruments to measure angles. In such cases, an equilateral triangle made of a smooth board, having sights placed on its three angles at A, B, and C, and a plummet hung from C; also two of its sides bisected at D and E by indented lines, and a sight placed occasionally at D or E. If such a wooden triangle be made, each side about 12 inches long, and some means be thought of to fix it on the top of a staff, either in a horizontal or vertical position, many problems may be solved by it. *Fig.* 3. *plate* 5.

In order to apply the triangle to the solution of problems, it must be known, that half the side AD is to the perpendicular DC, as 1000 is to 1732; and that the side AC is to the perpendicular CD, as 1000 is to 866.

To measure the altitude of an accessible object, such as a rock, tower, tree, &c. by an equilateral triangle. Fig. 42.

Find the station E by trials, where the base of the triangle being placed in the level line, by the plummet, you can see the top of the high object A through the sights, placed on the side of the triangle; then measure from your station E to the high object, and EB will be known And as 1000 is to 1732, so is EB to BA, the altitude required.

EXAM.

EXAM. Let EB=96 feet; required the height of the tower? *Ans.* 166.27 feet.

PROB. 2. *To measure the height of any inaccessible object* AB, *as also its distance* CB. *Fig.* 45.

Chuse two convenient stations C and D, from whence the highest point A can be seen, and measure the two angles ACB, ADB, and the distance of the stations C and D.

Then, because the angle ACB is equal to both the angles CAD and ADC, subtract the angle at D from the angle ACB, the remainder is the angle DAC. Now, in the oblique angled triangle ADC, all the angles, and the side DC, are given; to find AC. And then, in the right angled triangle ABC, the angles, and the hypothenuse AC, are given; to find AB and BC.

EXAM. Let the angle ACB=50° 39′, the angle at D=33° 30′, and the distance of the stations CD=360 feet; required AB and BC? *Ans.* AB=521, and BC=427.2.

To measure an inaccessible altitude AB *by the equilateral triangle. Fig.* 45.

Find the station C by trials, where the base of the triangle being placed in the level-line, you can see the top of the high object through the sights on the side of the triangle; then find the station D, where the triangle being placed as before, you can see the top of the high object through the sights, which are placed, one at the extremity of the base, and the other in the middle of the opposite side at E. Measure CD, the distance of the stations,

stations, and CD is equal to CA; which is therefore known. Then, as 1000 is to 866, so is CD, or CA to AB, the altitude; and, as 1732 is to 1000, so is AB to BC, the distance of the high object from the station C.

The only difficulty in applying the equilateral triangle, is to find the stations exactly, which a little practice will render easy: But, unless you can come so near the high object, that the angle ACB is 60°, the triangle cannot be applied.

PROB. 3. *To measure the height of a tower* AE, *standing on a high rock or mountain* EB. *Fig.* 46.

Take two stations C and D, from whence the points E and A can be seen, and find the quantities of the angles ADB, ACB, and ECB; and measure DC. Then calculate AC, AB, and BC as in the last problem. Lastly, In the right angled triangle ECB, there is given CB, and the angle ECB; to find BE; which being subtracted from AB, the remainder is AE.

EXAM. Let the angle ECB = 50° 10′, ACB = 67° 50′, and the angle at D = 43° 40′, also the distance of the stations CD = 134 feet; required AE and EB.

Ans. AE = 107.07, and EB = 102.22.

This problem may be solved by the equilateral triangle in the same manner as the last.

PROB. 4. *To find the perpendicular height of a hill* AB, *and reduce the slope side* AC *to the horizontal level* BC. *Fig.* 47.

If AC be such an ascent as can be measured with a chain; and, if the point A can be seen from C; then measure

measure the angle ACB, and the slope side AC; and, in the right angled triangle ABC, the hypothenuse AC, and the angle at C being known, AB and BC may be found, by Case 4th of right angled triangles. But, if AC cannot be measured with a chain, on account of cliffs, or other obstacles, it is an inaccessible height, and must be measured by the two last problems.

EXAM. Let AC=2496 links, and the angle of ascent ACB=21° 49′; required AB and BC? *Ans.* AB=927.6, BC=2317.2 links.

PROB. 5. *To measure the altitude of the Sun, Moon, or any Star.*

When the sun is visible, he will either appear in the horizon, where he hath no altitude, or in the zenith, (as in the torrid zone at noon day) where his altitude is 90°; or somewhere between the zenith and horizon, and then the number of degrees between the sun's center and horizon at any time of the day, is called his *altitude*: Or, the sun's altitude is the angle contained between one right line drawn from some point in the plane of the horizon, and another right line drawn from the same point to the center of the sun. This angle is measured the same way as any other vertical angle; but here there is need for a greater degree of accuracy, than in measuring other angles; for, unless the sun's altitude be taken true to half a minute, the observation can be of no use in finding the latitude of the place, the hour of the day, or the sun's azimuth, which are the usual *quæsita* depending on the sun's altitude.

To

To perform the problem without an inſtrument to meaſure an angle, chuſe ſome high wall, ſuch as the end of a houſe; take a pole nearly equal to the height of your eye, and, by trial, find out a ſtation, where the pole being ſet perpendicular to the plane of the horizon, your eye applied to the top of the pole, may ſee the top of the wall, and the upper limb of the ſun in one right line. Then find a point in the wall on a level with the top of the pole. Meaſure the diſtance of the pole from the wall, and the height of the wall above the pole: And then,

As the diſtance of the pole from the wall,
Is to the height of the wall above the pole;
So is radius,
To the tangent of the ſun's altitude.

From which ſubtract 16′ for the ſun's ſemidiameter; the remainder is the apparent altitude of the ſun's center.

This problem may be ſolved by the two poles, deſcribed in pages 77, 78, 79, *et ſeq.*

PROB. 6. *To meaſure the diſtance between any two objects* A *and* B, *the point* A *being acceſſible.* *Fig.* 48.

Chuſe ſome convenient ſtation C, from whence the objects A and B can be ſeen, and place poles at A and C. Meaſure the angles BAC, ACB, and the diſtance of the ſtations AC.

The ſum of the angles BAC, ACB, being ſubtracted from 180°, the remainder is the angle at B; then, in the triangle ABC, the angles and the ſide AC being known, the ſide AB may be found by caſe 1ſt, of oblique angled triangles.

EXAM. Let the angle BAC=69° 24′, and the angle ACB=75° 24′, and the diſtance of the ſtations AC =496 feet; required the diſtance AB? *Anſ.* 832.68.

N. B. When it is not convenient to meaſure the angles BAC, and ACB directly, their ſupplements DAC, ACF, may be meaſured.

2. The diſtance of the ſtations A and C muſt bear ſome reaſonable proportion to AB. It ought not to be leſs than $\frac{1}{5}$ or $\frac{1}{10}$ of it, if poſſible.

This is the general method of meaſuring a diſtance; but there are other methods which may anſwer very well in particular caſes, eſpecially when a ſmall diſtance is to be meaſured, ſuch as the breadth of a river.

To meaſure a diſtance, acceſſible at one extremity, by four poles. Fig. 48.

Place poles at A and C, ſo that BAC may be a right angle. Meaſure AC, and take AD in the ſame right line with AB, equal to one chain, or any convenient length, and place a pole at D. Place another pole at E, making ADE a right angle; then DE will be parallel to AC. Take DE equal to AC, and meaſure forward in the ſame right line with DE, until you come to the point F, where you ſee the pole at C, and the point B in the ſame right line; and then by ſimilar triangles, as FE: EC: :CA: AB.

EXAM. Let AC=426 links, AD or CE=100 links, and EF=57 links; required the diſtance AB? *Anſ.* 747.3 links.

2dly,

2dly, *The same thing may be done by an equilateral triangle. Fig.* 49.

Place the triangle horizontally at A, so that along one side of it you can see the point B; and, in the direction of the other side, place a pole at C; then measure from A to C, (the point C being found by trial), where the triangle being placed horizontally, you can see the point A along one side of it, and the point B along the other. Then ABC is an equilateral triangle; and so AB is equal to the measured distance AC.

If the perpendicular distance between the line AC and the point B be required: Having placed the triangle at the point A as before, and marked out the line AC by a pole, measure along that line until you find the point E, where the base of the triangle being placed in the direction of AC, you can see the point B through the sight E in the middle of the base, and that on the opposite angle; and then, as 1000 is to 1732, so is AE to EB, the distance required.

3dly, *By measuring the sides of a triangle equal to the triangle* ABC. *Fig.* 50.

Place poles at A and C, so that BAC be a right angle; measure AC, and in the same right line measure CD equal to AC; and from D, set off DE at right angles to DC. Measure along DE until you come to the point E, in a right line with C and B; then ED is equal to AB, which therefore is known. This method is good enough in small distances. But, when the distance AB is large, take CD equal to some part of AC, such as $\frac{1}{4}$, $\frac{1}{5}$, or $\frac{1}{10}$; and then DE being measured, and multiplied by 4, 5, or

10, the product will be equal to AB. This method will apply to any distance within your view; and, if you measure EC, you may find the distance CB by the same kind of operation. In this manner, the diagonals, or sides of fields, may be found out, when obstacles hinder the actual mensuration.

PROB. 7. *To measure an inaccessible distance* AB *from two stations* C *and* D, *from whence its extremities can be seen. Fig.* 51.

Measure the angles ACD, BCD, CDB, and CDA, with the distance of the stations CD. Then, in the triangle ACD having the two angles ACD, ADC, and the side CD, we can find the side AC; and, in the triangle BCD, having the two angles BCD, BDC, and the side CD, we can find BC. Lastly, in the triangle ABC, having the two sides AC and CB, with the contained angle ACB, we can find the angles CAB, ABC, and the side AB.

EXAM. Let the angles ACD=100° 30′, BCD=38°, CDB=108°, CDA=42° 30′, and ACB=62° 30′, and let the distance of the stations CD=398 links; required AB? *Ans.* 615 links.

To measure an inaccessible distance AB *by the equilateral triangle. Fig.* 4, *plate* 5.

Place the equilateral triangle horizontally at E, so that you can see the point B along one side of it; and, in the direction of the other side, mark out the line EDCF by poles, and leave a pole at E. Then measure from E until you come to the point D, where EDB is a right angle,

angle, and mark down the length of ED. Measure forward in the same right line until you come to the point C, where ACD is a right angle, and mark the length of DC. Measure on in the same line until you come to the point F, (which must be found by trials), where the triangle being placed horizontally, one side of it in the direction of FCD, you can see the point A along the other side of it, and mark down the length of CF. Then, if FC be equal to ED, the distance CD is equal to AB; which therefore is known; but, if FC be not equal to ED, calculate DB and CA thus; as 1000 is to 1732, so is ED to DB; and so is FC to CA Take the difference between AC and BD, and the square of this difference added to the square of CD, gives the square of aB, which, by extracting the square root, will be known.

This method of measuring an accessible distance is both easy and accurate, and may be applied with success to the distances of mountains, and the sides of fields, where the ground does not admit of applying the measure to each side; but is not so proper for large distances.

The equilateral triangle has been proposed as the most simple instrument, which can be made in a short time, with very little trouble; but, if we use a square, and divide two of its sides into some number of equal parts, we may solve problems more conveniently.

Description *of the* Geometrical Square. *Fig. 5. plate 5.*

The *Geometrical Square* may be made of wood, brass, or any other matter, either solid throughout, or consisting of four plain rulers joined together at right angles. A is

A is the center, from which hangs a plummet; each of the fides BE, DE, is divided into 100 or 1000 equal parts; C and F are two fights fixed on the fide AD; there is alfo an index AL, which turns round the center A; and thereon are two fights K and L. The fide DE is called the upright fide, and BE the reclining fide of the fquare.

To meafure an acceffible height AB *by the Geometrical Square. Fig. 6, plate 5.*

Let BR be a horizontal plane, on which the high object AB ftands perpendicular; and let BP be the diftance of the obfervator=96 feet; and let the height of the obferver's eye be 5 feet.

Set the fquare perpendicular to the horizon, fo that the eye being at D, and looking through the fights, fees A the top of the high object; and the plummet hanging freely, fuppofe the thread cuts off 80 equal parts from the fide, viz. DN. Then the triangles LDN, ADC being fimilar, as ND:DL::DC:CA; that is, as 80:100::96:120=CA.

But, if the plum-line fall on the angle oppofite to the center, as at the ftation E; then becaufe LD is equal to DP, DC is equal to CA; for the triangles LDP, DCA are fimilar; therefore, CA will be known by meafuring DC.

If the diftance of the obfervator BF be greater than the altitude, fuch as 300 feet, then the plum-line will cut the fide of the fquare in N: Suppofe NE=40 equal parts, the triangles LNE, DCA, are fimilar; for the angles LEN, DCA, are right, and the angle LNE

ENE is equal to the angle DAC, and consequently the angle ELN is equal to the angle ADC; therefore, as LE:EN::DC:CA; that is, as 100:40::300:120; to which add 5 feet, the height of the observator's eye, and AB=125 feet.

To measure a distance AF, *accessible at one extremity, by the Geometrical Square. Fig. 7. plate 5.*

Place the square horizontally on a support at A, so that through the sights on the side of the square you see the point F; turn the index, until it coincides with the other side of the square, and in the direction of that side place a pole at B, any convenient distance from A. Leave a pole at A, and measure AB. Then place the square horizontally at B, so that through the fixed sights you see the pole at A; and turn the index until through the sights on it you see the point F. Suppose the index cuts the right side of the square in G, then the triangles BDG, BAF are similar; and, as BD:DG::BA:AF.

But, if a large distance, as AH, is to be measured, the square being placed at B, the index will cut the reclined side of the square, suppose at L; then the triangles BIL, BAH, are similar, for the angles at A and I are right; the angle BLI is equal to the angle ABH, and the angle IBL to the angle AHB; therefore, as LI:IB::BA:AH.

If the square be well made, and properly applied, most problems may be solved by it; but, because it requires the same trouble and expence to fix the square on a support, and to set it level or perpendicular to the horizon as any circular instrument, without any peculiar advantage, it is now almost out of use.

PROB.

PROB. 8. *To measure the distances of any number of inaccessible objects from each other, by means of two stations, from whence the objects can be seen.*

The method of performing this is in all respects the same as in problem 7th, p. 92; for, it is only taking a greater number of angles at each station, and solving a greater number of triangles.

EXAM. Suppose A, B, C, *Fig.* 52. are three inaccessible objects, whose distances from each other are required, and E and D two stations, whose distance is known, and from whence the objects A, B, and C can be seen. The angles being measured, suppose them to be,

At the station D.	*At the station* E.
ADB= 29° 15′	CEB= 40° 30′
BDC= 39° 30′	BEA= 25° 30′
CDE= 51° 45′	AED= 46° 45′
ADE= 120° 30′	CED= 120° 45′

And the distance of the stations DE=740 links; required AB, BC, and AC? *Ans.* AB=1243, BC=1702, and AC=2822.

PROB. 9. *To measure the distance of any inaccessible high object from a given station, and its altitude above the level of that station.*

Find out the distance by problem 6th; then measure the angle of ascent at the given station, and, by means of this, and the distance before found, calculate the height.

EXAM. Let BE, *Fig.* 53. be any high object, and A the given station from whence it can be seen; chuse another station C; then measure the angle BAC=75° 12′, and the angle ACB=64° 29′, and let the distance of the stations

stations AC=425 links. At A measure the angle of ascent EAB=11° 19′; required the distance AB, and height BE? *Ans.* AB=592.7, and BE=118.6.

PROB. 10. *The distances of three objects* A, B, C, *in the same plane being given, and the angles observed at a fourth place as at the station* S; *to find the distances of these three objects from that station. Plate* 5. *Fig.* 8, 9, &c.

CASE 1. If the station S be taken without the triangle ABC in one of its sides produced. *Fig.* 8. *plate* 5.

Find the angle ACB by the 4th Case of oblique triangles, and then the angle ACS will be known: Now, in the triangle ASC, the angles ACS, CSA, and consequently the angle CAS, with the side AC being given, the sides SA, SC, and consequently SB, may be found by Case 1st of oblique triangles.

CASE 2. If the station S be in one side of the triangle ABC, as in AC. *Fig.* 9.

Find the angle ACB as before; and, in the triangle SBC, the angles BCS, CSB, and the side BC being known, SB, SC, and consequently SA, may be found. To find the point S, make the angle CAD equal to the observed angle CSB, and through B draw BS parallel to AD.

CASE 3. If the three objects lie in a right line, as A, C, B. *Fig.* 10.

1. Suppose a circle described through A, B, and the station S. From S draw the right line SC, which will meet the circle in the point D; join BD, DA, AS, and SB. Then, in the triangle ABD, all the angles and the side AB are known; for the angle ABD is equal to the

N observed

observed angle ASC, because they stand on the same arc AD; and the angle BAD is equal to the angle CSB for the same reason: Hence the side AD may be found.

2. Now, in the triangle ACD, the sides CA, AD, and the contained angle CAD, are known; to find the angle ACD, the supplement whereof is the angle SCA; and then in the two triangles ASC, BSC, all the angles, and a side in each triangle are known, from which SA, SC, and SB, may be found.

Case 4. If the station S be without the triangle ABC, but not in any of its sides produced; describe a circle through the points A, B, S, as before. *Fig.* 11.

1. Since the angle ASC is equal to the angle ABD, and BSC equal to BAD; in the triangle ABD all the angles and the side AB being known, AD may be found.

2. All the sides of the triangle ABC being known, by these find the angle CAB; hence the angle CAD will be known.

3. In the triangle ACD, the sides CA, AD, and the contained angle CAD are given; by which find the angles ACD, ADC, and then the angle ACS will be known.

4. In the triangle ACS, all the angles, and the side AC are given, to find the sides SA and SC.

5. In the triangle ABS, all the angles, and the side AB are given, to find SB.

Case 5. If the station S be within the triangle ABC. *Fig.* 12.

1. A circle being described through A, S, B, as before, the supplement of the angle CSB is equal to the angle

angle BSD; that is, BAD; and the ſupplement of the angle ASC is equal to the angle ASD, or ABD; hence the angle ADB will be known.

2. In the triangle ABD, all the angles, and the ſide AB are known; from which the ſide AD may be found.

3. In the triangle ABC, calculate the two angles BAC, ACB, from the three given ſides, and the angle CAD will be known.

4. In the triangle ACD, the ſides CA, AD, with the contained angle CAD, are given; and by theſe the angle ACS may be found; which being ſubtracted from the angle ACB, there will remain the angle SCB.

5. In the triangle ACS, all the angles, and the ſide AC being known, the ſides AS and SC may be found; and, in the triangle CSB, the ſide CS and the angles being known, SB may be found.

Case 6. If the ſtation S be ſo ſituated with reſpect to the objects A, B, C, that the point D in the circumference of the circle is not in a right line with S and C, as in the three laſt caſes; ſuppoſe a circle deſcribed through the ſtation S, and two of the given objects as B and C; then make the angle CBD equal to the obſerved angle CSA, and make the angle BCD equal to the ſupplement of the angle ASB; draw ADS and SC. *Fig.* 13.

1. Since the angle BDC is equal to the obſerved angle BSC; in the triangle BCD, all the angles, and the ſide BC, are known; by which the ſide BD may be found.

2. Calculate the angle ABC, and ſubtract it from the angle CBD, the remainder is the angle ABD; and in the triangle ABD the two ſides AB, BD, and the angle

ABD being known, the angles BAD, ADB, may be found: Hence the angle SDB, which is equal to the angle SCB, will be known.

3. In the triangle ABS, the angles and the side AB being known, the sides AS and SB may be found.

4. In the triangle SCB, the angles, with the side BC, are known; from which the side SC may be found. Q. E. I.

PROB. 11. *Let* B *and* C *be two objects whose distance* BC *is known; and let* A *and* E *be two stations from whence these objects can be seen, and the angles* BAC, BAE, AEB, *and* AEC *measured; it is required to find the distances* AB, AC, AE, *and* EC? *Plate* 5. *Fig.* 14.

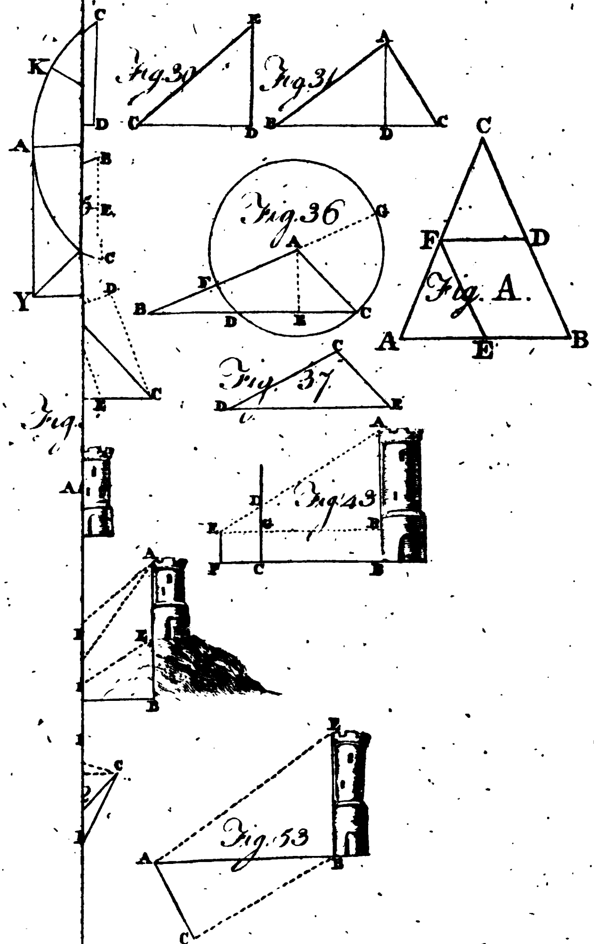
Since in each of the triangles ABE and ACE two angles are known by observation, the third angle in each triangle will be known.

Take a right line GH at pleasure, *Fig.* 15. and thereon constitute the triangles GHD, GHF, equiangular to the triangles EAB, EAC; join DF. Then take BC of the given length, and upon it constitute the triangles BCA, BCE, equiangular to the triangles DFH, DFG; and the figure ABCE is similar to the figure HDFG.

Calculate HD, HF, and FD, as in Problem 7th, and then, as DF is to BC, so is DH to BA; and so is HF to AC, and HG to AE; and, in the triangle ACE, there is sufficient data to find CE.

MIS-

PLATE 2. Page 110.



MISCELLANEOUS PROBLEMS.

1. *Suppose a man stands on the Terraqueous Globe, the height of his eye being 6 feet; and his sensible horizon free of all obstructions to the sight; it is required to find the extent of his view in every direction? Fig. 54.*

Let CA be the semidiameter of the earth=21024960 feet, and CB represent the same, increased by the given height of the eye, and consequently=21024966: Let BA be the visual ray meeting the curve surface of the earth in A. Now, supposing the earth to be a perfect globe, the triangle ABC is right angled at A, and the curve line EA will be nearly equal to AB, the tangent.

The square of AB is equal to the difference of the squares of BC and CA.

The square of CB =442049195301156
The square of CA =442048943001600

The difference is $\overline{AB}^2=$ 252299556 the square root of which is 15883 feet = 3 miles 43 feet; the extent of a man's view on a plain horizon, the height of his eye being 6 feet. Hence it is evident, that a man whose eye is 6 feet high, standing on a plain, such as a frozen sea, can see another man of the same height, at the distance of 6 miles 86 feet.

By a like calculation, if the height of the eye be 12 feet, the extent of the view will be 4 miles 447 yards; and such a spectator would see an object of the same height at the distance of 8½ miles.

If

If the height of the eye be 24 feet, the extent of the view will be 6 miles 29 yards; and such a spectator would see another object of the same height at the distance of 12 miles 58 yards.

If the height of the eye be 48 feet, as on the mast of a ship at sea, the extent of the view would be 8 miles 895 yards; and a spectator in this position would see an object of the same height with his eye, at the distance of 17 miles 30 yards.

2. *To measure the distance of any object by an explosion, such as the firing of a cannon.*

It has been found by many experiments, that the motion of sound is about 1142 feet in one second of time; and hence large distances may be measured pretty exactly, thus: Let there be two men, one placed at each extremity of the distance to be measured, and let one of them fire off a great gun, and the moment that the other sees the flash, let him look on a watch which shows the seconds until he hears the report. Then multiply 1142 by the number of seconds, and the product is the distance in feet.

If a watch be not at hand, set a pendulum of $39\frac{1}{5}$ inches long a-going, and count the vibrations, or number of times it crosses the perpendicular, and this will show the number of seconds.

By this method, we can measure the distance of thunder from the place where we are, by counting the seconds between the time of seeing the lightning and hearing the report.

3. To

3. *To measure the velocity of running waters.*

Put a piece of cork, or other light matter, into the water, and place a mark in the bank where the cork was put in. Then follow the cork as it goes down the stream for one minute, or any other space of time; and place another mark in the bank at the place where the cork is at the end of the time. Measure the distance between the marks, and then, as the time the cork was in the water is to one hour, so is the space between the marks to the velocity of the water in an hour.

4. *To measure the velocity of the wind.*

When the sun shines, observe when the shadow of a cloud is at some particular place on the ground, and then count the seconds as it goes along until it come to some other place which you can remember. Measure the distance between the two places, and then say, as the number of seconds is to one hour, so is the distance measured to the velocity of the wind in one hour.

N.B. The greatest velocity of the wind is near 60 miles *per* hour.

OF LEVELLING.

Those who understand the mensuration of heights and distances, will not find much difficulty in levelling, as it consists in finding the height or depression of any part of the earth's surface above or below any other part. I shall, therefore, only explain some things whereon the practice depends, and conclude with an example.

The earth is a round body, and, in the business of levelling, may be considered as a perfect globe.

All heavy bodies, such as earth, water, and air, gravitate towards the center of the earth.

A level

A level is either true or apparent.

A true level is any part of the earth's surface, every point of which is equally distant from the center, such as a body of smooth water. Any line drawn in such a surface, is called a *true level line*, and is therefore an arc of a great circle, described on the terraqueous globe.

An apparent level, is properly the sensible horizon, or a plane touching the earth's surface in one point, *viz.* in the place where the spectator stands. Any right line drawn in this plane, is the line of apparent level, and is a tangent to the curve, which is a line of true level passing through that place. Hence, the lines of true and apparent level coincide only in one point; and in every other point will be distant from one another, more or less, according to their extent. But, as the earth is a great globe, the lines of true and apparent level will lie very near each other in small distances.

PROB. *To find the distance between the apparent and true level at the distance of one mile from the point of contact. Fig.* 54.

In the right angled plain triangle ABC, there is given the side CA, the semidiameter of the earth $=3982$ miles, and the side $AB=1$ mile; to find the hypothenuse CB.

The square of CA =	15856324
The square of AB =	1
Sum of the squares of CA and AB =	15856325

The square root of which is $3982.0001255 =$ CB.

Therefore, the apparent level at the distance of one mile from the observer's station, is higher than the true level by .0001255 of a mile, or very near 8 inches.

A TABLE,

A TABLE, showing the number of Feet, Inches, and Decimals of an Inch, which are to be subtracted from the apparent, to find the true level for the several distances therein mentioned.

Miles.	F.	inches.	Miles.	F.	inches.	Miles.	F.	inches.
$\frac{1}{4}$	0	0.49	9	53	8.4	20	265	2.3
$\frac{1}{2}$	0	2.	10	66	3.48	21	292	4.5
$\frac{3}{4}$	0	4.47	11	80	2.56	22	320	10.6
1	0	7 95	12	95	5.6	23	350	8.6
2	2	7.8	13	112	0.5	24	381	10.5
3	5	11.6	14.	129	11.33	25	414	4.37
4	10	7.3	15	149	2.	26	448	2.
5	16	6.9	16	169	8.68	27	483	3.77
6	23	10.57	17	191	7.2	28	519	9.3
7	32	5.60	18	214	9.6	29	557	6.8
8	42	5.10	19	239	4.	30	596	8.2

To find what must be subtracted for any distance not found in the table. Multiply .6629832 by the square of the distance in miles. The product is the number of feet, and decimals of a foot, to be subtracted from the apparent level to find the true.

Note. .6629832 feet is the quantity to be subtracted from the apparent level, at the distance of one mile

The instrument commonly made use of in levelling, is a little glass tube filled with spirits of wine, all to a very little space, which contains a small quantity of air; and, when the tube is perfectly level, the bubble of air rests in the middle of it. To this level are fitted sights of some kind or other. A little telescope, with cross hairs in its focus, is best.

In levelling, two poles of ten or twelve feet length, divided into inches, and tenths of an inch, with pieces of

O paper

paper or pasteboard fitted to them, to slide up or down, are necessary. On the pasteboard should be drawn a horizontal black line, which may be seen through the sights of the levelling instrument, in order to adjust the height or depression exactly.

Exam. Let ABCED, *Fig.* 55. represent an uneven ground; it is required to find how much the point D is higher than A?

Chuse a station B between C and A, and there place your levelling instrument. Place poles in A and C, and directing the sights to the pole in A, move the pasteboard up or down, until the horizontal black line appear in the level. Then direct the sights to the pole in C, moving the pasteboard as before, until the horizontal black line appear through the sights. Measure the heights of the horizontal black lines on the pasteboards above the ground, and their difference is the height of C above A. In the same manner, take some station E between C and D, and find the height of D above C; the sum of these two is the height of D above A. Proceed in the same manner when more stations are necessary, remembering to reduce the apparent to the true level, when the distance is above 400 yards.

N. B. The several stations may be chosen according to conveniency. It is not necessary that they be all in the same right line.

SECT.

SECT. IV.

MENSURATION OF SURFACES.

AS every length or diſtance is meaſured by a line of ſome determinate length, expreſſed in inches, feet, yards, or decimal parts of itſelf, as the ſurveying chain: So every ſuperficies, or magnitude of two dimenſions, is eſtimated by ſome known ſquare, as a ſquare inch, foot, yard, chain, or mile.

If we grant the common poſtulate, that magnitudes may be generated by motion, *viz.* a line by the motion of a point, and a ſurface by the motion of a line, we ſhall eaſily diſcover thoſe properties of right lined figures, on which the menſuration of ſurfaces depend.

Let AB, *Fig.* 56. be a right line of 5, and AD of 3 equal parts, ſuch as feet, inches, or any other meaſure, containing the right angle BAD; and let AD be ſuppoſed to move along the line AB until it comes to the ſeveral poſitions EF, GH, KL, MN, and BC, and by this motion generate the right angled parallelograms AF, EH, GL, KN, and MC. It is evident at ſight, that each of theſe parallelograms contains three equal ſquares, that is, as many as AD contains equal parts; and that the number of parallelograms is equal to the number of equal parts in AB. Therefore, the whole rectangle ABCD contains 3 times 5, or 15 equal ſquares, whoſe common ſide is one of the equal parts in AB or AD. Or the rectangle contains a number of ſquares equal to the product of the length multiplied by the breadth.

 Hence,

Hence, the reaſon of the common rule for finding the area of a rectangle is abundantly evident.

And, becauſe effects are always proportional to their cauſes; it may be inferred, that parallelograms generated by equal right lines are equal, in whatever poſition the generating right lines proceed, provided that theſe lines move equal diſtances from their firſt poſition.

Fig 57. Let the right line BC generate the parallelograms ABCD, EBCF, by moving from its firſt poſition BC, until it reſt in AD or EF; theſe parallelograms muſt be equal, as being produced by the ſame cauſe, acting in the ſame circumſtances.

Hence, the rule for finding the area of any parallelogram, is,

1. Multiply the baſe by the perpendicular height; the product is the area in ſquares of the ſame name with the dimenſions taken.

Draw the diagonal DB, *Fig*. 56. and the triangle DAB is equal to the triangle DCB; for, the parallelogram being cut in two by the line DB, the two triangles properly applied will coincide. Hence, every triangle is half of its circumſcribing parallelogram; and the rule for finding the area of any triangle is, Multiply the baſe by the perpendicular height, and half the product is the area. Or,

2. Multiply the baſe by half the perpendicular, or the perpendicular by half the baſe; the product is the area.

The menſuration of all plain ſurfaces depends on theſe few principles. For every ſurface bounded by right lines, is either a parallelogram or a triangle, or a figure which can be reduced to triangles, by drawing lines

lines within it; and therefore, the rules for finding the area of the parallelogram and the triangle, will ſerve for finding the area of any right lined figure whatever.

And the areas of figures, whoſe boundaries are curve lines, ſuch as the circle, are computed upon the ſame principles. The particular rules given for computing them, are derived from the ſuppoſition, that they conſiſt of a great number of very ſmall rectilineal figures, which, taken all together, approach infinitely near to the true area; and therefore the menſuration of all plain ſurfaces depend on, and ariſe from, the rules for meaſuring the parallelogram and triangle.

Plain ſurfaces, which men have occaſion to meaſure, are thoſe of timber and ſtone; as in buying and ſelling of wood and marble, or computing the extent of artificers work; ſuch as maſons, brick-layers, carpenters, and painters work; or ſome part of the earth's ſurfaces, as in buying, ſelling, and letting of land.

In timber, ſtone, and artificers work, the meaſures are commonly taken in feet, inches, and parts of an inch, by a foot-rule, yard, or tape rolled up in a box. In ſurveying of land, the meaſures are taken in links, of the ſurveying chain, already deſcribed.

1. TABLE of Engliſh Lineal meaſures.

12	Lines	make	1 Inch
12	Inches	make	1 Foot
3	Feet	make	1 Yard
2	Yards, or 6 feet	make	1 Fathom
$5\frac{1}{2}$	Yards, or $16\frac{1}{2}$ feet	make	1 Pole, or perch
4	Poles, or 66 feet	make	1 Chain
10	Chains, or 40 poles	make	1 Furlong
8	Furlongs, or 80 chains,	make	1 Mile
3	Miles	make	1 League.

A com-

A common pace is $2\frac{1}{2}$ feet. A geometrical pace is 5 feet. 5280 feet, or 1760 yards, is one English mile.

2. TABLE of Scots Lineal Measures.

37 English inches	make	1 Ell
6 Ells, or $18\frac{1}{2}$ feet		1 Fall, or rood
4 Falls, or 74 feet		1 Chain
10 Chains, or 40 falls		1 Furlong
8 Furlongs, or 80 chains		1 Scots mile

Note. 6 ells is a rood in mason-work.

If the Scots surveying chain be $74\frac{2}{5}$ English feet, as it ought to be, the Scots mile will be 5952 feet, or longer than the English mile by 672 feet.

3. TABLE of English Square Measures.

144 Square lines	make	1 Square inch
144 Square inches		1 Square foot
9 Square feet		1 Square yard

In Scotland, 36 square ells is one rood of mason-work.

4. ENGLISH Land Measures.

9 Square feet	make	1 Square yard
$30\frac{1}{4}$ Square yards		1 Square pole, or perch
40 Square poles		1 Square rood
4 Square roods		1 Square acre.

5. SCOTS Land Measures.

9.5 Square feet	make	1 Square ell
36 Square ells		1 Square fall
40 Square falls		1 Square rood
4 Square roods		1 Square acre

Note. The feet mentioned in this Table are English feet.

In

In computing the areas of plain surfaces, the most general and best method, is, to reduce the fractional parts to decimals; and, by working properly, the area will be found in squares and decimal parts, of the same name with the dimensions. But several artificers chuse rather to take the dimensions in feet, inches, and 12th parts of an inch, or lines; and, by multiplying these, get the area in square feet, and 12th parts of a square foot, (which are improperly called inches), and the lower denominations are 12th parts, each of the next above it. This operation is commonly called Cross Multiplication, and depends on the following rules:

1. Feet multiplied by feet produce square feet.

2. Feet multiplied by inches, and the product divided by 12, give square feet; and the remainder is 12th parts of a square foot, here called inches.

3. Feet multiplied by lines, and the product divided by 144, give square feet; the remainder divided by 12, gives 12ths of a square foot; and the second remainder 12ths of the former, here called lines.

4. Inches multiplied by inches, and the product divided by 12, give 12ths of a square foot; the remainder is lines.

5. Inches multiplied by lines, and the product divided by 144, give 12ths of a square foot; the remainder divided by 12, gives lines, and the second remainder, 12ths of these.

6. Lines multiplied by lines, and the product divided by 12, give 12ths of a line; the remainder is 12ths of the former.

EXAM.

EXAM. Multiply 29 feet 11 inches by 13 feet 10 inches.

F.	I.			
29	11	$13\times11=143$		$10\times11=110$
13	10	$29\times10=290$		and $\frac{110}{12}=9.\ 2$
87		12)433		
29				
36	1	36 1		
	9 2			
Prod. 413	10 2			

1. Multiply 29 by 13, the product is square feet.

2. Multiply 13 by 11, and 29 by 10, and divide the sum of the products 433 by 12, the quotient is 36 square feet, and $\frac{1}{12}$ of a square foot.

3. Multiply 11 by 10, and divide the product by 12, the quotient is 9 inches 2 lines. The several articles being properly placed and added, the whole product is 413 square feet $\frac{10}{12}$ of a square foot, and $\frac{2}{12}$ of $\frac{1}{12}$ of a square foot, which, in the tradesmens language, is 413 square feet, 10 inches, 2 lines.

EXAM. 2. Multiply 27 feet, 9 inches, 4 lines, by 19 feet, 8 inches, 11 lines.

F.	I.	L.				
27	9	4			$19\times9=171$	
19	8	11			$27\times8=216$	
143					12)387	
27						
32	3				32	3
2	7	1				
	6	0			$19\times4=76$	
		10	11		$27\times11=297$	
			3	8		
					373	
Prod. 548	5	0	2	8		

F.

The method of operation in this is nearly the same as in the last example, and may be understood by reading over the several steps of the work.

```
          F.  I.  L.
144)373(2   7   1
    288
 12) 85
      7   1
8× 9=72 and 72/12=6 inc.
   8× 4=32
   9×11=99
      12)131
         10 11
11× 4=44 and 44/12=3 8
```

3. Multiply 39 feet, 10 inches, 9 lines, by 19 feet, 9 lines. Product, 760 feet 6 2 0 9.

4. Multiply 31 feet 7 inches, by 9 feet 4 inches 6 lines. Product, 296 feet 1 1 6.

5. Multiply 28 feet 10 inches, 9 lines, by 19 feet, 2 inches, 3 lines. Product, 554 feet 5 3 2 3.

6. Multiply 49 feet 2 inches, by 39 feet 5 inches. Product, 1937 feet 11 10.

7. Multiply 12 yards, 2 feet, 11 inches, by 13 feet, 9 inches, 5 lines. Product, 59 yards 5 feet, 5 5 7.

Otherwise. Reduce each term to inches, and multiply the one by the other, the product will be square inches; which may be reduced to square feet or yards by the table. If there be lines in both or either term, reduce them to decimals of an inch.

EXAM. 1. Multiply 25 feet 6 inches, by 18 feet 9 inches.

25 feet 6 inches=306 inches, and 18 feet 9 inches=225 inches. 306×225=68850 square inches, which, being divided by 144, the quotient is 478 square feet 18 inches.

2. Multiply 31 feet 7 inches, by 9 feet, 4 inches, 6 lines.

379×112.5=42637.5 square inches, which being reduced, is 296 square feet 13.5 square inches.

1. *To measure any right angled quadrilateral figure, and compute its area.*

Meaſure its length and breadth, and multiply the one by the other; the product is the area in ſquares of the ſame denomination with the meaſures of the ſides.

What is the area of a ſquare, each of its ſides being 63 yards, 1 foot, 9 inches?

Anſw. 4042 ſquare yards, 7 feet, 81 inches.

```
      Y.  F.  I.
      63   1   9
       3
     ---
     190
      12
     ---
    2289 inches.
    2289
   -----
    20601
   18312
   4578
  4578
  -------
144)5239521(36385.
 Rem. 81    4042  7  81
            yds. ft. in.
```

In this and ſuch like queſtions, the beſt method of calculation is to reduce the given number, or numbers, to the loweſt denomination in either, and then multiply according to the rule; the product is the area in ſquares of that denomination, which may be brought to a higher denomination by the Tables.

More EXAMPLES *of* SQUARES.

1. What is the area of a ſquare, each of its ſides being 12 feet?

2. What is the area of a ſquare whoſe ſide is 16 feet 10 inches?

Anſ. 283 ſquare feet 52 inches.

3. Re-

3. Required the area of a ſquare, its ſide being 12 yards 2 feet 7 inches?

Anſ. 165 ſquare yards, 3 feet, 97 inches.

4. Required the area of a ſquare pavement, each ſide of it being 21 feet, 9 inches, and 10 lines?

Anſ. 476 ſquare feet $12\frac{25}{100}$ inches.

Of Rectangles.

1. The length of a graſs-plot is 75 yards 2 feet, and its breadth 49 yards $1\frac{1}{2}$ feet; what is its area?

Anſ. 3 roods, 3 poles, 25 yards.

```
Y.  F.      Y.  F.
75  2       49  1½          148.5
 3           3                227
----        -----           ------
227         148.5           10395
                            2970
                           2970
                          ---------
                          9)33709.5 ſqr. feet
                          ---------        40
                   30.25) 3745.9      (123
                                    --------
                   Remr.   25.15     3  3  25
```

2. The floor of a room is 25 feet 9 inches long, and 18 feet 11 inches wide; what is its area?

Anſ. 487. feet 15.

3. The circumference of a room is 90 feet 11 inches, and its height 9 feet 8 inches; what is the area of its walls in ſquare yards? Multiply the circumference by the height.

Anſ. 97 yards, 5 feet, 124 inches.

4. The length of a dry ſtone dyke is 5280 feet 9 inches, and its height 4 feet 7 inches; how many roods

 of

of mason-work are in it? and, what does the building come to, at 10s. 7½d. *per* rood?

Ans. 74 roods, 25 yards, &c. and the price comes to L. 39 : 13 : 8½.

The following questions occur frequently in business:

1. What length of a deal, whose breadth is 9¼ inches, will make a square foot?

Divide 144 by 9¼, the quotient 15.56 inches, is the answer.

2. The length of a pavement is 47 feet 9 inches, and its breadth 13 feet 11 inches; how many stones, each 17 inches long, and 10½ inches broad, will serve to pave it? and, what will the expence amount to, at 7¾d. *per* square foot?

Find the area of the pavement, as also of one stone in inches, and divide the former by the latter; the quotient is the number of stones.

Ans. The number of stones is 536; and the whole expence comes to L. 21 : 9 : 2.

3. The length of a house is 50 feet 9 inches, and its breadth 23 feet 5 inches, within the walls; how many deals, each 18 feet 10 inches long, and 9¼ inches wide, will floor it? *Ans.* $81\frac{3597}{4181}$ or 82 deals nearly.

4. Suppose the length of a house is 40 feet 9 inches, and the sloping height of the roof above the walls 19 feet 5 inches; how many slates will cover the roof, supposing each slate to cover 17½ square inches?

Ans. $13021\frac{13}{35}$.

Of

Of Oblique Angled PARALLELOGRAMS.

2. *To measure any oblique angled parallelogram, and compute its area.*

Measure one side, and the perpendicular falling from the opposite angle upon that side, and then multiply the one by the other, the product is the area.

EXAM. 1. What is the area of an oblique angled parallelogram, its base being 49 feet 3 inches, and the perpendicular falling from the opposite angle 36 feet 6 inches; also its acute angle, being 70°?

Ans. 199 yards, 6 feet, 90 inches.

2. How many square yards are there in an oblique angled parallelogram, its base being 37 feet 10 inches, and the perpendicular height 28 feet 9 inches; also its acute angle being 63° 30′?

Ans. 120 yards, 7 feet, 102 inches.

OF TRIANGLES.

3. *To measure any triangular plain surface, and compute its area.*

Measure one side for a base, and also the perpendicular which falls from the opposite angle upon that base; and then multiply the base by half of the perpendicular; or, multiply the perpendicular by half of the base; the product is the area.

EXAM. 1. What is the area of the triangle ABC, (*Fig.* 31.) its base BC being 49 feet 6 inches, and the perpendicular AD 42 feet 9 inches; also the segment BD=36 feet?

Ans. 117 square yards, 5 feet 9 inches.

2. What is the area of the upper part of the gavel of a house, the breadth of the house being 23 feet 9 inches, and

and the perpendicular height of the gavel above the side-walls being 14 feet 6 inches?

Ans. 19 square yards, 1 foot, 27 inches.

4. To find the area of a plain triangle, its three sides being given.

Add the three sides, and take half of the sum; from the half sum subtract each side; then multiply the half sum by the three remainders continually, and extract the square root of the product; the root is the area of the triangle.

Demonstration, Fig. 31.

Put b=AB, a=BC, and c=AC. From A let fall the perpendicular AD, and put x=BD, then $a-x$=CD.

$$b^2-x^2=c^2-\overline{a-x}^2=\overline{AD}^2$$

that is $b^2-x^2=c^2-a^2+2ax-x^2$

$$2ax=b^2-c^2+a^2 \text{ and } x=\frac{b^2-c^2+a^2}{2a}.$$ Substitute the square of this for x^2 in the first step, and we have $$b^2-\frac{b^4+2b^2c^2-2b^2a^2-c^4+2a^2c^2-a^4}{4a^2}=AD^2$$

that is $$\frac{2a^2b^2+2b^2c^2+2a^2c^2-b^4-c^4-a^4}{4a^2}=AD^2$$

therefore $$\frac{\sqrt{2a^2b^2+2b^2c^2+2a^2c^2-b^4-c^4-a^4}}{2a}=AD$$

This multiplied by $\frac{a}{2}$, gives the area of the triangle, viz.

$$\frac{\sqrt{2a^2b^2+2b^2c^2+2a^2c^2-b^4-c^4-a^4}}{4}=$$

$$\sqrt{\frac{a+b+c}{2}\times\frac{a+b-c}{2}\times\frac{a+c-b}{2}\times\frac{b+c-a}{2}}$$

which is the words of the above rule expressed in symbols.

EXAM.

EXAM. Let the three sides of a triangle be 186, 276, and 146 feet; required the area?

186+276+146=608, whose half is 304.

From 304 subtract each side, the three remainders are 118, 28, and 158.

304×118×28×158=158697728, whose square root is 12597.52 square feet, the answer.

By logarithms. Add the logarithms of the half sum, and three differences together, and take half of the sum; the number answering to this is the area of the triangle.

	Numb.	*Log.*
	304	2.4828736
	118	2.0718820
	28	1.4471580
	158	2.1986571
		2)8.2005707
Ans.	12597.52	4.1002853

2. What is the area of a triangular field whose three sides are 1900, 1620, and 1230 links of the Scots Surveying chain?

Ans. 987540 square links.

3. Required the area of a triangular field, its three sides being 136, 247, and 296 yards in English land measure?

Ans. 16673 square yards, or 3 acres, 1 rood, 31 poles.

OF TRAPEZIUMS.

A trapezium is a quadrilateral figure, whose opposite sides and angles are unequal.

When two opposite sides of a trapezium are parallel, as ABCF, (*Fig.* 57.) its area may be found thus: Multiply

ply the half-sum of its parallel sides by the perpendicular distance between them, the product is the area.

Exam. Let AF=42 yards, BC=34 yards 2 feet, and CD=30 yards, 1 foot, 4 inches; what is the area of the trapezium ABCF?

Ans. 1167 yards, 48 inches.

5. To find the area of any trapezium.

Divide it into two triangles by a diagonal line, to which, draw perpendiculars from the opposite angles; measure the diagonal and the two perpendiculars, and multiply the diagonal, by the sum of the perpendiculars, and half of the product is the area of the trapezium.

Note. This rule is short and easy, and when it can be applied, is the best for practice.

Exam. *Fig.* 58. Let the diagonal DB=236 feet, the perpendicular Ce=137 feet, and the perpendicular Af=95 feet; what is the area of the trapezium ABCD?

Ans. 27376 square feet.

Exam. 2. Let the diagonal DB=349 feet 9 inches; the perpendicular Ce=152 feet 6 inches, and the perpendicular Af=112 feet 2 inches; also Be=94 feet, and Bf=192 feet. Required the area of the trapezium ABCD? *Fig.* 58.

F.	*I.*	*F.*	*I.*
152	6	112	2
12		12	
1830		1346	
1346			
2)3176			
1588			

F. I.

F.	I.	
349	9	*Ans.* 5142 yds. 5 ft. 84 in.
12		

4197
1588

33576
33576
20985
4197

144)6664836 (46283 9 F. 84
Remr. 84

5142 5 84
yd. ft. in.

2. What is the area of a trapezium, whose diagonal is 119 yards, 2 feet, 6 inches; and the perpendicular, 50 yards, distant from one end of the diagonal, is 62 yards; also the perpendicular, 36 yards distant from the other end of the diagonal, is 84 yards, 2 feet, 3 inches?

Ans. 8792 yards, 6 feet, 135 inches.

Or, the area of any trapezium may be found thus: Measure the four sides and the diagonal, and find the area of each triangle from its three sides; the sum of these is the area of the trapezium.

Exam. What is the area of a trapezium, whose sides are as follow, viz. AB=46, BC=37, CD=79, and DA=63 feet; also the diagonal AC=73 feet? *Fig.* 58.

Ans. 2823.52 square feet.

6. To find the area of an irregular polygon, or multilateral figure. Draw diagonals, and let fall perpendiculars on them from the opposite angles. Measure the diagonals and perpendiculars, and by these find the area of each triangle; the sum of these areas is the answer. Or,

Measure all the sides and diagonals, and find the area of each triangle from its three sides; the sum of these is the answer.

This is the most proper method in surveying of land.

Exam. *Fig.* 59. Let the diagonal AC=245 feet, AD=208 feet, the perpendicular E*f*=110 feet, D*g*=130 feet, and B*h*=97 feet; what is the area of the irregular polygon ABCDE?

Ans. 39247.5 square feet.

2. Let AB=174, BC=150, CD=152, DE=148, and EA=160 feet; also the diagonals AC=246, and AD=220 feet; it is required to find the area of the irregular polygon ABCDE?

Ans. 41234 square feet.

7. To find the area of any regular polygon, such as a heptagon, hexagon, &c. Every regular polygon is equal to a plain triangle, whose base is equal to the sum of the sides of the polygon, and height equal to the perpendicular falling from the centre on any of its sides. Therefore, measure one of its sides, and the perpendicular falling from the centre on that side; and multiply half the sum of the sides by the perpendicular, the product is the area of the polygon.

Exam. *Fig.* 60. Let the side of the hexagon AB=329 feet, and the perpendicular falling from the center on that side CD=284 feet; what is its area?

Ans. 280308 square feet.

2. What is the area of a pentagon, each of its sides being 45 feet 9 inches, and the perpendicular falling from the center on any side=31 feet 8 inches?

Ans. 3621.875 square feet.

3. What

3. What is the area of an octagon, whose side is 13 yards, 1 foot, 3 inches; and the perpendicular falling from the center on any of its sides is 16 yards 6 inches?

Ans. 867 square yards, 5 feet, 72 inches.

OF THE CIRCLE, *Fig.* 61.

In all questions concerning the circle, it is considered as a regular polygon of a great number of sides, which, taken altogether, are equal to the circumference.

The length of a right line, nearly equal to the circumference of a circle, whose diameter is known, is found in this manner: A regular polygon of a great number of sides is supposed to be inscribed within the circle, and another of as many sides described about it. The circumference of the circle lying between both, is greater than the one, and less than the other; but the difference is very small; and therefore, the sum of the sides of each polygon being calculated, the arithmetical mean between them is taken for the true circumference.

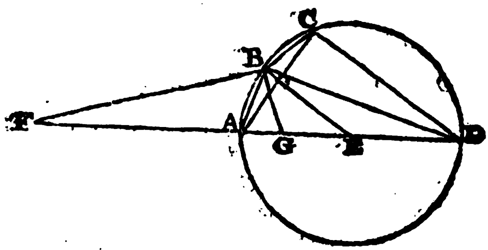
By this method, it has been found, that the diameter being 1, the circumference will be 3.1415926535, or 3.1416, and its area .7853981633, or .7854.

To exemplify this in the plainest manner.

DEFINITION. If any point B be taken in the circumference of a circle ACD, and from that point straight lines BA, BD, be drawn to the extremities of the diameter; then AB is the chord of the arch AB, and BD is the supplemental chord.

The diameter of a circle being given to find the circumference. Let AD be the diameter, E the center, and AB, BC two equal arches: join AB, BC and CD;

produce DA to F, ſo that AF=DC, and draw the ſtraight lines FB, BD: then DB is a mean proportional between ED and DF.



Demonstration. The triangle ABF is equal to the triangle CBD, for AF is equal to CD, AB to BC, and the angle FAB is equal to the angle BCD; the angle CDB is equal to the angle at F, and the angle CBD to the angle ABF. But the angle CDB is equal to the angle BDE or EBD, and therefore the triangle EDB is equiangular and ſimilar to the triangle BDF. therefore as ED : DB :: DB : DF=AD+DB, then $\overline{DB}^2 = ED \times DF$ or $DB^2 = \overline{ED \times AD + DB}$.

If the radius ED = 1, then $\overline{DB}^2 = AD + DB$, and $DB = \sqrt{\overline{AD+DB}}$.

If DG be taken equal to DC; join BG, and the triangles EAB, BAG are ſimilar; and EA : AB :: AB : AG=AD−DG.

Hence the chord of any arch is a mean proportional between the radius and the difference between the diameter and the ſupplemental chord of double the arch.

If the diameter AD = 2, the radius EA = 1 then $\sqrt{\overline{2+CD}}=BD$ the ſupplemental chord of half the arch AC: alſo $\sqrt{\overline{2-CD}} = AB$ the chord of half the arch AC.

To

To find the circumference of a circle, its diameter being 2.

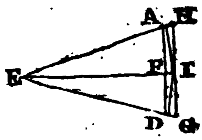
If AC be the ſide of a hexagon inſcribed in the circle; then AC = the radius = 1, and the ſupplemental chord CD $=\sqrt{3}$; for CD $=\sqrt{DA^2-AC^2}=\sqrt{4-1}=\sqrt{3}=a$: alſo the ſupplemental chord of the arch AB, the 12th part of the circumference $=\sqrt{2+\sqrt{3}}=b$. And by calculating the ſupplemental chord of the half of every ſucceeding arch, we ſhall have one as ſmall as we pleaſe, and from thence the chord of this ſmall arch, which will be the ſide of the polygon of as many ſides as the denominator of the fraction contains units; then multiply this ſide by the number of ſides, the product ſhall be the perimeter of the polygon, which is nearly the circumference of the circle.

The ſteps of the operation are in the following Table:

Parts of the CIRCUMFERENCE.		SUPPLEMENTAL CHORDS.
$\frac{1}{6}$	$\sqrt{3}$	$1.73205080757 \text{68}=a$
$\frac{1}{12}$	$\sqrt{2+a}$	$1.931851652557=b$
$\frac{1}{24}$	$\sqrt{2+b}$	$1.982889722742=c$
$\frac{1}{48}$	$\sqrt{2+c}$	$1.995717846472=d$
$\frac{1}{96}$	$\sqrt{2+d}$	$1.998929174951=e$
$\frac{1}{192}$	$\sqrt{2+e}$	$1.999732275818=f$
$\frac{1}{384}$	$\sqrt{2+f}$	$1.999933067834=g$
$\frac{1}{768}$	$\sqrt{2+g}$	$1.999983266888=h$
$\frac{1}{1536}$	$\sqrt{2+h}$	$1.999995816717=k$

Add 2 to the ſupplemental chord k and ſubtract the ſum 3.999995816717 from 4, the ſquare of the diameter, and there remains .000004183283, which is the ſquare of the ſide of a polygon of 3072 ſides inſcribed in

in the circle, and the square root of .000004183283 is .0020453071 the side of the interior polygon.



To find the side of a polygon of 3072 sides circumscribed about the circle. Let AID be the arch; EI the radius, AD the side of the inscribed polygon, and HG the side of the circumscribed polygon. The perpendicular $EF = \sqrt{EA^2 - AF^2}$

From $EA^2 =$ 1.

Subtr $AF^2 =$ .0000010458207

$FE^2 =$ .9999989541793 then EF = .999999477

As EF : EI :: AD : HG = the side of the exterior polygon; that is, .999999477 : 1 :: .0020453071 : .0020453082 = HG.

Interior side	.0020453071
N° of sides	3072
	40906142
	143171497
	61359213 0
perim. of the inter.	6.28318834112
—— of the exter.	6.28318867904
	2)12.5663702016
	6.2831851 = the circumf. of the circle, the diameter being 2; true to the 7th decimal place.

Exterior side	.0020453082
N° of sides	3072
	40906164
	143171574
	61359246 0
perim. of the exterior polygon.	6.28318867904

Hence

Hence the diameter of a circle being 1, the circumference is 3.1415925, or 3.1416 nearly; and its area, .7854.

8. The diameter of a circle being given, to find its circumference.

Because the circumferences of circles are to each other as their diameters, and the circumference of a circle, whose diameter is 1, is 3.1416; therefore, as 1 is to 3.1416, so is the diameter of any circle to its circumference. And, because the first term of the proportion is 1, therefore, multiply 3.1416 by the given diameter, the product is the circumference.

EXAM. 1. What is the circumference of a circle, whose diameter is $19\frac{1}{2}$ inches?

Ans. 61.2612 inches.

2. The diameter of the earth is 7964 miles; what is the circumference of one of its great circles?

Ans. 25019.7 miles.

9. The diameter of a circle being given, to find its area.

A circle is equal to a plain triangle, whose base is equal to the circumference, and its perpendicular height equal to the radius; but similar triangles are to each other as the squares of their correspondent sides; therefore, circles are to each other as the squares of their *radii*; or, (because the radius is half of the diameter), as the squares of their diameters. And, since the area of a circle, whose diameter is 1, is .7854; as the square of 1 is to the square of any diameter so is .7854 to the area; and, because the first term of the proportion is 1,

Multiply the square of any diameter by .7854, the product is the area of the circle.

EXAM.

Exam. 1. What is the area of a circle whose diameter is 12 inches?

Anf. 113.0976 square inches.

2. What is the area of a circle whose diameter is 7964 miles?

Anf. 49814227.4784 square miles.

There are other numbers expressing the proportion of the diameter of a circle to its circumference. *Archimedes* found, that the diameter being 7, the circumference would be 22; and therefore, as 7 is to 22, so is the diameter of any circle to the circumference; and, if the area of the circumscribed square be 14, the area of the circle would be 11; and therefore, as 14 is to 11, so is the square of any diameter to the area of the circle. But these numbers are not very accurate.

Metius found, that the diameter being 113, the circumference would be 355; therefore, as 113 is to 355, so is any diameter to the circumference; which is nearer the truth than 7 to 22.

10. To find the length of any arc of a circle, the number of degrees in that arc, and the radius of the circle being given.

Multiply the number of degrees in the given arc, the radius of the circle, and the number .01745329 continually; the product is the length of the arc.

Exam. Required the length of an arc of a circle of 121° 26′, the radius of the circle being 25.455?

Anf. 53.9496, &c.

Note. .01745329 is the length of an arc of 1 degree, the radius being 1.

11. The area of a circle being given, to find the diameter and circumference.

Divide

Divide the given area by .7854; the square root of the quotient is the diameter; which, being multiplied by 3.1416, the product is the circumference.

EXAM. 1. What is the diameter and circumference of a circle whose area is 10000 square feet?

Ans. Diameter 112.8 feet. Circumference 354.372 feet.

2. What length of a cord tied to a cow's tail, the other end being fixed in the ground, will allow her to eat a Scots acre of grass; the length of the cow and tail being $3\frac{1}{2}$ yards?

Ans. 39.3 yards.

Note. A Scots acre of land is 5760 square yards.

12. The circumference of a circle being given, to find its area.

Multiply the square of the circumference by .0795775, the product is the area.

EXAM. 1. What is the area of a circle, whose circumference is 54 inches?

Ans. 232.04799 square inches.

2. What is the area of a circle whose circumference is $42\frac{1}{2}$ inches?

Ans. 143.7368, &c.

To bring all the problems about the Circle into one view.

Put $d=$ the diameter, $p=$ the circumference, $A=$ the area: and let $a=3.1416$. $b=\frac{a}{4}=.7854$

1. $1:a::d:p$ or $ad=p$.
2. $1:b::d^2:A$ or $bd^2=A$.
3. $d=\frac{p}{a}$ or $d=.3183p$ because $\frac{1}{3.1416}=.3183$.

4. $a^2d^2=p^2$, or $d^2=\frac{p^2}{a^2}$, that is $d^2=.10132\,p^2$.

for $\frac{1}{9.86965}=.10132$.

5. $d^2=\frac{A}{b}=1.2732$ A for $\frac{1}{.7854}=1.2732$.

6. $ad^2=\frac{aA}{b}=\frac{p^2}{a}$ then $\frac{a^2A}{b}=p^2$, and by

substituting $\frac{a}{4}$ for b, we have

7. $\frac{4a^2A}{a}=p^2$ that is $4aA=p^2$; therefore

8. $A=\frac{p^2}{4a}$ that is $A=\frac{p^2}{12.5664}$ or $.0795775\,p^2=A$.

9. $d=\sqrt{\frac{A}{b}}=\sqrt{1.2732\,A}$.

10. $p=\sqrt{4aA}$ or $p=\sqrt{\frac{A}{.0795775}}$

13. To find the area of any sector of a circle; the length of the arc and the radius of the circle being given.

Multiply the radius by half the length of the arc, the product is the area of the sector.

Exam. *Fig.* 61. It is required to find the area of the sector BCR, the length of the arc BC being 52.994555, and the radius RC=25.455?

Ans. 674.488, &c.

14. To find the area of the segment of a circle less than a semicircle, its base and height being given. *Fig.* 61.

Let CPO be the segment, whose area is to be found.

1. Divide the square of CI by OI, the quotient is IB, which, added to OI, gives BO, the diameter.

2. Having found the diameter, find the circumference of the circle by problem 9. p. 127.

3. In

3. In the right angled triangle CRI, all the sides are known, from which the angle CRI may be found, whose double is the angle CRP, or the arc COP.

4. As 360° is to the arch COP in degrees, so is the circumference of the circle to the length of the arch COP.

5. Multiply the radius RO by half of the arch COP, the product is the area of the sector CRPO.

6. Find the area of the triangle CPR, and subtract it from the area of the sector CRPO; the remainder is the area of the segment COP; but when the area of a segment greater than a semicircle is required, such as, CBP. To the area of the sector RCBP, add the area of the triangle CPR, the sum is the area of the segment CBP.

Exam. Let the chord or base of the segment CP= 44.4, and the versed sine or height OI=13; what is the area of the segment CPO? *Ans.* 410, &c.

Otherwise put r=the radius RO. c=half the chord =CI and d=RO—OI: and the area of the segment COP will be $\frac{2\frac{1}{3}rr-1\frac{1}{3}rd-dd}{1\frac{1}{2}r+d}\times C$

And the last example wrought by this rule comes out 409.49 square inches.

2. It is required to find the area of a segment of a circle, its base being 25.455, and its height 5.28?

Ans. 92.469.

15. To find the area of any zone of a circle included between two parallel chords; the diameter of the circle, the including chords, and the breadth of the zone being given. *Fig.* 61.

Case 1. When the including chords are equally distant from the center, as CP and EF.

 Since

Since IG, the breadth of the zone, is given, RI will be known; which being ſubtracted from the ſemidiameter of the circle, the height of the ſegment cut off, OI will remain: Therefore, find the area of the whole circle, and alſo the area of the ſegment COP: Subtract twice the area of the ſegment COP from the area of the circle, and there will remain the area of the zone CEFP.

CASE 2. When the zone is included by a diameter and any chord, as AD, CP.

Find the area of the ſegment COP, and ſubtract it from the area of the ſemicircle, the remainder is the area of the zone ACPD.

CASE 3. When the including chords are unequally diſtant from the center, and both on the ſame ſide of it.

In this caſe, the diſtance of each of the including chords from the center of the circle muſt be known; and then having found the area of each of the ſegments cut off by the chords, the difference of theſe areas is the area of the zone.

CASE 4. When the including chords are unequal, and on different ſides of the center.

The diſtance of each chord from the center muſt be known; and in this caſe compute the area of each of the ſegments cut off, and ſubtract their ſum from the area of the whole circle, the remainder is the area of the zone.

The ſame rules will ſerve for computing the area of any part of a circle included between chord lines which are not parallel; the diſtance of each chord from the center being known.

EXAM. 1. Let the diameter of a circle be 16 inches; it is required to find the area of its middle zone, includ-

ed

ed between two parallel chords equally distant from the center, each chord being 12.49 inches, and the breadth of the zone 10 inches?

Ans. 148.867 square inches.

2. Let the diameter of a circle be 25, it is required to find the area of a zone of it contained between two parallel, but unequal chords, lying on different sides of the center; the greatest chord being 20, and its distance from the center $7\frac{1}{2}$; and the least chord 15, and its distance from the center 10?

Ans. 395.4369.

16. The diameter of a circle being given, to find the diameter of another circle, such, that the area of the given circle shall be to the area of the other in a given ratio, suppose of 2 to 3.

Because the areas of circles are to each other as the squares of their diameters; therefore, as 2 is to 3, so is the square of the given diameter to the square of the diameter of the circle required.

Exam. Let the diameter of a circle be $19\frac{1}{2}$ inches; it is required to find the diameter of another circle, whose area shall be to the area of the first circle in the ratio of 3 to 2?

Ans. 23.882 inches.

2. The area of a circle is 50 square feet; it is required to find the diameter of another circle, whose area shall be to the area of the given circle, as $3\frac{1}{2}$ to 5?

Ans. The diameter is 6.675 feet.

Of the ELLIPSE.

Definition 1. An ellipse is a plain figure bounded by a curve line, like a circle; but has different properties. *Fig.* 24.

2. There is a point in an ellipse called its center, through which its diameters pass; but every part of its circumference is not equally distant from this point, as in the circle.

3. All the diameters of a circle are equal to each other, but the diameters of an ellipse are unequal.

4. The ellipse hath two principal diameters, called axes, which cut each other at right angles in its center.

5. The longest diameter AB is called the transverse axis, and the shortest diameter GH is called the conjugate axis.

17. The two axes of an ellipse being given, to find its circumference.

1. Take half the sum of the two axes, and multiply it by 3.1416, the product is the circumference nearly. Generally too little.

2. Square the two axes, and multiply the square root of half their sum by 3.1416, and the product is the circumference nearly. Generally too much.

3. Take the half sum of these two products, it is the circumference, very near the truth.

Exam. Let the transverse axis AB=31½ inches, and the conjugate axis GH=23¾ inches; what is the circumference of the ellipse?

Ans. 87.2114 inches.

AB

AB=31.5
GH=23.75

2) 55.25

Half sum of the two axes=27.625
then 27.625 × 3.1416=86.7867 the 1st product.
The square of AB = 992.25
the square of GH = 564.0625

Sum of the squares of AB and GH = 1556.3125

half ditto 778.15625

and the square root of 778.15625 is 27.8954
now 27.8954 × 3.1416=87.63618864=the 2d product
to which add the 1st product 86.7867

2) 174.4228

Ans. The circumference is 87.2114 inches.

6. The area of any ellipse, is equal to the area of a circle, whose diameter is a mean proportional between the two axes of the ellipse. Or,

The rectangle, or product of the two axes of any ellipse is equal to the square of the diameter of a circle, whose area is equal to the area of the ellipse.

18. To find the area of an ellipse, its two axes being given. *Fig.* 24.

Multiply the longer axis by the shorter, and the product by .7854; the last product is the area of the ellipse?

EXAM. Let the longest axis AB=31½ inches, and the shortest GH=23¾ inches; what is the area of the ellipse?

Ans. 587.577 square inches.

SUR-

SURVEYING OF LAND.

Small parts of the earth's ſurface are eſtimated in acres, and parts of an acre.

A right angled parallelogram, whoſe length is 10 chains, and breadth 1 chain, or any ſpace containing 10 ſquare chains, or 100,000 ſquare links, is an acre.

TABLE *of Scots Land Meaſure.*

Sq. Links.		Sq. Feet.		Sq. Ells.						
$17\frac{13}{36}$	=	$9\frac{73}{144}$	=	1						
625	=	$342\frac{1}{4}$	=	36	=	1 Sq. Fall.				
25000	=	13690	=	1440	=	40	=	1 Sq. Rood.		
100000	=	54760	=	5760	=	160	=	4	=	1 Sq. Acre.

N. B. In this table the Scots ell is 37 inches, and the feet are Engliſh.

The ſquare ell expreſſed in ſquare links and decimal parts is 17.36111, or 9.50694 Engliſh ſquare feet.

TABLE *of Engliſh Land Meaſure.*

Sq. Links.		Sq. Feet.		Sq. Yards.						
$20\frac{80}{121}$	=	9	=	1						
625	=	$272\frac{1}{4}$	=	$30\frac{1}{4}$	=	1 Sq. Pole.				
25000	=	10890	=	1210	=	40	=	1 Sq. Rood.		
100000	=	43560	=	4840	=	160	=	4	=	1 Sq. Acre.

The Engliſh ſquare yard expreſſed in ſquare links, and decimals of a link, is 20.661157 ſquare links of the Engliſh chain.

When the dimenſions of a field are taken with the ſurveying chain, they are commonly expreſſed in links; and the area computed from theſe dimenſions is ſquare links.

To reduce ſquare links to acres, roods, &c. If it be Scots meaſure, divide the ſquare links by 100000, which may

may be done by pointing off five figures to the right hand. The figures on the left of the point are acres, and those on the right are decimals of an acre. Multiply these decimals by 4, 40, and 36; and, from each product, point off five figures to the right hand, and you have the roods, falls, and ells.

If it be English measure, divide the square links by 100000 as before, and multiply the remaining decimals by 4, 40, and $30\frac{1}{4}$; and from each product point off five decimals, and you have the area in English acres, roods, poles or perches, and yards.

EXAMPLES.

1. To find the area of a rectangular field, bounded by four straight lines. *Fig.* 56

Measure its length and breadth, and multiply the one by the other.

What is the area of a rectangular field whose length is 2496 links, and breadth 1689 links of the Scots chain?

$2496 \times 1689 = 4215744$ square links.

				Ac.	*R.*	*F*	*E.*
Acres	42.15744		*Ans.*	42	0	25	6
	4						
Roods	0.62976						
	40						
Falls	25.19040						
	36						
	114240						
	57120						
Ells	6.85440						

EXAM. 2. What is the area of a rectangular field, whose length is 2798 links, and breadth 1894 links of the English chain?

2798 × 1894 = 5299412 square links.

			Ac.	R.	P.	Y.
Acres	52.99412	*Anſ.*	52	3	39	1
	4					
Roods	3.97648					
	40					
Poles	39.05920					
	$30\frac{1}{4}$					
	177600					
	1480					
Yards	1.79080					

3. How many acres are there in a square field whose side is 1976 links of the Scots surveying chain?

Anſ. 39 acres 7 falls.

4. How many acres are there in a square field whose side is 2675 links of the English chain?

Anſ. 71 acres, 2 roods, 9 poles.

5. What is the area of a rectangular field, the length being 1868 links, and the breadth 1496 links of the Scots chain?

Anſ. 27 acres, 3 roods, 31 falls.

6. What is the area of a rectangular field, its length 2867 links, and its breadth 1984 links of the English chain?

Anſ. 56 acres, 3 roods, 21 poles.

7. How

7. How many English acres are there in a rectangular field, its length being 2468 links, and breadth 1426 links of the Scots chain?

Ans. 35 acres, 30 poles, 29 yards.

8. How many acres are there in a square mile?

Ans. 640 acres.

For 80 chains, or 8000 links is a mile, and the square of 8000, is 64000000 square links, which is 640 acres.

The English acre is to the Scots acre as 1089 to 1369; or 1369 English acres are equal to 1089 Scots acres.

1. To reduce Scots land measure to English, and the contrary.

Because the English chain is 66 feet, and the Scots 74; as 66 is to 74, so is any number of Scots links to the number of English links, in the same line or length; and, as 74 is to 66, so is any number of English links to the number of Scots links, in the same line.

EXAM. 1. How many links of the English chain are equal to 2496 links of the Scots chain?

66 : 74 : : 2496 : 2798 the answer.

2. How many links of the Scots chain are equal to 1894 links of the English chain?

74 : 66 : : 1894 : 1689 the answer.

3. To reduce Scots acres to English acres; as 1089 is to 1369, so is any number of Scots acres to their equivalent English acres; and, as 1369 is to 1089, so is any number of English acres to their equivalent Scots acres.

EXAM. 1. Reduce 42.15744 Scots acres to English acres.

1089 : 1369 : : 42.15744 : 52.99681 the answer; or, 52 acres, 3 roods, 39 poles, &c.

2. Reduce 500 English acres to Scots acres.

1369 : 1089 :: 500 : 397.73559 the answer;

or, 397 acres, 2 roods, 37 falls, 25 ells, nearly.

3. To reduce lineal links of the Scots chain to feet. As 100 is to 74, so is any number of links to the feet equal to them.

And to reduce links of the English chain to feet; as 100 is to 66, so is any number of links to the feet.

Lineal feet may be reduced to links of either kind, by inverting the proportions.

4. To reduce square feet to square links, Scots measure. 2500 square links are equal to 1369 square feet; therefore, as 1369 is to 2500, so is any number of square feet to square links.

In English measure, 2500 square links are equal to 1089 square feet; therefore, as 1089 is to 2500, so is any number of square feet to the square links.

5. When the dimensions of a field are taken in feet, and the area computed in square feet; to reduce them to Scots acres. Divide the square feet by 9.50694, or by 9.5, the quotient is square ells; divide the ells by 36, the quotient is falls; divide the falls by 40, the quotient is roods; and divide the roods by 4, the quotient is acres.

6. To reduce square feet to English acres. Divide them by 9, $30\frac{1}{4}$, 40, and 4, the quotients are square yards, poles, roods, and acres.

Exam. What is the area of a rectangular space of ground, whose length is 596 feet, and breadth 156 feet, in Scots and English acres?

596 × 156 = 92976 square feet.

9.50694)92976.00000(9779 square ells.

36)9779(271 falls. 40(271

remainder 23.

4)6 31

	Ac.	R.	F.	E.				
Ans.	1	2	31	23 Scots.		1	2	31

9)92976

30.25)10330 6

40)341 14

4)8 21

2 0 21 14

	Ac.	R.	P.	Y.	
Ans.	2	0	21	14	*Eng.*

2. How many Scots acres are there in a square field, each of its sides being 716 feet?

Ans. 9 acres, 1 rood, 18 falls, 35 ells.

3. How many English acres are there in a square field, its side being 596 feet?

Ans. 8 acres, 24 poles.

4. What is the area of a rectangular field, its length being 598 feet, and breadth 386 feet, in Scots and English acres?

Ans. 4 acres, 34 falls, 33 ells, Scots; and
5 acres, 1 rood, 7 poles, 25 yards, English.

5. It is required to find the area of a rectangular field in Scots acres; its length being 374 ells, and breadth 275 ells?

Ans. 17 acres, 3 roods, 16 falls, 34 ells.

6. *Fig.* 57. What is the area of a field in form of an oblique angled parallelogram, whose longest side is 2368 links, and the perpendicular falling from the opposite angle

angle upon that side 1648 links of the Scots chain; the acute angle of the parallelogram being 63° 30′?

Ans. 39 acres, 0 roods, 3 falls, 33 ells.

7. *Fig* 31. What is the area of a triangular field, whose base is 2896 links, and the perpendicular falling upon it from the opposite angle 1896 links of the English chain? See the rule, p. 117.

Ans. 27 acres, 1 rood, 32 poles.

8. What is the area of a triangular field whose base is 1768 links, and the perpendicular distant 795 links from the nearest extremity of the base, is 1674 links of the English chain?

Ans. 14 acres, 3 roods, 7 poles.

9. What is the area of a triangular field, whose three sides are 2464, 1968, and 1764 links of the Scots chain?

Ans. 17 acres, 33 falls. See page 118.

10. A farm of a triangular form, has its three sides $2\frac{1}{2}$ miles, 3 miles, and $1\frac{1}{2}$ miles; how many acres does it contain? *Ans.* $1197\frac{1}{3}$ acres, nearly.

2.5	3.5	3.5	3.5
3.	2.5	3.	1.5
1.5	1.	.5	2.
2)7.0			
3.5			

3.5 × 2 × .5 = 3.5, Extract the square root,

then $\sqrt{3.5} = 1.870826$ square miles

multiply by 640

74833040
11224956

1197.328640 Acres.

11. The

11. The kingdom of England, and the principality of Wales together, are of a triangular form, the south coast being the base, and the north point of Northumberland the vertex of the triangle: Now, supposing the length from North to South is 360 miles, and the breadth from East to West is 300 miles: How many acres are in the whole?

Length	360 miles
Multiply by ½ the base	150
area	54000 square miles
	640
	2160000
	324000
Ans.	34,560,000 acres.

12. *Fig.* 58. What is the area of the quadrilateral field ABCD, whose diagonal BD=3096 links, the side AB=1976 links, BC=1760 links, CD=2016 links, and DA=2340 links?

Ans. 39 acres, 3 roods, 5 falls. See p. 120.

13. What is the area of a field of five sides, (*Fig.* 59.) the side AB being 1740, BC=1500, CD=1520, DE=1480, and EA=1600 links; also the diagonal AC=2460, and AD=2200 links?

Ans. 41.23654 square links, or 41 acres 37 poles.

14 To measure a polygonous field, such as ABCDEF, by the chain. *Fig.* 62.

Having walked over and taken a view of the field, draw a random, or eye-draught of it, in which draw proper diagonals.

Measure all the sides and diagonals with the chain, and the whole field is divided into triangles; in each of which

which all the sides are known from, whence their areas may be found, and consequently the area of the whole field.

EXAM. Let AB=2175, BC=1925, CD=1275, DE=1675, EF=1575, FA=1725, AC=3715, FC=4085, and FD=3025 links of the Scots chain; what is the area of the field? *Ans.* 69 acres, 1 rood, 37 falls.

8. To survey a field, by measuring every angle and side. *Fig.* 63.

Begin at some convenient angle, which call your first station, and mark it in your field-book thus, $\odot^{1}$.

Having chosen your first station, place poles in proper places, and measure the angle at this station; which write in your field-book below the former character.

When you begin to measure the sides with the chain, proceed with your left hand to the dyke, or boundary of the field; and, if the line which you measure does not coincide with the side of the field, you must measure the distance between them at proper places; these distances are called *offsets*, and must be entered in your field-book; and over against them, in the proper column, write down the distances from the beginning of the line you are then measuring.

In the same manner, measure every angle and side, until you have gone round the whole field, and are arrived at your first station.

To know if the angles have been measured truly. From twice the number of sides of the field, subtract 4, and multiply 90° by the remainder. Add together the angles of the field, as they stand in your field-book; and, if the sum be equal to the product, they have been truly measured.

If

If there be any considerable difference, such as a degree or more, it must be corrected by measuring the angles again.

FORM *of the* FIELD-BOOK.

Offsets.		
	95	N. E. corner.
	0^1	
32	84° 50′	
0	400	
	1090	
	0^2	
	85° 15′	
	1339	
	0^3	
	68° 55′	
90	90	
113	130	
65	400	
25	888	Goes on 145, as in next offset.
145	0^4	
	121°	
0	328	
200	450	
232	558	
0	620	
pond.	700	
	956	
	0^1	

To draw a Plan of the Field on Paper. Fig. 63.

1. Draw the right line EAB. Set 95 links from E to A, and make the angle BAC=84° 50′.

Having raised a perpendicular at the point A, on it set the offset 32 links from A to *b*; then set 400 links from E to *t*; and because you find 0 offset in the field-book, the line EB coincides with the boundary of the field from that place; set 1090 links from E to B.

2. At B make the angle ABD=85° 15′, and set 1339 links from B to D.

3. At D make the angle BDC=68° 55′. Set the several numbers of links found in your field-book, between o^3 and o^4, from the point D on the line DC; and from the points *l*, *n*, *m*, raise perpendiculars, on which set the offsets as they stand in your field-book; and set 145 links from C to *f*.

4. Set the several numbers of links in the field-book between o^4 and o^1 from C towards A; and at the points *r* and *s*, raise perpendiculars, on which set the offsets 200 and 232 links. The extremities of the offsets being every where joined, will give the outlines, or boundaries of the field.

To find the area of the Field.

Join AD; and in the triangle ABD, there is given the side AB=995 links, the side BC=1339 links, with the angle ABD=85° 15′; and, by the third case of oblique triangles, the side AD will be 1600 links. Then, in each of the triangles ABD and ADC, all the sides are known; and so their areas may be found.

For

For the areas of the parts where the offsets were taken. Multiply Df by half of mu, the product is the area of the triangles Dfn. In like manner, find the areas of the other triangles fCg, bgr, iKs, EKA, and Atb; and in the quadrilateral figure $hrsi$; multiply the half sum of the offsets rh and si by rs. The sum of the areas of all these different parts is the area of the field, which is near 11 acres 3 roods.

Note 1. In surveying a quadrilateral field, it is not necessary to measure all the angles. If one angle and the four sides are known, the diagonal can be calculated, and from thence the area.

2. Although a field has more than four sides, the greatest part of it may, by taking proper stations, be brought within four straight lines; and the rest measured by taking offsets. Hence,

The measuring of all the angles of a field, although much practised, and generally proposed by writers on this subject, is not necessary in surveying, except in cases where some of the sides are inaccessible.

When a four-sided field has two of its angles at the extremity of any side, equal to two right angles, but the other sides unequal, as in *Fig.* 57. the area may be found thus:

Having measured the sides, multiply the half sum of the two parallel sides by the breadth of the field, the product is the area; that is, $\frac{AF+BC}{2} \times AB$ = the area.

9. To survey any small field by measuring diagonals, and the perpendicular falling from the opposite angles on them. *Fig.* 59.

Having made choice of the most proper diagonals, such as AC and AD, measure the diagonal AC, beginning at the point A; and, when you come to the point *b*, measure the perpendicular *b*B; also, when you come to the point *g*, measure the perpendicular *g*D, marking in your field-book, or on an eye-draught of the field, the lengths of *Ab*, *b*B, *Ag*, *g*D. In like manner, measure AD; and, when you come to the point *f*, measure *f*E, marking down their lengths as before.

To find the area of the field, multiply AC by the half sum of *b*B and *g*D; also multiply AD by half of E*f*, and the sum of these products is the area.

In the same manner, a field of four, or any number of sides, wherein the ground is not incumbered with great ascents or descents, may be accurately measured.

The points *b*, *g*, and *f*, where the perpendiculars meet the diagonals, are to be found by trials; or, more accurately, by a cross, which is a square board, whereon sights are placed at the extremities of two right lines cutting each other at right angles. This being fixed on a staff stuck in the ground, at the place where the perpendicular is supposed to meet the diagonal, by looking through one pair of sights in the direction of the diagonal, the other pair will be in a line perpendicular to it. When this line falls on the angle, the station is right; but, if not, by moving the cross backward or forward, the point where the perpendicular falling from any angle on the diagonal may be accurately found.

This method of measuring fields is both accurate and easy, and therefore, preferable to any other, when it can be applied.

10. When

10. When two sides of a triangular field, AB and AC can be measured; but the third side BC is inaccessible; to find the length of that side, and the area of the field. *Fig.* 70.

Measure AB and AC, and take AD and AE in the same right lines produced; so that AD be to AE, as AB to AC, and measure ED; then, because the two triangles ADE and ABC are similar, as AD : DE :: AB : BC. And the three sides being known, the area of the triangle may be found.

By this method, the diagonal of any four-sided field may be found, without measuring it.

Note. In order to have AD and AE in just proportion to AB and AC, take AD equal to some part of AB, such as $\frac{1}{4}$, $\frac{1}{5}$, or $\frac{1}{10}$; and take AE the same part of AC.

11. To find the perpendicular and area of a triangular field, by measuring only that side of it on which the perpendicular falls. *Fig.* 64.

Let ABC represent a triangular field, and only its side AC accessible; it is required to find the perpendicular BD, and the area of the field.

Measure CA, and mark the point D, where the perpendicular meets it; then take AF in the right line CA produced, of any length, and make the right angle AFE. Measure along the line FE, until you find the point E in the same right line with A and B. Then the triangles AFE and ABD being similar, as AF : FE :: AD : DB the perpendicular; which being multiplied by half of AC, the product is the area of the triangle ABC.

By this method, the perpendicular BD, whether it be that of a triangular field, or a distance of any moderate magnitude,

magnitude, not exceeding 2 or 3 miles, may be accurately investigated by the surveying chain.

12. To survey a triangular field, one of whose sides is inaccessible.

Let ABC, *Fig.* 64. represent a triangular field, whose side BC is inaccessible; measure the two sides AB=769 links, and AC=1178 links, and the angle BAC=37° 45′. Then, supposing a perpendicular to fall from B upon AC, as R : Sine A :: AB : the perpendicular. Multiply AC by half of the perpendicular, the product is the area of the field: which, in this example, is 2.77243, or 2 acres, 3 roods, 3 falls.

Otherwise. Having measured the two sides AB and AC, with the contained angle BAC, work thus:

As radius		10.0000000
Is to the sine of BAC	37° 45	9.7869056
So is the product of AB and AC	{ 769	2.8859263
	{ 1178	3.0711453
To twice the area of the triangle	554596	5.7439772

the half of which is 277298 square links, the area of the triangle.

The following problem may be wrought by this rule.

13. To survey a field, by taking a station within it, from whence all its angles can be seen, and the lines drawn from the station to the angles measured.

Let ABCDE, *Fig.* 65. represent the field to be surveyed. Chuse some convenient station, as at O, from whence you can see all the angles of the field; place poles in A, B, C, D, E, and measure all the angles AOB, BOC, &c. at the point O; and measure all the lines AO, OB, OC, OD, OE; then the field is divided

into

into as many triangles as it hath sides; in each of which, two sides with the contained angle are given, to find the area, which may be done by the last problem; and the sum of all these is the area of the field.

EXAM. Let the angle AOB=78°, BOC=62° 50', COD=79° 20', DOE=77° 30', and EOA=62° 20'; also, let AO=580 links, BO=580, CO=420, DO=395, and EO=512 links; what is the area of the field ABCDE?

Ans. 5 acres, 3 roods, 15 falls.

Note 1. The station may be taken any where, as at one of the angles, if all the other angles of the field can be seen from thence.

2. A field may also be surveyed from two stations; but this method is seldom practised.

14. When the boundary of a field is an irregular curve line, *Fig.* 66. the space between it and the station-line AB must be measured, by taking the offsets *bc*, *de*, and *fg*. To find the area of the space AB*ge*C, multiply A*b* by half of *b*C, and B*f* by half of *fg*, the products are the areas of the triangles A*b*C, B*fg*; then, if *bd*=*df*, add *b*C, *de*, and *fg*; take a third part of the sum, and multiply it by *bf*, the product is the area of *bdfge*C; to which add the areas of the two triangles A*b*C, B*fg*, the sum is the area of the space AB*ge*C If *bd* be not equal to *df*, compute the areas of C*d* and *dg* separately.

15. To reduce Scots or English acres to square links; multiply them by 100000.

EXAM. Reduce 28 acres to square links.

Ans. 2800000.

When acres, and parts of an acre are to be reduced to square links; reduce the parts to decimals of an acre, Write down the acres on the left hand of the decimals, and multiply the whole by 100000, which is done by removing the decimal point five places toward the right hand; the product is square links.

EXAM. 1. Reduce 32 acres, 3 roods, 19 falls, and 30 ells, to square links.

36)30.00000 *Ans.* 3287395.

40)19.83333

4)3.49583

32.87395.

2. Reduce 40 acres, 2 roods, 23 poles, English measure, to square links?

Ans. 4064375.

OF LAYING OUT GROUND.

1. To find the side of a square field, which shall contain any given quantity of land; reduce the given quantity to square links; the square root of these is the side of the square required.

EXAM. 1. What is the length of the side of a square field, which shall contain 30 acres? *Ans.* 1732.

30 acres=3000000 square links.

And the square root of 3000000 is 1732.

2. What

2. What is the length of the side of a square field that shall contain 27 acres, 1 rood, 19 falls?

Ans. 1654 links.

2. One side of a rectangular field, and the quantity of land it must contain being given; to find the other side.

Reduce the given quantity of land to square links, and divide them by the given side, the quotient is the other side.

EXAM. It is required to lay out 29 acres, 3 roods of land, in form of a right angled parallelogram, one of whose sides is 1128 links; what is the length of the other side? *Ans.* 2637.

2. It is required to lay out 31 acres, 2 roods, 15 falls of ground, in form of an oblique angled parallelogram, whose acute angle is 57°, one of its sides being 1672 links; required the perpendicular falling from the opposite angle upon that side?

Ans. 1889 links; found as in the last example.

3. To lay out any quantity of land in form of a triangle; its base being given.

Divide the square links in the given area by one half of the base; the quotient is the perpendicular.

EXAM. It is required to lay out 31 acres, 2 roods of land, in form of a triangle, whose base is 1726 links, and the acute angle at the base 51° 20′; required the perpendicular? *Ans.* 3650.

2. To lay out one acre of land in an equilateral triangle; it is required to find the length of its side?

Ans. 480.56 links.

Put $a = 100{,}000$ square links, the area of the triangle, $x =$ the length of the side; then the triangle being equilateral, the sum of its sides is $3x$, and the half sum $= \frac{3x}{2}$, from which subtracting each side, the three remainders are $\frac{x}{2}$, $\frac{x}{2}$, $\frac{x}{2}$, therefore $\frac{3x}{2} \times \frac{x}{2} \times \frac{x}{2} \times \frac{x}{2} = \frac{3x^4}{16}$ the square of the given area.

$\frac{3x^4}{16} = a^2$ or $3x^4 = 16a^2$, and by extracting the square root of both sides $x^2 \times \sqrt{3} = 4a$.

$$x^2 = \frac{4a}{\sqrt{3}} = \frac{400000}{1.73205} = 230940.21.$$

$$x = \sqrt{230940.21} = 480.56 \text{ links the side.}$$

Hence, Having the area of an equilateral triangle given, to find the length of its side. Multiply the given area by 4; divide the product by the square root of $3. = 1.7320508$, and the square root of the quotient is the length of the side of the triangle.

Exam. It is required to find the side of an equilateral triangle, whose area shall be one square mile, or 640 acres of land? *Ans.* 12157.37 links.

Given area = 64000000 square links
4

1.73205) 256000000 (147801737.82

and the square root of 147801737.82 is 12157.37 links.

Note. For the angles of the polygons mentioned in the following problems, *See* page 13.

4. The area of a regular pantagon being given, to find its side. *Plate* 1. *fig.* 21.

Exam.

Exam. Let the area of the pentagon = 3601 square feet, what is the length of its side AB? *Ans.* 45.74 feet.

The angle of the pentagon FAB is 108°, its half CAG=54°.

Put $a=3601$, $b=$the tang. of $54°=1.3763819$. $x=$AB.

In the triangle CAG: As rad. : tan. CAG : : AG : GC; that is, as $1 : 1.3763819=b : : \frac{x}{2} : \frac{bx}{2}=$ CG = the perpendicular.

$5x=$the sum of the sides, and $\frac{5x}{2}=$the half sum

then $\frac{5x}{2}\times\frac{bx}{2}=a$ or $\frac{5bx^2}{4}=a$

$$5bx^2=4a$$

$$x^2=\frac{4a}{5b}=\frac{14404}{6.88190}=2092.8813$$

And the square root of 2092.8813 is 45.74 feet, the side of the pentagon.

5. The area of a regular hexagon being given, to find its side?

Exam. Let the area of the hexagon $a=100000$ square feet; required the length of its side?

Ans. 196.188 feet.

Let fall a perpendicular from the centre upon one of its sides; the angle of a hexagon is 120° its half is 60°, the tangent of 60° is $1.73205=b$. Put $x=$the side.

as $1 : b : : \frac{x}{2} : \frac{bx}{2}=$the perpendicular.

The half sum of the sides is $3x$, then $3x\times\frac{bx}{2}=a$

$$\frac{3bx^2}{2}=a \text{ and } x^2=\frac{2a}{3b}=\frac{200000}{5.19615}=38490.0358$$

then $x=\sqrt{38490.0358}=196.188$ feet.

 6. The

6. The area of a regular heptagon being given, to find the length of its ſide.

Exam. Let the area of a heptagon be 100,000 ſquare yards: What is the length of its ſide?

Anſ. 165.894 yards.

Half the angle of the heptagon is $64^\circ\ 17'\frac{1}{7}$, of which the tangent is $2.0763215=b$, $a=100000$ and $x=$the ſide.

$$\text{as } 1:b::\frac{x}{2}:\frac{bx}{2}=\text{the perpendicular}$$

$$\frac{7x}{2}\times\frac{bx}{2}=\frac{7bx^2}{4}=a$$

$$x^2=\frac{4a}{7b}$$

$$\text{and } x=\sqrt{\frac{4a}{7b}}=\sqrt{\frac{400000}{14.53425}}=165.894 \text{ yards.}$$

7. The area of a regular octagon being given, to find the length of its ſide.

Half the angle of an octagon is $67^\circ\ 30'$, whoſe tangent is $2.4142136=b$. the given area $=a$, and $x=$ the ſide.

Half the ſum of the ſides of an octagon is $4x$.

Then $4x\times\frac{bx}{2}=2\,bx^2=a$, and $x^2=\frac{a}{2b}$

$$\text{or } x=\sqrt{\frac{a}{2b}}$$

Exam. It is required to lay out 4 acres of land in an octagon: What muſt be the length of its ſide?

Anſ. 287.824 links.

In this example $a=400000$ ſquare links. $2b=4.8284272$

$$x^2=\frac{400000}{4.8284272}=82842.71 \text{ and } x=\sqrt{82842.71}=$$

287.824 links, the ſide.

8. The

8. The area of a regular nonagon being given, to find the length of its ſide.

$x =$ the ſide of the nonagon.

Half the angle of a nonagon is 70°, of which the tangent is $2.7474774 = b$. The half ſum of the ſides is $\frac{9x}{2}$, and the perpendicular falling from the centre is $\frac{bx}{2}$

$$\frac{9x}{2} \times \frac{bx}{2} = \frac{9bx^2}{4} = a. \quad x^2 = \frac{4a}{9b.} \quad \text{and } x = \sqrt{\frac{4a}{9b.}}$$

Exam. The area of a nonagon is 10,000 ſquare feet, what is the length of its ſide?

Anſ. 40.214 feet.

$a = 10,000 \qquad 9b = 24.7272966$

Then $\sqrt{\frac{4a}{9b}} = \sqrt{\frac{40,000}{24.7272966}} = 40.214$ feet.

9. The area of a regular decagon being given, to find the length of its ſide.

The angle of a decagon is 144°, its half 72°, whoſe tangent is $3.0776835 = b$. $\frac{bx}{2} =$ the perpendicular as before. The half ſum of the ſides is $5x$.

Then $\frac{5bx^2}{2} = a$ and $x^2 = \frac{2a}{5b.}$ alſo $x = \sqrt{\frac{2a}{5b}}$

Exam. The area of a decagon is 50,000 ſquare feet, what is the length of its ſide? *Anſ.* 80.208 feet.

$a = 50000.\ 5b = 15.3884175.$

$$x^2 = \frac{100000}{15.3884175} = 643\,3.4106$$

and $x = \sqrt{6433.4106} = 80.208.$

10. The area of an undecagon being given, to find the length of its ſide.

The

The angle of an undecagon is 147° $16'\frac{4}{11}$; its half $= 73° \ 38'\frac{2}{11}$ whose tangent is $3.4051423 = b$. The perpendicular $= \frac{bx}{2}$ and the half sum of the sides is $\frac{11x}{2}$

$$\text{Then } \frac{11x}{2} \times \frac{bx}{2} = \frac{11bx^2}{4} = a$$

$$x^2 = \frac{4a}{11b} \qquad x = \sqrt{\frac{4a}{11b}}$$

EXAM. The area of an undecagon is 5280 square yards, what is the length of its side?

Ans. 23.747 yards.

$a = 5280. \quad 11b = 37.4565653$

then $x^2 = \frac{21120.}{37.4565653} = 563.8530$ and $x = \sqrt{563.8530}$ $= 23.747$ yards.

11. The area of a dodecagon being given, to find the length of its side.

Let $a =$ the area, $x =$ the side as before. Half the angle of a dodecagon is 75° whose tangent is $3.7320508 = b$.

The perpendicular $= \frac{bx}{2}$, and $6x$ is the half sum of the sides.

$$6x \times \frac{bx}{2} = 3\,bx^2 = a \qquad x^2 = \frac{a}{3b} \text{ and } x = \sqrt{\frac{a}{3b}}$$

EXAM. Let the area of a dodecagon be 360 square yards, what is the length of its side?

Ans. 5.67043 yards.

$$x = \sqrt{\frac{360}{11.19615\cdot24}} = 5.67043$$

12. To reduce any quadrilateral figure ABCD to a triangle. *Fig.* 67.

Join AC; through B draw BE parallel to AC; produce DC to E, and join AE; then the triangle AED is equal to the given figure ABCD.

13. To

13. To reduce any five sided figure ABCDE to a triangle. *Fig.* 68.

Join AC and AD; and through B and E draw BF and EG parallels to AC and AD; join AF and AG; then the triangle AFG is equal to the pentagon ABCDE.

14. To divide a triangular field ABC into any number of equal parts, by lines drawn from one of its angles, suppose A. *Fig.* 69.

Divide the opposite side BC into the number of equal parts required, in the points *e* and *f*; and lines drawn from A to *e* and *f* will divide the field.

If it were required to divide the field into two parts, having a given ratio, such as 3 to 2; divide BC in the point *g*, so that B*g* is to *g*C as 3 to 2; and a line drawn from A to *g* will divide the field into the parts required.

15. To divide a triangular field ABC into two parts, having a given ratio to each other, suppose 6 to 5, by a line drawn parallel to one of its sides AC. *Fig.* 70.

Divide the area of the whole field into parts in the given proportion, by the rule of Fellowship, in common arithmetic, and determine which of the parts must lie next the angle B; then, because similar triangles are to each other as the squares of their like sides, as the area of the whole field is to the area of the triangle BFD, so is the square of BC to the square of BD; and, by extracting the square root, BD will be known. Through D draw DF parallel to AC, and DF divides the field into the parts required.

16. To divide a triangular field ABC into two parts, having a given proportion to each other, such as 3 to 4, by a line drawn from a given point E in one of its sides BC. *Fig.* 71.

Draw

Draw AE, and divide BC in D, so that BD be to DC as 3 to 4. Through D draw DF parallel to EA; and join EF, and FE will divide the field into the parts required.

17. To divide a four or five sided field into parts, having any given proportion to each other.* The general rule is, Reduce the given field to a triangle; divide the triangle into the parts required, and the same lines will divide the field. For the particular cases, see *Wilson's Trigonometry.*

In the rectangular field ABCD, *Fig.* 72. whose length is 3619 links, and breadth 2649 links, there is a well at E, distant from A 2230 links, and from D 1950 links; it is required to divide the field into three equal parts, by right lines drawn from E.

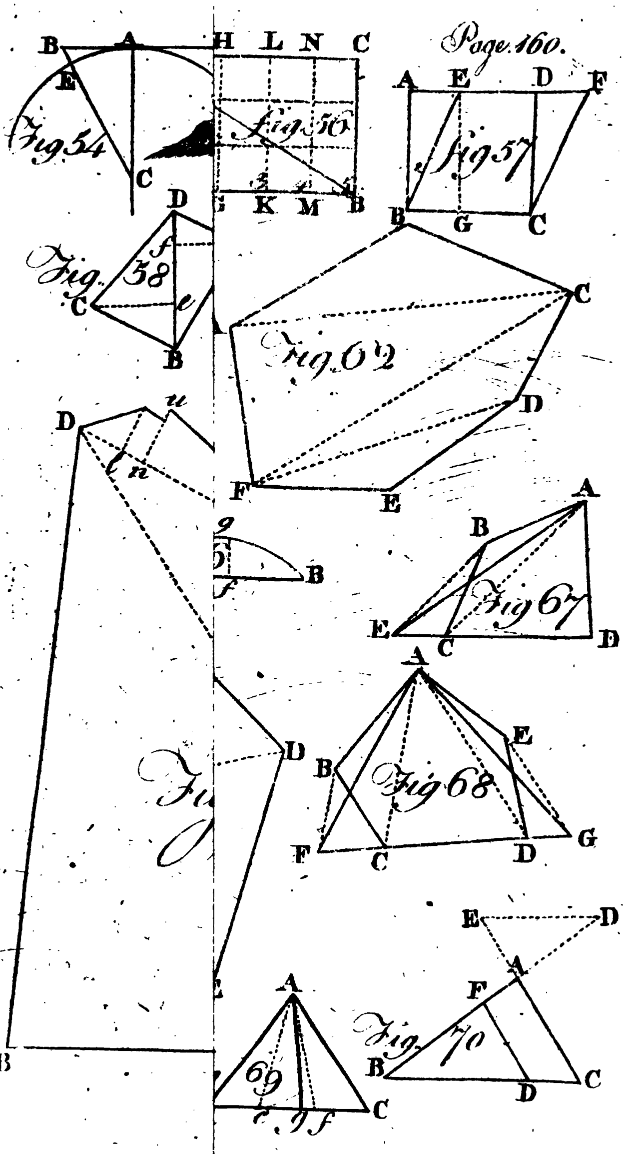
To solve this problem; describe the figure accurately; find the point E; join EA, ED, and EB; likewise through E draw the perpendiculars *hl* and *ik*. Measure the perpendiculars E*h* and E*i*; which being subtracted from the length and breadth of the field, will leave E*l* and E*k*.

2d, Multiply AD by half of the perpendicular E*h*, the product is the area of the triangle AED. If this be equal to the third part of the field, the problem will be solved, by dividing the remaining part of the field into two equal parts. But, if the triangle AED be less than the third part of the field, subtract it from the third part, the remainder is an area which must be added to the triangle AED.

3d, Divide the remainder by half of the perpendicular E*i*; the quotient being taken from the scale, and set from

D to

Page.160.



D to F, join EF, and AEFD is one third part of the field.

4th, Multiply AB by half of E*l*, the product is the area of the triangle AEB; if this be less than the third part of the field, subtract it from the third part, and divide the remainder by half of E*l*; the quotient being taken from the scale, and set from B to G; join EG, and the field is divided into three equal parts by the right lines EA, EF, and EG.

When either of the triangles AED or AEB is greater than the third part of the field, the difference between the area of the triangle and the third part of the field must be divided by half of the perpendicular of that triangle; and the quotient being taken from the scale, must be set the contrary way from what it is in this example.

In this manner, any field of whatever number of sides, may be divided into any number of equal parts; or parts having a given ratio to one another, by right lines drawn from a given point, either within the field, or in any of its sides or angles.

SECT. V.

MENSURATION OF SOLIDS.

A CUBE is a solid body bounded by six equal squares.

As every superficies is measured by a square, whose side is unity, (as 1 inch, 1 foot, 1 yard, &c.) so every solid is measured by a cube, whose side is also a unit, as a solid or cubic inch, foot, yard, &c.

TABLE *of Solid or Cubic Measure.*

1728	Cubic lines	=	1	Cubic inch.
1728	inches	=	1	foot.
27	feet	=	1	yard.
216	yards	=	1	rood.

If the base of a rectangular solid be one square inch, and its altitude 5 inches, the whole will contain 5 cubic inches; but, if the base consist of 15 square inches, and the altitude of 5, the whole solid will contain 15 times 5, or 75 cubic inches. Hence,

1. The rule for computing the solid content of a cube, parallelopipedon, and prism, is, multiply the area of the base by the altitude; the product is the solid content.

For the cube and parallelopipedon, the rule may be expressed thus.

Multiply the length by the breadth, and the product by the thickness or height; the last product is the solidity.

EXAM-

EXAMPLES.

1. What is the solid content of a cube, whose side is 3 feet $9\frac{1}{2}$ inches? *Fig.* 73.

Ans. 54 cubic feet, $884\frac{3}{8}$ inches.

2. What is the solidity of a log of wood, whose length is 17 feet 6 inches, breadth 2 feet 3 inches, and thickness 1 foot 9 inches? *Ans.* 68 cubic feet, 1566 inches.

3. A cellar was dug, whose length was 49 feet 6 inches, breadth 23 feet 9 inches, and depth 9 feet 5 inches; how many solid yards were taken out of it; and what was the expence, at 3d. *per* cubic yard?

Ans. 410 yards, which comes to L. 5 : 2 : 6.

2. Every solid body, whose opposite ends are equal and similar rectilineal figures, parallel to each other, and its other sides parallelograms, is called a *prism*. *Fig.* 74.

Exam. What is the solidity of a triangular prism; one side of its triangular base being 19.5 inches, and the perpendicular falling from the opposite angle upon that side 16.89 inches; also the length of the prism 39.25 inches?

Ans. 3 cubic feet 1279.59 inches.

Perpendicular 16.89
Side 19.5

8445
15201
1689

2)329.355

Area of the base = 164.6775
Multiply by the length 39.25

Ft. inches.
1728)6463.591875(3 1279.59

2. What

2. What is the solidity of a triangular prism, its base being an equilateral triangle, having each of its sides 12 inches, and the length of the prism $48\frac{1}{2}$ inches?

Ans. 1 foot 1296 inches.

Area of the base $= 62.3538$

Then $62.3538 \times 48.5 = 3024.1593$ cubic inches.

Ft. inches.

and 1728)3024.1593(1. 1296

3. What is the solidity of a prism whose bases are regular pentagons, one side being 2 feet $9\frac{1}{2}$ inches, and the perpendicular falling from the center on that side 23.05 inches; also the length of the prism 6 feet 8 inches?

Ans. 89 cubic feet 643 inches.

2. To find the solidity of a pyramid. *Fig.* 75.

Because every pyramid is the third part of a prism of the same base and altitude, multiply the area of its base by one third part of its height, the product is the solid content.

1. What is the solid content of a triangular pyramid, one side of its triangular base being 24.5 inches, and the perpendicular falling upon that side 21.217 inches; also the height of the pyramid 35.5 inches?

Ans. 3075.58 cubic inches.

Perpendicular 21.217
Side 24.5

106085
84868
42434

2)519.8165

Area of the base = 259.90825

259.

$$\begin{array}{r} 259.90825 \\ 35.5 \\ \hline 129954125 \\ 129954125 \\ 77972475 \\ \hline 3)9226.742875 \\ \hline \end{array}$$

Anſ. 3075.580958 cubic inches.

2. What is the ſolidity of a pyramid, its baſe being a ſquare whoſe ſide is 12 feet 4 inches, and its height 29 feet 7 inches?

Anſ. 1499 cubic feet $1701\frac{1}{3}$ inches.

3. To find the ſolidity of a cylinder, the diameter of its baſe and height being given. *Fig.* 76.

Find the area of its baſe, and multiply it by the height; the product is the ſolidity.

Exam. 1. What is the ſolid content of a cylinder, whoſe length is 5 feet 5 inches, and the diameter of its baſe 2 feet 3 inches?

Anſ. 21 cubic feet, 928 inches.

2. What is the ſolidity of a cylinder, its length being 21 feet 10 inches, and the diameter of its baſe 5 feet 6 inches? *Anſ.* 518 cubic feet, 1251 inches.

4. To find the ſolid content of a cone, the diameter of its baſe and height being given *Fig.* 77.

Becauſe every cone is the third part of a cylinder of the ſame baſe and altitude.

Multiply the area of its baſe by one third part of its height; the product is the ſolidity.

Exam. 1. What is the ſolid content of a right cone, whoſe axis is 40 inches, and the diameter of its baſe 32 inches? *Anſ.* 6 ſolid feet, 355 inches.

2. What

2. What is the solidity of a cone, whose height is 3 feet 9 inches, and the diameter of its base 3 feet 11 inches? *Ans.* 15 cubic feet 104 inches.

5. To find the superficies of any solid body bounded by plains; find the area of each bounded plain; the sum of these is the superficies of the solid.

EXAM. A squared piece of timber is 21 feet 9 inches long; the greater end is 25 inches by 19.75 inches, and the less end 19 inches by 15: It is required to calculate the superficies of the whole? *Ans.* 148 feet $20\frac{1}{2}$ inches.

In this example, the perimeters of the two ends may be considered as the parallel, but unequal sides of a trapezium, and the length of the piece of timber, as the perpendicular breadth of the trapezium; therefore, multiply the half sum of the perimeters by the length, the product is the superficies of the sides, to which add the superficies of the two ends, and the sum is the superficies of the whole.

	inches.		
Perimeter of the greater end	89.5		
of the less	68.		
Sum	157.5		
F. I. half	78.75		
Length 21.9 =	261 inches		
		19.75	19
	7875	25	15
	47250		
	15750	9875	95
		3950	19
Superficies of the sides	20553.75		
—— of the greater end	493.75	493.75	285
—— of the less	285.		
Superficies of the whole	21332.50 square inches		
	or 148 square feet $20\frac{1}{2}$ square inches.		

6 To

6. To find the superficies of a cylinder, the diameter of the base, and height of the cylinder being given. *Fig.* 76.

Find the circumference of the base, and multiply it by the length of the cylinder, the product is the curve superficies: Then find the areas of the two bases; the sum of these three is the superficies of the cylinder.

EXAM. 1. What is the superficies of a cylinder, its length being 7 feet 9 inches, and the diameter of its base 3 feet 5 inches?

Ans. 14619.43 square inches, or 101 square feet, 75.43 square inches.

2. Required the superficies of a cylinder, its length being 6 feet $9\frac{1}{2}$ inches, and the diameter of its base 5 feet $1\frac{1}{2}$ inches?

Ans. 21687.6429 square inches, or 150 square feet, 87 inches.

7. To find the superficies of a right cone, its axis and the diameter of its base being given. *Fig.* 77.

Find the area and circumference of the base, and multiply the circumference of the base by half the length of the conic superficies; the product is the curve superficies, to which add the area of the base, the sum is the superficies of the cone.

Note. The length of the conic superficies is equal to the square root of the sum of the squares of the axis of the cone and the semidiameter of the base.

EXAM. 1. What is the superficies of a right cone, the diameter of its base being 36 inches, and its axis 48 inches.

Ans. 3916.796 square inches, or 27 square feet, 28.796 inches.

2. Required

2. Required the superficies of a cone, the diameter of its base being 12 feet 4 inches, and its axis, or perpendicular height, 24 feet 9 inches?

Ans. 88360.39 square inches, or 613 square feet, 88.39 inches.

8. To find the superficies of a globe, its diameter being given. *Fig.* 61.

1. The superficies of a globe is equal to four times the area of one of its great circles: Wherefore, find the area of a circle of the same diameter with the globe, and multiply it by 4, the product is the superficies of the globe. Or,

2. Because the area of a circle is equal to the product, made by multiplying the circumference by half of the radius, or one fourth part of the semidiameter; if the circumference of a circle be multiplied by its diameter, the product will be four times the area of that circle: Therefore, multiply the circumference of any globe by its diameter, and the product shall be the superficies of the globe.

EXAM. 1. What is the superficies of a globe whose diameter is 12 inches?

Ans. 452.39 square inches.

2. Required the superficies of a globe, its diameter being 7 feet 6 inches?

Ans. 176 square feet, 102.96 square inches.

3. The diameter of the terraqueous globe is 7964 miles; what is its superficies in square miles?

Ans. 199256909.9136 square miles.

9. To find the curve superficies of any frustum, or zone of a globe or sphere, the height of the frustum, or zone, and the circumference of the sphere being given.

Multiply

Multiply the circumference of the sphere by the height of the frustum, or zone, h product is the curve superficies. *Vide MacLaurin's Fluxions*, p. 13, and 14.

EXAM. 1. The diameter of the terraqueous globe being 7964 miles, and consequently its circumference 25019.7024 miles; it is required to find the superficies of either frigid zone? *Plate* 5. *Fig.* 16.

Let PÆSQ represent the solstitial colure, ÆQ the equator, PS the earth's axis, P the north pole, and *ar* the diameter of the arctic circle. Join C*a*; then, in the right angled triangle *a*C*n*, we have C*a*=3982 miles, and the angle *a*C*n*=23° 28′; to find C*n*, which being subtracted from CP, the semidiameter, there will remain *n*P, the height of the frigid zone.

As radius	10.0000000
Is to cosine *a*C*n*=23° 28′	9.9625076
So is C*a*=3982	3.6001013
To C*n*=3652.65	
	3.5626089

*n*P=329.35 miles, the height of the zone.

Then 25019.7024 × 329.35=8240238.98544 square miles, the surface of either frigid zone.

EXAM. 2. The same things being given, it is required to find the superficies of either temperate zone?

In the right angled triangle C*to*, there is given C*t*=3982 miles, and the angle *t*C*o*=66° 32′, to find C*o*, which being subtracted from C*n*, the remainder is, *no*, the height of the zone.

As radius	10.
Is to cosine $tCo=66^\circ\ 32'$	9.6001181
So is $tC=3982$	3.6001013
To $Co=1585.69$	3.2002194

From $Cn=3652.65$
Subtr. $Co=1585.69$

Remain $no=2066.96$ miles, the height of the zone, and $25019.7024 \times 2066.96=51714724.072704$ square miles, the surface of each temperate zone.

Exam. 3. It is required to find the superficies of the torrid zone?

Since by the last operation $Co=1585.69$, the double of this is 3171.38 miles, the height of the zone; and $25019.7024 \times 3171.38=79346983.797312$ square miles, the surface of the torrid zone.

	Square Miles.
The surface of the torrid zone is	79346983.797312
of the temperate zones	51714724.072704
	51714724.072704
of the frigid zones	8240238.985440
	8240238.985440
The surface of the whole globe	199256909.913600

10. If the whole superficies of the polar segment, or frustum of a globe, be required; having found the curve superficies, find the area of its circular base, and add it to the curve superficies, the sum is the answer. Or work by the following rule.

11. To find the superficies of the polar segment, or frustum of a globe COPI, its height OI, and the diameter of its base CP being given. *Fig.* 61.

The

The curve superficies is equal to the area of a circle whose radius is CO; and CO is equal to the square root of the sum of the squares of CI and OI. Wherefore,

Find the area of a circle whose diameter is double of CO. Find also the area of the circular base, whose diameter is CP; the sum of these is the superficies of the segment.

Exam. It is required to find the superficies of the segment of a globe, its diameter CP being 44.4 inches, and its height OI = 13 inches?

Ans. 3627.5344 square inches.

12. To find the solidity of a globe, its diameter being given.

The solidity of a globe, whose diameter is 1, has been found to be .5236 nearly; and globes are in proportion to one another as the cubes of their diameter; therefore, as 1 is to the cube of the given diameter, so is .5236 to the solidity; or, because the first term of the proportion is 1,

Multiply the cube of the given diameter by .5236, the product is the solid content of the globe.

Exam. 1. What is the solidity of a globe, its diameter being 12 inches?

Ans. 904.78 cubic inches.

2. Required the solidity of a globe, its diameter being $18\frac{1}{2}$ inches.

Ans. 3315.23 cubic inches.

3. What is the solidity of the terraqueous globe, its diameter being 7964 miles?

Ans. 264480338425.3184 cubic miles.

 A globe

A globe may be considered as a body consisting of an infinite number of small cones, whose bases are in the superficies of a globe, and their vertices at its center; and, because the solidity of a cone is the product of its base into the third part of its height, the solidity of the whole globe will be the product of its superficies into the third part of its radius, or into the sixth part of its diameter. Therefore,

To find the solidity of a globe, its diameter being given.

Find the superficies of the globe, by Prob. 8. and multiply the superficies by the 6th part of the diameter, the product is the superficies of the globe.

In the last example, the diameter is 7964 miles, its sixth part is $1327\frac{1}{3}$, and the superficies of the globe is 199256909.9136; and

$199256909.9136 \times 1327\frac{1}{3} = 264480338425.3184$, the same as before.

13. To find the solidity of the polar segment, or frustum of a globe COPI, its height OI, and the diameter of its base CP being given. *Fig.* 61.

Divide the square of CI by OI, the quotient is IB, which being added to OI, the sum is the diameter of the globe. And then,

From the triple product of the diameter of the globe (OB) into the square of the frustum's height (OI), subtract twice the cube of its height, and multiply the remainder by .5236, the product is the solidity of the segment.

EXAM.

EXAM. It is required to find the solidity of the segment of a globe, the diameter of its base CP being 44.4 inches, and its height OI = 13 inches?

Ans. 11214.1349 solid inches.

2. It is required to find the solidity of either frigid zone, the axis, or height of it being 329.35 miles, and the diameter of the earth (taken as a perfect sphere) being 7964 miles.

(1) $329.35 \times 329.35 = 108471.4225$,

and $108471.4225 \times 329.35 \times 2 = 71450126.$

(2.) $108471.4225 \times 7964 \times 3 = \underline{2591599226.37}$

Difference $= 2520149100.37$

(3.) $2520149100.37 \times .5236 = 1319550068.9537$ cubic miles the answer.

14. To find the solidity of the middle zone of a globe CPFE, the diameter of the globe, the diameter of the zone's base, and the height of the zone being given. *Fig.* 61.

Multiply the square of the globe's diameter by 2, and to the product add the square of the diameter of the zone's base; divide the sum by 3.8197, and multiply the quotient by the height of the zone; the product is the solidity of the zone.

EXAM. 1. The diameter of a globe AD is 16 inches, and two polar segments being cut off, the diameter of the base of the middle zone CP or EF is 13.8564 inches, and the height 8 inches: What is the solid content of the zone?

Ans. 1474.46 cubic inches.

2. It is required to find the solidity of the torrid zone, the diameter of the earth being 7964 miles, the diameter

diameter of either tropic=7305.3 miles, and the axis, or height of the zone=3171.38 miles.

(1.) 7964×7964×2	=	126850592
7305.3×7305.3	=	53367408.09
		180218000.09

(2.) $\frac{180218000.09}{3.8197}\times 3171.38 = 149629472782.1242$ cubic miles, the anfwer.

15. To find the folidity of any zone of a fphere contained between two parallel and unequal plains, (fuch as *artd*, *Plate* 5. *Fig.* 16.) the diameters of the bafes and height of the zone being given.

To the fum of the fquares of the femidiameters of the two bafes, add one third part of the fquare of the zone's height; multiply the fum by the height, or axis of the zone, and multiply the product by 1.5708; the laft product is the folidity of the zone.

EXAM. It is required to find the folidity of either temperate zone, the femidiameter of the polar circle being 1585.69 miles; the femidiameter of the tropic =3652.65 miles; and the height, or axis of the zone =2066.96 miles?

(1.) The fquare of 1585.69 is	2514412.7761	
Ditto of 3652.65	13341852.0225	
Sum	15856264.7986	add
The fquare of 2066.96 is 4272323.6416		
The third part whereof is	1424107.8805	
	17280372.6791	

(2.) 17280372.6791 × 2066.96 × 1.5708 =56105581678.3745, the folidity of the temperate zone in cubic miles.

From

From the three last problems, we have

		Cubic Miles.
The solid content of the torrid zone	=	149629472782.1242
Of the two temperate zones	{	56105581678.3745
		56105581678.3745
Of the two frigid zones	{	1319550068.9537
		1319550068.9537

Solidity of the terraqueous globe = 264479736276.7806

Note. The reason why this sum falls short of the solid content of the terraqueous globe (in Exam. 3. of Art. 12) is, in this calculation, the diameters of the polar circles and tropics, and the heights or axes of the several zones are computed only to two places of decimals. If these lines had been taken nearer to the truth, the sum of the solidities of the zones would have come nearer to the solidity of the whole globe.

If a semi-ellipse be revolved round its longer axis, the solid generated by that revolution is called an *oblong spheroid*; but, if it be revolved round its shorter axis, the solid is called an *oblate spheroid*.

16. To find the solidity of any spheroid, its two axes being given.

Multiply the axis round which the generating ellipse was revolved by the square of the other axis, and multiply the product by .5236; the last product is the solidity of the spheroid, whether it be oblong or oblate.

EXAM. It is required to find the solidity of an oblong spheroid, whose axes are 42 and $19\frac{1}{2}$ inches?

Ans. 8362.15. cubic inches.

2. What is the solid content of an oblate spheroid, whose axes are $19\frac{1}{2}$ and 42 inches?

Ans. 18010.79 cubic inches.

17. The

17. The solidity of the middle zone of a spheroid is found by the same rule as that of a globe or sphere. *Art.* 14 *of this Section.*

EXAM. It is required to find the solidity of the middle zone of an oblong spheroid, (*Fig.* 24). the conjugate diameter GH being 43.4 inches, the diameter of either base NP or MO=32.6 inches, and the length of the zone *kl*=38 inches?

Ans. 48049.67 cubic inches.

For twice the square of GH is 3767.12, to which add the square of NP=1062.76, and divide the sum 4829.88 by 3.8197; the quotient 1264.465 being multiplied by 38 gives 48049.67 for the answer.

18. To find the solidity of any frustum, or segment of a spheroid, cut off by a plain perpendicular to one axis, and parallel to the other, the diameter of its base and its height being given.

Find its solidity as if it were the frustum of a globe; and then, as the solidity of a globe whose diameter is equal to the axis of the spheroid perpendicular to the base of the frustum, is to the solidity of the spheroid; so is the solidity of the frustum of the globe to the solidity of the like frustum of the spheroid. Or,

The frustums are to one another as the cube of the fixed diameter of the spheroid is to the product of the same diameter into the square of its other diameter.

EXAM. It is required to find the solidity of the frustum of an oblong spheroid, whose axes are 50 and 30 inches; the diameter of its base (parallel to the lesser axis) being 18 inches, and its height 5 inches?

Ans. 659.736 cubic inches.

For

For $50\times5\times5\times3=3750$, from which
Subtract $5\times5\times5\times2=250$

Remains 3500

And $3500\times.5236=1832.6=$ the solidity, if it was the segment of a sphere whose diameter is 50 inches.

The cube of 50 is 125000, and $50\times30\times30=45000$.

Lastly, as $125000:45000::1832.6:659.736$, the answer.

If a semi parabola be revolved round its axis, the solid generated by that revolution is called a *parabolic conoid*. Fig. 78.

19. To find the solidity of a parabolic conoid.

Multiply the area of its circular base by one half of its altitude, the product is the solid content; because this solid is one half of a cylinder, having the same base and altitude.

Exam. Required the solidity of a parabolic conoid, the diameter of its base being 31 inches, and its height 43 inches? *Ans.* 16227.54 cubic inches.

20. To find the solidity of the lower frustum of a parabolic conoid, cut off by a plain parallel to its base, its height and the diameters of its bases being given.

Find the areas of the two bases, and divide the sum by 2; then multiply the quotient by the height or length of the frustum; the product is the solidity.

Exam. It is required to find the solidity of the lower frustum of a parabolic conoid, the diameter of its greater base being 24 inches, and of its lesser 16 inches, and the length of the frustum 35 inches?

Ans. 11435.42 cubic inches.

If a parabola be revolved round one of its ordinates, the solid generated is called a *parabolic spindle*.

Z 21. To

21. To find the solidity of a parabolic spindle, its greatest diameter and its length, or axis, being given. *Fig.* 79.

Find the solidity of a cylinder, the diameter of whose base is equal to the greatest diameter of the spindle, and its height equal to the length of the spindle, and $\frac{8}{15}$ of this cylinder is the solid content of the spindle.

Exam. Required the solidity of a parabolic spindle, its length being 60 inches, and its greatest diameter 24 inches? *Ans.* 14476.4928 cubic inches.

22. To find the solidity of the middle zone of a parabolic spindle, its greatest diameter, the diameter of its base, and the length of the zone being given.

To twice the square of its greatest diameter, add the square of the diameter of its base; and, from the sum subtract $\frac{4}{10}$ of the square of the difference between these two diameters; divide the remainder by 3.8197, and multiply the quotient by the length of the zone; the product is the solidity.

Exam. It is required to find the solidity of the middle zone of a parabolic spindle, its greatest diameter being 32 inches, the diameter of either base = 24 inches, and the length of the zone = 40 inches?

Ans. 27210.48 cubic inches.

When a pyramid or cone is cut by a plain parallel to its base, the part between that plain and the base is called a *frustum*.

23. To find the solidity of a frustum of a pyramid or cone, its length, and the dimensions of its bases, being given.

Find the areas of its bases, and multiply the one by the other. Extract the square root of the product, and

add

add the root to the areas of the bases; then multiply the sum by one third part of the length of the frustum; the product is the solidity.

EXAM. 1. It is required to find the solidity of the frustum of a pyramid, the greater base being a rectangle, whose sides are 12 and 9 inches; and the lesser base, the same figure whose sides are 10 and $7\frac{1}{2}$ inches; and the length of the frustum 19 inches? *Ans.* 1729 solid inches.

2. How many cubic feet of timber are there in a log of wood 18 feet long, the base at the greater end being 32 inches by 20, and at the lesser end 16 inches by 10? *Ans.* $46\frac{2}{3}$ cubic feet.

Note. This rule is general, and serves to find the solid content of the frustum of any pyramid or cone; but in some cases the problems may be solved with less labour. For example,

1. When the bases of the frustum are squares. Multiply the side of the greater base by the side of the less, and add the product to the squares of these sides, then multiply the sum by one-third part of the length of the frustum, the product is the solid content.

EXAM. 1. Suppose the frustum of a pyramid hath square bases; the side of the greater base being 12 inches, and the side of the less, 9 inches, and its length 17 inches: What is the solid content?

Ans. 1887 cubic inches.

Product of the sides	108	333
Square of the greater base	144	17
of the less	81	
		2331
Sum	333	333
		3)5661
		Ans. 1887

 2. What

2. What is the solid content of the frustum of a pyramid having square bases; the side of the greater base being 30 feet 9 inches, and of the less 20 feet 10 inches, and the height, or length of the frustum 42 feet 3 inches?

Ans. 28451 cubic feet, 631 inches.

2. The solid content of the frustum of a cone may be found thus; multiply the diameter of the greater base by the diameter of the less, and to the product add the squares of the two diameters; multiply the sum by .2618 (the 3d part of .7854), and multiply the product by the length of the frustum, the last product is the solid content.

EXAM. What is the solid content of the frustum of a cone, its length being 19 inches, the diameter of the greater base = 16 inches, and of the lesser base 9 inches?

Ans. 2392.59 cubic inches.

Product of the diameters =	144	.2618
Squares of the diameters	256	481
	81	2618
Sum	481	20944
		10472
		125.9258

Then 125.9258 × 19 = 2392.5902 cubic inches.

2. Let the length of the frustum of a cone be 6 feet 9 inches; the diameter of the greater base 3 feet 8 inches, and the diameter of the lesser base 2 feet 7 inches: What is the solid content of the frustum?

Ans. 52 cubic feet 501.9 inches.

A solid resembling the frustum of a pyramid, having parallel bases, but not similar to one another, is called a *prismoid*.

24. To find the solidity of a prismoid, its length and the sides of its bases being given.

1. To

1. To the longeſt ſide of the greater baſe add half the longeſt ſide of the leſſer baſe, and multiply the ſum by the breadth of the greater baſe.

2. To the longeſt ſide of the leſſer baſe add half the longeſt ſide of the greater baſe, and multiply the ſum by the breadth of the leſſer baſe. Add this product to the former, and multiply the ſum by one third part of the given length, the product is the ſolidity of the priſmoid.

Note. This rule holds only when the baſes are quadrilateral figures. If the baſes conſiſt of more than four ſides, uſe the following rule:

To the ſum of the areas of the two baſes, add four times the area of a ſection parallel to the baſes, taken in the middle of the ſolid; multiply the ſum by the given length, and one ſixth part of the product is the ſolidity of the priſmoid.

A ſolid reſembling the fruſtum of a cone, and having parallel baſes, and theſe baſes both elliptical, (or the one baſe an ellipſe and the other a circle), but diſproportional; the diameters of the one baſe not having the ſame proportion to one another as the correſpondent diameters of the other baſe, is called a *cylindroid.*

25. To find the ſolidity of a cylindroid, its length, and the dimenſions of its baſes, being given.

1. To the longeſt diameter of the greater baſe, add half the longeſt diameter of the leſſer baſe, and multiply the ſum by the ſhorteſt diameter of the greater baſe.

2. To the longeſt diameter of the leſſer baſe, add half the longeſt diameter of the greater baſe, and multiply the ſum by the ſhorteſt diameter of the leſſer baſe. Add this product to the former, and multiply the ſum by

·7854,

.7854, then multiply the product by one third part of the given length; the last product is the solidity of the cylindroid.

Note. However useful the two last rules may be on some occasions, they are seldom applied in practice; for trees have generally their bases, either perfectly, or nearly similar; and, therefore, are frustums of pyramids or cones: And, when they are not similar, their contents, when considered as frustums of pyramids or cones, come so near to the same, when considered as prismoids and cylindroids, that the difference is quite inconsiderable; not amounting to half a cubic foot in sixty.

Of MEASURING TIMBER.

When large trees are of any regular form, as a parallelopiped, prism, cylinder, pyramid, or cone, their solid contents are found by the foregoing rules. When their forms are different, they must be considered as frustums of some regular solid.

1. *Of Equal* SQUARED TIMBER.

By equal squared timber, is understood all trees cut into the form of a parallelopiped, whose bases are equal squares or rectangles; and the solid content is found by *Art.* 1. *of this section.*

EXAM. 1. Required the solid content of a squared piece of wood, its length being 19 feet, and the side of its square base 16 inches?

Ans. 33 cubic feet, 1344 inches.

2. In-

3. In a piece of squared timber, 32 inches broad, 18 inches thick, and 14 feet 6 inches long; how many solid feet?

Ans. 58 cubic feet.

Note. In measuring squared timber, unskilful measurers take a fourth part of the circumference for the side of a mean square base; this quarter girt being multiplied by itself, and the product by the length, they account the solid content; but this method gives an answer always greater than the truth, except when the breadth and the thickness are nearly equal; and the greater difference there is between the breadth and thickness, the greater will the error be. In the last example $32+18=50$, and half of 50 is 25, the side of the mean square base; and $25\times25=625$ for the area of the base. Lastly, $\frac{625\times14.5}{144}$ $=62$ solid feet, 1614 inches; which is near 5 feet above the true solidity.

2. *Of Unequal* Squared Timber.

By unequal squared timber is meant, any piece of squared timber whose bases are unequal, whether these bases be squares or rectangles; and such are most trees when hewn into what is called a Square Form. Any solid of this kind is truly, or very nearly, the frustum of a pyramid; and its solid content may be found by *Art.* 23. *of this section.*

Exam. It is required to find the solid content of a tree 18 feet 6 inches long, the base at the greater end being (a rectangle) 32 inches by $19\frac{1}{2}$, and at the lesser end 17 inches by 9? *Ans.* 46.506 cubic feet.

The

The usual customary way of measuring such timber is, to take the square or rectangle at the middle of the tree for a mean base, and multiply this by the length.

The last Example wrought by this method.

(1.) $\frac{32+17}{2}=24.5$, and $\frac{9+19\frac{1}{2}}{2}=14.25$

(2.) $\frac{24.5\times14.25\times18.5}{144}=44.852$ cubic feet, too little by 1.654 feet.

Hence the error of this customary way of measuring is abundantly evident.

Exam. 2. How many solid feet are there in a tree 24 feet long, its bases being squares; the side of the greater = 15 inches, and the side of the less = 6 inches?

Ans. 19 cubic feet, 864 inches.

3. What is the solid content of a tree 27.36 feet long; the breadth at the greater end being 1.78 feet, and the thickness 1.23 feet; and the breadth at the lesser end 1.04 feet, and thickness .91 feet.

Ans. 41.723 solid feet.

3. *Of Round* Timber.

1. When round trees are equally thick from one end to the other, they are cylinders; and their lengths being measured in feet, and their circumferences in inches, their solidities may be found thus:

Multiply the square of the circumference, or girt, by .07957775; multiply the product by the length, and divide the last product by 144, the quotient is the solid content of the tree in cubic feet.

Exam. 1.

EXAM. 1. What is the solidity of a round tree, its length being 17 feet 6 inches, and its circumference 41 inches?

Ans. 16.25 cubic feet.

2. Required the solidity of a round tree, its circumference being 52 inches, and its length 30 feet?

Ans. 44.828 solid feet.

3. The length of a round tree is 25 feet 5 inches, and its girt 4 feet $5\frac{1}{2}$ inches; required its solidity?

Ans. 40.202 cubic feet.

2. When round trees are smaller at one end than the other, (commonly called Tapering Timber), they are frustums of cones; and their solid contents may be found by *Art.* 23. *of this section*; but, because it has been always thought best to take the circumference and length of a round tree, to find its solidity,

Measure the circumference at both ends, and in as many places between the two as may be thought necessary; divide the sum of the circumferences by their number, the quotient is the mean girt; then measure the length, with which, and the mean girt, find the solidity by the last rule.

EXAM. 1. It is required to find the solidity of a round tree, its length being 17 feet 3 inches, and the circumference being taken in five places, the several girts are 113.16, 95.04, 73.8, 56.88, and 37.92 inches, and consequently the mean girt=75.36 inches?

Ans. 54.1375 cubic feet.

2. What is the solidity of a round tree, its length being 19 feet 9 inches, and mean girt =47.124 inches?

Ans. 24.237 cubic feet.

 3. The

3. The customary way of measuring round timber is this: Take a fourth part of the mean girt in inches, and square it; multiply this square by the length in feet, and divide the product by 144, the quotient is the solidity of the tree.

But this rule, although universally used, gives the answer always too little by about one fourth part of the whole.

EXAMPLES.

Length.		Mean Girt.	Solidity in
Feet.	Inches.	Inches.	Cubic Feet.
17	9	$30\frac{1}{2}$	7.1666
21	7	$41\frac{1}{4}$	15.94
19	6	50	21.1588
22	10	49	23.7946
23	11	$67\frac{3}{4}$	47.6471
25	0	39	16.5039
27	3	59	41.1706

These examples are all wrought by the common rule, and the answers are therefore less than the truth. It may be a very good exercise for a learner to work them by the former, or following rule, namely,

4. Multiply the square of one fifth part of the circumference by twice the length; the product is the solidity, very near the truth.

EXAM. The length of a round tree is 24 feet 9 inches, and its mean girt $57\frac{1}{2}$ inches; what is the solidity?

Ans. 45.46 cubic feet.

One 5th of 57.5 is 11.5, and twice the length is 49 feet 6 inches; then $\frac{11.5 \times 11.5 \times 49.5}{144} = 45.46$, true to the fourth part of a foot.

More

More EXAMPLES *wrought by this Rule.*

Length. Feet.	Inches.	Mean Girt. Inches.	Solid content in Cubic Feet.
25	5	$48\frac{1}{2}$	33.214
27	6	51	39.7375
26	9	$59\frac{3}{4}$	53.055
23	3	65	54.5729
21	4	70	58.074
19	10	75	61.9791
18	6	$81\frac{1}{4}$	68.2675

Of the Five REGULAR BODIES, *viz.*

1. The *Tetrahedron*, or regular triangular pyramid, contained under four equal equilateral triangles.
2. The *Hexahedron*, or Cube, contained under six equal squares.
3. The *Octahedron*, contained under eight equal equilateral triangles.
4. The *Dodecahedron*, contained under twelve equal equilateral pentagons.
5. The *Icosahedron*, contained under twenty equal equilateral triangles.

1. *Of the* TETRAHEDRON.

To find the superficies of the Tetrahedron. Let the side

ſide of one of the containing triangles be a, then the ſum of the three ſides will be $3a$, and the half ſum $\frac{3a}{2}$; from which each ſide being ſubtracted, the three remainders are $\frac{a}{2}$, $\frac{a}{2}$, $\frac{a}{2}$. The continual product of the half ſum and three remainders is $\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2} = \frac{3a^4}{16}$, and the ſquare root of $\frac{3a^4}{16}$ is $\frac{a^2 \times \sqrt{3}}{4}$ = the area of one of the containing triangles; and this root being multiplied by 4, gives $a^2 \times \sqrt{3}$ for the ſuperficies of the tetrahedron. Therefore, multiply the ſquare of the ſide of one of the containing triangles by the ſquare root of 3; the product is the ſuperficies of the tetrahedron.

Exam. What is the ſuperficies of a tetrahedron, each ſide of its triangular faces being 12 feet?

Anſ. 249.4152 feet.

To find the ſolidity of a Tetrahedron.

1. Meaſure the perpendicular height of the ſolid, and multiply the area of the baſe by one third part of the perpendicular, the product is the ſolidity. Or,

2. Multiply the cube of a ſide of one of the containing triangles by the ſquare root of 2, and divide the product by 12, the quotient is the ſolidity.

Exam. Required the ſolidity of a tetrahedron, each ſide of its triangular faces being 12 feet, and the perpendicular height of the ſolid being 9.8 feet?

Anſ. 203.68 cubic feet, by rule 1ſt; and by rule 2d, the anſwer is 203.64 cubic feet.

2. Of

2. *Of the* Hexahedron, *or* Cube.

Six times the square of its lineal side is the superficies of the cube; and the cube of the same side is the solid content.

3. *Of the* Octahedron.

To find the superficies of the Octahedron.

Since this solid is contained under eight equal equilateral triangles, find the area of one of these triangles, (by the rule for the tetrahedron), and multiply it by 8, the product is the superficies of the Octahedron. Or,

Multiply twice the square of the lineal side by the square root of 3; the product is the superficies of the Octahedron.

Exam. Required the superficies of an octahedron, each side of its triangular faces being 12 feet?

Ans. 498.83 square feet.

To find the solid content of an Octahedron.

The octahedron consists of two equal pyramids, joined together on the same square base; namely, the square of a side of one of the triangular faces: Therefore, measure the length of the octahedron, and multiply two thirds of it by the square of a side of one triangular face; the product is the solid content.

Exam. What is the solidity of an octahedron, each side of its triangular faces being 12 feet, and the length of the solid 8.485 feet?

Ans. 814.579 cubic feet.

Otherwise:

Otherwise: Multiply one third part of the cube of the side of one triangular face by the square root of 2; the product is the solid content of the octahedron.

4. *Of the* DODECAHEDRON.

To find the superficies of a Dodecahedron.

Because this solid is contained under 12 equal regular pentagons, multiply the area of one of these pentagons by 12; the product is the superficies of the dodecahedron.

Find the area of one of the containing pentagons thus: The angle of a pentagon is 108°, its half is 54°; therefore, as the radius is to the tan. 54°, so is half of the side of the pentagon to the perpendicular falling from the center on that side. Multiply half the sum of the sides by the perpendicular, the product is the area of the pentagon.

EXAM. Required the superficies of a dodecahedron, each side of its pentagonal faces being 12 feet?

Ans. 2972.984 square feet.

To find the solid content of a Dodecahedron.

The dodecahedron consists of 12 equal pyramids, their bases being pentagons, and their vertices in the center of the inscribed sphere. When the solid is set on one of its sides, its height will be the diameter of the inscribed sphere, and the half of this diameter will be the common height of the pyramids. Therefore,

Measure the diameter, and multiply the area of one of the containing pentagons by one sixth part of the diameter,

meter, and multiply the product by 12; the last product is the solidity of the dodecahedron. Or,

Multiply the area of one base by twice the diameter, the product is the solidity of the dodecahedron.

Exam. Required the solid content of a dodecahedron, the diameter being 26.72424, and each side of the containing pentagons 12?

Ans. 13241.79.

5. *Of the* Icosahedron.

To find the superficies of an Icosahedron.

This body is contained by 20 equal equilateral triangles; therefore, find the area of one of these triangles, and multiply it by 20; the product is the surface of the Icosahedron.

Exam. What is the superficies of an Icosahedron, each side of its triangular faces being 12 inches?

Ans. 1247.076.

To find the solidity of an Icosahedron.

The Icosahedron consists of twenty equal triangular pyramids, whose vertices are in the center of the inscribed sphere; therefore, measure the diameter of the solid, and multiply the area of one face by one sixth part of the diameter, and multiply the product by 20; the last product is the solid content of the icosahedron.

Exam. Required the solidity of an icosahedron, its diameter being 18.138264, and the side of each triangular face 12 inches?

Ans. 3769.96 cubic inches.

The

The ſuperficies and ſolidity of any of the above bodies may be eaſily had from the following table, wherein the ſide of each of the containing plains is 1.

Names.	Superficies.	Solid content.
Tetrahedron	1.732050	0.1178511
Hexahedron	6.000000	1.0000000
Octahedron	3.464101	0.4714045
Dodecahedron	20.645728	7.6631189
Icoſahedron	8.660254	2.1816950

To uſe this Table, obſerve the following rules:

1. When the ſuperficies is wanted, multiply the tabular number by the ſquare of the lineal ſide; the product is the ſuperficies.

2. When the ſolidity is required, multiply the tabular number by the cube of the lineal ſide; the product is the ſolidity.

Exam. Required the ſuperficies and ſolidity of a dodecahedron, the lineal ſide being 3?

Anſ. The ſuperficies is 185.81, and the ſolidity is 206.904.

OF GAUGING.

1. To gauge any vessel, or to find the quantity of liquor it can contain.

Find how many solid inches will fill the cavity of the vessel, and divide these by the number of solid inches which make a pint or gallon; the quotient is the content of the vessel in pints or gallons.

TABLE *of Cubic Inches in several Measures.*

104 Cubic inches	=	1 Scots pint
2210 Ditto	=	1 Scots wheat firlot
282 Ditto	=	1 English ale gallon
231 Ditto	=	1 English wine gallon
268.8 Ditto	=	1 English corn gallon
2150.4 Ditto	=	1 English corn or malt bushel.

From these divisors, multipliers may be found thus: Divide 1 by any number in this table, the quotient is a constant multiplier, by which the content of any vessel in cubic inches being multiplied, the product is its content in that measure for which the multiplier was found.

TABLE *of Divisors and Multipliers.*

104	.009615 Scots pints
282	.003546 ale gallons
231	.004329 wine gallons
268.8	.0037202 corn gallons
2150.42	.00046502 malt bushels.

N. B. In gauging vessels, the dimensions are always taken in inches and decimals of an inch.

EXAM. 1. Suppose a vessel in form of a parallelopiped, the length of its base 27 inches, breadth $16\frac{1}{2}$ inches, and the depth of the vessel $32\frac{1}{4}$ inches; required the content in Scots pints, and English ale gallons?

$27 \times 16.5 \times 32.25 = 14367.375$ cubic inches.

Now, if you divide 14367.375 by 104, or multiply them by .009615, the result will be 138.14 Scots pints, the content of the vessel.

And, if you divide 14367.375 by 282, or multiply them by .003546, the result will be the content of the vessel in English ale gallons, viz. 50.946.

In the same manner, the content of any vessel may be found; but those who practise gauging proceed thus:

In any vessel equally wide from top to bottom, they compute the area of its base in square inches; and, by dividing or multiplying these by the numbers in the tables, get what they call the area in gallons, (that is, the number of gallons which the vessel contains when the liquid is only one inch deep), which multiplied by the depth of the liquid, gives the quantity contained in the vessel.

EXAM. 2. Suppose a trough, or cistern, in form of a right angled parallelopiped, its base 27 inches long, and $16\frac{1}{2}$ inches wide, and the height of the vessel $32\frac{1}{4}$ inches, but the depth of liquor only 20 inches; required its content in English ale gallons?

Ans. 31.594.

$27 \times 16.5 \times .003546 = 1.579743$ the area of the base in gallons, which multiplied by 20, produces 31.594 ale gallons.

3. Suppose

3. Suppose a cylindric vessel hath the diameter of its base 20 inches, and its height 30 inches; required the content in wine gallons? *Ans.* 40.8.

$$20 \times 20 \times .7854 \times 30 \times .004329 = 40.8.$$

Suppose a tub having circular bases, the diameter of the mouth is 60 inches, and the diameter of the bottom is 36 inches, and the perpendicular depth from top to bottom is 30 inches; required its content in Scots pints and English ale gallons?

This vessel is to be considered as the frustum of a cone; and, on this supposition, its content will be 55417.8 cubic inches; which, by reduction, is 532.8 Scots pints, or 196.5 English ale gallons.

The calculation is tedious by common arithmetic, but may be easily performed by logarithms; thus,

To twice the logarithm of the diameter of one base add the logarithm of .7854, the sum is the logarithm of the area of that base. Do the same for the area of the other base, and find the numbers answering to each. Then add the logarithms of the areas of the two bases, and take half of the sum, and find the number answering thereto.

Add the areas of the two bases, and the last found number, and multiply the sum by one third part of the depth, the product is the content in cubic inches.

Operation for the laſt Example.

	Log.		Log.
Diameter 60	1.7781513	Diameter 36	1.5563025
	1.7781513		1.5563025
.7854	9.8950909	.7854	9.8950909
2827.44	3.4513935	1017.8784 =	3.0076959
1017.8784		2827.44	3.4513935
1696.464			2)6.4590894
5541.7824			
Mult. by 10		1696.464 =	3.2295447
Anſ. 55417.824			

The content of any veſſel of this form may be found with leſs trouble by this rule.

To the product of the diameters of the two baſes, add one third part of the ſquare of their difference; the ſum is the ſquare of a mean diameter; which being multiplied by .7854, and the product by the depth of the veſſel, gives the content in cubic inches.

Exam. Let the diameter of the greater baſe be 60, and of the leſſer baſe 57.6. and the perpendicular height of the tub 29.976 inches; required its content?

Anſ. 81410.3 cubic inches.

The content of a veſſel of this form may be found without meaſuring the diameter of the bottom, or leaſt baſe; thus, Meaſure the diameter of its mouth, or upper baſe, AB, the diagonal BC, and the length of the ſtave AC (*Fig.* B. *plate* 4) Then, in the triangle ABC, the

three

three sides are known; and, having drawn the perpendicular CE, we have,

As AB : BC+CA :: BC−CA : BE−EA, which is equal to CD; and hence AE and EC may be found. And, when AB, CD, and CE are known, the content of the vessel may be found by the last rule.

EXAM. Let the diameter AB=40, the diagonal BC =42, and the length of the stave AC=20 inches, required the content of the vessel?

Ans. 21371.58 cubic inches.

By the above rule, BE−EA, or CD=34.1 inches, and AE=2.95 inches; $\overline{AC}^2-\overline{AE}^2=\overline{EC}^2=391.2975$; hence EC=19.7812 inches; and the rest of the operation is the same as in the last example.

When vessels are not equally wide from top to bottom, gaugers consider them as frustums of some regular solid, and compute their contents at every inch of their depth, which contents they enter in a table, and, when they come to survey, they have only to take the depth; and, by comparing the wet inches with the table, have the content by inspection.

2. *To Gauge a* CASK.

Casks are distinguished into the following four varieties:

1. Such as resemble the middle frustum of a spheroid.
2. Such as resemble the middle frustum of a parabolic spindle.
3. Such as being cut through in the middle, the two parts are parabolic conoids.
4. Such as being cut through in the middle, the parts are the lower frustums of two equal cones.

Measure

Measure the head and bung diameters, and the length of the cask in inches; and then,

1. If the staves are very much curved, the cask is supposed to be the middle zone, or frustum of a spheroid; and its content may be found by this rule.

To twice the square of the bung diameter, add the square of the head diameter; multiply the sum by the length of the cask, and divide the product by 3.8197, the quotient is the content in cubic inches.

Exam. Suppose there is a spheroidal cask, its bung diameter = 31.5 inches, head diameter = 24.5 inches, and the length of cask = 42 inches; required its content in English ale gallons?

Ans. 100.78.

2. If the staves of a cask are less curved than was supposed in the last article, the cask is taken for the middle frustum, or zone, of a parabolic spindle; and its content is computed by this rule.

To twice the square of the bung diameter, add the square of the head diameter, and from the sum subtract four tenths of the square of the difference of the diameters; divide the remainder by 3.8197, and multiply the quotient by the length of the cask; the product is its content in cubic inches.

Exam. Let the bung diameter = 34 inches, the head diameter = 30, and the length of the cask = 40 inches; what is its content in English ale gallons?

Ans. 119.039.

3. When the staves of a cask are very little curved, the cask is supposed to consist of the two lower frustums of two equal parabolic conoids, their greatest bases joined together

together in the middle of the cask; the content of such a vessel may be found by this rule.

To the square of the bung diameter, add the square of the head diameter; multiply the sum by .3927, and the product by the length of the cask; the last product is the content in cubic inches.

Exam. Let the bung diameter of such a cask=32 inches, the head diameter=29, and the length of the cask=42 inches; required its content in cubic inches and ale gallons?

Ans. 30760.191 cubic inches, or 109.07 ale gallons.

4. If the staves of a cask are straight between the bung and the ends of the cask, the vessel is supposed to consist of the two lower frustums of equal cones; and its content is found by this rule.

To the sum of the squares of the head and bung diameters add their product; multiply the sum by the length of the cask, and divide the product by 3.8197; the quotient is the content in cubic inches.

Exam. Required the content of a cask in ale gallons, its bung diameter being 32 inches, its head diameter 24 inches, and the length 40 inches?

Ans. 87,93.

By these rules, the content of any cask may be found, it being known to which of the four varieties the cask belongs; but, in common practice,

A mean diameter, whereby the cask is reduced to a cylinder in either variety, is found thus:

5. Multiply the difference between the bung and head diameters by .7 for the spheroid, by .65 for the spindle, by .6 for the conoids, and by .55 for the cones; add the product

product to the head diameter; the sum is a mean diameter; or the diameter of the base of a cylinder equal to the cask, their lengths being the same. The mean diameter being squared and multiplied by .7854, and the product by the length of the cask, gives the content in cubic inches, which may be reduced to gallons by the table.

Exam. Suppose the bung diameter is 30 inches, the head diameter 20 inches, and the length of the cask 40 inches; required its content in ale gallons according to each variety?

Difference of Bung *and* Head Diameters *is* 10 Inches.

	M. D.	
$10\times.7=7.$	and $20+7.=27$	for the spheroid
$10\times.65=6.5$	and $20+6.5=26.5$	for the spindle
$10\times.6=6.$	and $20+6.=26$	for the conoids
$10\times.55=5.5$	and $20+5.5=25.5$	for the cones.

For the Content *in* Ale Gallons.

$27\times27\times.7854\times40\times.003546=81.21$ spheroid.
$26.5\times26.5\times.7854\times40\times.003546=78.23$ spindle
$26\times26\times.7854\times40\times.003546=75.3$ conoids
$25.5\times25.5\times.7854\times40\times.003546=72.43$ cones.

Mr. Ward, who had much practice in gauging, says, He never gauged a cask that contained so much as the first variety makes it; and therefore, recommends the 2d and 3d varieties as the best general rules for gauging casks.

To

To Gauge MALT.

1. If the malt lie on the floor in a rectangular form, multiply the length by the breadth, and the product by the depth, all taken in inches; the product is the number of cubic inches in the quantity; which being divided by 2150.42, the quotient is the number of bushels.

The same rule serves for finding the quantity of malt contained in any vessel in form of a parallelopipedon.

EXAM. Suppose a quantity of malt on the floor, 288 inches long, 144 inches broad, and $9\frac{1}{2}$ inches deep; required the number of bushels?

Ans. 183.21.

2. When malt is in a cistern, or any vessel, the content of the vessel is to be found in cubic inches, by some of the former rules, and then divided by 2150.42, the quotient is the number of bushels.

3. To find the solidity of any irregular solid, such as a thorn or whin-bush.

Put the irregular body into any vessel, and fill it with water; take out the body, and the water will fall lower, and leave a part of the vessel empty, equal to the solidity of the body to be measured; then measure so much water by a vessel of a known capacity as shall fill up the empty space, and the number of cubic inches in that space, and consequently in the irregular body, will be known.

4. To find the tonnage or burden of a ship, multiply the length of the keel, taken within board, by the breadth of the ship at the mid ship-beam, taken also within board, or from plank to plank, and multiply the product by the

 depth

depth of the ship taken from the plank below the keelson to the under part of the upper deck plank, and divide the product by 94, the quotient is the tonnage required.

5. To find the dimensions of a vessel that shall contain a given quantity.

General Rule. In vessels equally wide from top to bottom: If the base be given, divide the content in cubic inches, by the area of the base in square inches; the quotient is the depth or length of the vessel in inches.

If the length or depth of the vessel be given, divide the content by the depth, the quotient is the area of the base; and the area of the base being known, its side, if it must be a square; its length and breadth, if it must be a rectangle; or its diameter, if it must be a circle, may be found.

EXAMPLES.

1. It is required to make a vessel in form of a parallelopipedon, its base being 19 inches by 14, which shall contain 40 ale gallons; required its depth or length?

Ans. 42.4 inches.

For the area of the base is $19 \times 14 = 266$ square inches, and 40 ale gallons $= 282 \times 40 = 11280$ cubic inches; divide 11280 by 266, the quotient 42.4 is the length.

2. A vessel, equally wide from top to bottom, and that shall contain 40 ale gallons, is to be made, its depth 42.4 inches; required the area of its base?

Ans. $\frac{11280}{42.4} = 266$ square inches.

If

If the base be a square, its side will be the square root of 265, which is 16.3 inches.

If the base be a rectangle, and one of its sides 19 inches, the other will be $\frac{266}{19}=14$ inches.

If the vessel be a cylinder, its base is a circle; and, to find its diameter, divide 266 by .7854; the square root of the quotient, viz. 18.4 inches is the diameter.

6. To find the dimensions of a round vessel, having unequal bases, that shall contain a given quantity.

CASE 1. When the diameters of the two bases are given, to find its depth.

To the product of the diameters of the bases, add one third part of the square of their difference; multiply the sum by .7854, and divide the given content in cubic inches by the product; the quotient is the depth of the vessel.

EXAM. A tub is to be made that shall contain 196.5 English ale gallons, the diameter of its greater base=60 inches, and of its lesser base=36 inches; required its perpendicular depth?

Ans. 30 inches.

CASE 2. When the depth, and the diameter of one base are given, to find the diameter of the other base.

Multiply .7854 by the perpendicular depth, and divide the given content of the vessel in cubic inches by the product, the quotient is the square of a mean diameter, which consists of the product of the two diameters, and one third part of the square of their difference; and, when the diameter of one base is given, the other is found by the solution of a quadratic equation; thus,

Put a = the given diameter, x = the diameter sought, and S = the quotient of the given content, divided by the product of the depth into .7854.

Then $ax+\frac{\overline{a-x}^2}{3}=S$; that is, $ax+\frac{aa-2ax+xx}{3}=S$

$3ax+aa-2ax+xx=3S$

$xx+ax=3S-aa$

$xx+ax+\frac{aa}{4}=3S-aa+\frac{aa}{4}=3S-\frac{3aa}{4}$

$x+\frac{a}{2}=\sqrt{3S-\frac{3aa}{4}}$

$x=\sqrt{3S-\frac{3aa}{4}}-\frac{a}{2}$ which gives this rule.

Multiply the given depth by .7854, and divide the content of the vessel by the product; multiply the quotient by 3, and subtract $\frac{3}{4}$ of the square of the given diameter from the product; extract the square root of the remainder, and subtract one half of the given diameter from the root; the remainder is the diameter sought.

Exam. It is required to find the diameter of the lesser base of a tub, which shall contain 60 ale gallons; the diameter of the greater base being 40 inches, and the depth 24 inches?

Ans. The diameter of the lesser base is 18.637 inches.

1st, The content of the vessel is $282\times60=16920$ cubic inches.

2d, $.7854\times24=18.8496$ and $\frac{16920}{18.8496}=897.634$.

3d, $897.634\times3=2692.902$, and $\frac{3}{4}$ of $\overline{40}^2$ is 1200.

4th, $2692.902-1200=1492.902$.

5th, $\sqrt{1492.902}=38.637$, and $38.637-20=18.637$.

The

The content of a caſk, with the bung and head diameters being given, to find its length; the caſk being of the 3d variety.

Multiply the ſum of the ſquares of the bung and head diameters by .3927, and divide the content of the caſk in cubic inches by the product; the quotient is the length of the caſk.

Exam. A caſk is to be made which ſhall contain 109.07 Engliſh ale gallons, its bung diameter =32 inches, and its head diameter=29 inches; what muſt its length be?

Anſ. 41.99, or 42 inches.

SECT.

SECT. VI.

NAVIGATION.

DEFINITIONS and PRINCIPLES, *Fig.* 80.

1. NAVIGATION is the art of conducting a ſhip in the ſea from one port or place to another.

2. The earth and ſea conſtitute one ſpherical body, which differs very little from a perfect globe; its diameter is about 7964 miles, and its circumference 25020 miles.

3. The earth hath two motions, one round the ſun in a year, called its *annual motion*; and another round its own axis from weſt to eaſt, once in twenty-four hours, called its *diurnal motion*.

4. The annual motion of the earth, is the cauſe of the apparent annual motion of the ſun; and its diurnal motion is the cauſe of the apparent revolution of the ſun, moon, and fixed ſtars from eaſt to weſt in the ſpace of twenty-four hours.

5. Several circles are ſuppoſed to be deſcribed on the terraqueous globe, and alſo in the viſible heavens, which altogether are called circles of the ſphere; and theſe are either great or ſmall.

6. A great circle divides the globe into two equal parts, as the *equator*, *meridian*, *horizon*, *ecliptic*, and *vertical circles*.

7. A

7. A small circle divides the globe into two unequal parts, as the *polar circles*, *tropics*, and other parallels of latitude and declination.

8. The *poles* of the globe P and S are the extremities of its axis, or, they are the quiescent points, when the globe turns round its axis; and opposite to these are the celestial poles in the visible heavens.

9. The *equator* is a great circle surrounding the globe in the middle between the two poles, as ÆQ.

10. *Meridians* are great circles passing through the two poles, and cutting the equator at right angles, as P*m*S, &c.

11. The *ecliptic* EC is the path of the earth's annual motion round the sun, or the path of the sun's apparent annual motion round the earth; one half of it lies on the north, and the other half on the south side of the equator. The sun appears always somewhere in this circle; for, this being the earth's orbit, in whatever point the earth is, the sun is seen in the opposite point. The sun's daily progress is near one degree.

12. The intersections, or points where the ecliptic crosses the equator represented by the point A, are called *equinoctial points*; because, when the sun appears in either of these, the day and night are equal.

13. The points of the ecliptic, which are farthest from the equator at E and C, are called *solstitial points*; because, while the sun passes over a small part of the ecliptic on each side of these points, he is for some time nearly at the same distance from the equator, and so seems to stand still.

14. The

14. The meridian which passes through the equinoctial points is called the *equinoctial colure*, as PAS; and that which passes through the solstitial points is called the *solstitial colure*, as PÆS. These are the only two meridians which divide both equator and ecliptic into four equal parts.

15. The ecliptic is divided into twelve equal parts, called *signs*, each containing 30°, namely,

♈	♉	♊	♋	♌	♍	♎
Aries,	Taurus,	Gemini,	Cancer,	Leo,	Virgo,	Libra,

♏	♐	♑	♒	♓
Scorpio,	Sagittarius,	Capricornus,	Aquarius,	Pisces.

16. The *Zenith* Z, is that point in the visible heavens directly over the spectator's head, and the Nadir N, is the opposite point.

17. There are two horizons, the *rational* and *sensible*. The rational horizon HO, is a great circle surrounding the globe in the middle between the zenith and nadir, which are its poles.

18. The *sensible*, or *visible horizon*, is the boundary of the spectator's view, and is properly a plane touching the earth in the place where the spectator stands.

19. *Latitude* on the earth, is the distance of any place north or south from the equator, reckoned on the meridian. Thus ZÆ is the latitude of Z.

20. *Longitude* on the earth, is the distance of any place east or west from the first meridian, reckoned on the equator; or the longitude of any place is the arch of the equator between the first meridian and the meridian of that place.

21. The

21. The first meridian is not fixed to any particular place, but chosen according to the fancy of different authors and nations. In English maps and globes, the meridian of London is the first. In French maps, the meridian of Ferro Island, &c.

22. In the visible heavens, circles passing through the poles of the ecliptic, and cutting it at right angles, are called *circles of latitude*, as *rt*.

23. *Latitude* in the heavens, is the distance of any star from the ecliptic, reckoned on the circle of latitude passing through the star.

24. *Longitude* of any star in the heavens, is an arch of the ecliptic between the vernal equinox and the circle of latitude passing through the star, reckoning in the order of the signs.

25. *Declination*, is the distance of the sun, or any star, north or south from the equator, reckoned on the meridian.

26. The obliquity of the ecliptic, or the angle it makes with the equator CAQ, is 23° 28′, and the same is the sun's greatest declination.

27. Circles parallel to the equator, are called *parallels of latitude on the earth*, and *parallels of declination in the heavens*. One of these is supposed to pass through every point in the meridian; but four are distinguished by particular names, viz. the two *tropics*, and the two *polar circles*.

28. The *tropics* E*d* and *b*C are distant from the equator 23° 28′. That on the north is called the *Tropic of Cancer*, because it touches the ecliptic in the beginning

of the sign Cancer; and that on the south is called the *Tropic of Capricorn*, because it touches the ecliptic in the beginning of the sign Capricorn. These circles are the apparent paths of the sun on the solstitial days, viz. on the 21st of June, and 22d of December.

29. The tropics are the boundaries of the sun's declination north or south from the equator. The space between them is called the *torrid zone*; because the sun is always vertical to some place within this tract of the earth's surface.

30. The arctic, or north polar circle *ar*, is 23° 28′ distant from the north pole; the space within it is called the *north frigid zone*. The antarctic, or south polar circle, *te*, is 23° 28′ distant from the south pole, and the space within it is called the *south frigid zone*.

31. The space between the tropic of Cancer and the arctic circle, is called the *north temperate zone*; and the space between the tropic of Capricorn and the antarctic circle, is called the *south temperate zone*.

32. When the sun appears in either equinox, he hath no declination. In all other points of his apparent path, the arch of the ecliptic between the vernal equinox and the sun's place, is called the *sun's longitude*. The arch of the equator between the same equinox and the meridian passing through the sun's place, is called the *sun's right ascension*, and the arch of the meridian between the equator and the sun's center, is called the *sun's declination*. The same is to be understood of the moon, or any star.

33. When the sun is in either equinox, he rises due east, and sets due west; but in all other points of the

ecliptic,

ecliptic, he rises and sets at some distance from the east and west points; and this distance is called the *sun's amplitude.* The greatest amplitude A*m* or A*n*, is when the sun is in either tropic.

34. The amplitude is always of the same name with the declination; that is, when the sun hath north declination, he rises between east and north, and sets between west and north; but, when he hath south declination, he rises between east and south, and sets between west and south. The same is true of the moon, or any star.

35. Vertical or azimuth circles Z*m*N and Z*n*N pass through the zenith and nadir, and cut the horizon at right angles. Of these, the meridian cuts the horizon in the north and south points; and the prime vertical crosses the meridian at right angles in the zenith, and cuts the horizon in the east and west points.

36. An azimuth circle is supposed to pass through the sun, moon, or any star which appears above the horizon. That part of the azimuth circle between the horizon and the sun at any time, is called his *altitude;* and the other part between the sun and the zenith, is called his *zenith distance;* and the arch of the horizon, between the point where the azimuth circle cuts it, and the north or south points, is called the *sun's azimuth.* This is reckoned either from north or south, the one being always the supplement of the other.

37. The arch of the equator between the vernal equinox and that point of it which rises with the sun, is called *the sun's oblique ascension;* and the arch of the equator between the same equinox and that point of it which sets with the sun, is called the *sun's oblique descension.*

 38. The

38. The difference between the sun's right and oblique ascension, is called the *ascensional difference*. This quantity reduced to time, at the rate of 15° to an hour, shews how long the sun riseth before or after 6 o'clock.

39. Oblique ascension and descension, with ascensional difference, apply only to places between the equator and polar circles. For, at the equator, the right and oblique ascension are always the same; and within the frigid zones, the sun shines for several days and nights together without setting.

40. The mariner's compass, *Fig*. 81. is a representation of the sensible horizon of that place, where it is, by means of a circular piece of paper, called a *card*, its circumference being divided into 32 equal parts, called *points*. This card being properly fixed to a piece of steel, called a *needle*, (which, by the touch of a loadstone, is endued with the property of pointing north and south) and placed on a supporter on which it can turn freely round; the lines drawn from the center to north, south, and all other points on the card, will tend towards the same points of the horizon; and therefore, by the help of this card or compass, a ship may be directed in any proposed tract or course.

41. The several parts of the earth's surface are most conveniently represented on the surface of a small globe, because this body is similar to the earth; but this does not answer the purposes of navigators, and therefore other representations on plain surfaces have been contrived.

42. The first and most simple representation of the earth's surface, is the *plain chart*, wherein the meridians are

are parallel to each other, and the parallels of latitude are all of the same length with the equator; but this does not agree with the figure of the earth, and therefore, the extent of lands and seas, the situations, distances, and bearings of places, and consequently the solution of questions in navigation on the principles of the plain chart, cannot be true. The error, however, is not equally great in all places; for, within the torrid zone, the plain chart agrees nearly with the globe: And in maps representing a small part of the earth's surface, of about 2 or 3 degrees of latitude, and as many of longitude, the error is not very considerable, if the place represented lies between the equator and 40° of latitude. The error increases with the latitude, and is the greatest of all towards the poles.

43. The resolution of questions in navigation, on the principles of the plain chart, is called *plain-sailing*. The other methods, known by the names of Middle Latitude, and Mercator's Sailing, being only attempts to correct the errors of the plain chart, cannot be understood until plain sailing is learned.

44. A ship's course is known by the compass, and the distance she sails in an hour is found by the log-line thus: A small cord, of a sufficient length, is marked with knots at the distance of $\frac{1}{120}$ part of a nautical mile, or minute, from each other; and, to the end of the cord, a piece of wood is fixed, called the *log*. The line thus prepared is wound up on a reel, which turns easily round. They heave out the log, and, at the same time, set a half-minute sand-glass a-running, and so find how many knots the ship sails in half a minute; and, because half a minute

nute is the same part of an hour, as a knot is of a mile, the number of knots shows the number of miles the ship runs in an hour.

Note. A nautical mile is 6120 feet; and consequently the distance between the knots on the log-line should be 51 feet, viz. $\frac{6120}{120}$; but, as it is safer to have the reckoning rather before the ship than after it, therefore 50 feet may be taken as the proper length between knot and knot on the log-line.

45. A rhumb line, is any right line drawn from the center of the compass; such as N.E. S.S.W. &c.

46. The course is the angle which the rhumb line makes with the meridian; and may be reckoned either in degrees, or points of the compass.

47. There are 32 points in the compass, and each of these is divided into halfs and quarters.

48. Because the whole circumference of any circle consists of 360°, and the same is divided into 32 points: to find how many degrees one point contains, divide 360° by 32, and the quotient will be $11° 15' =$ one point.

49. Distance is the number of miles or leagues sailed on any rhumb in a given time,

50. Difference of latitude is the distance a ship has made to the northward or southward of the place she set out from, and it is reckoned on the meridian.

51. Departure is the distance east or west, which a ship has made from the meridian of the place she sailed from, reckoned on the parallel of latitude which she

has

has arrived in; and in the plain chart, it is the same as difference of longitude.

52. If a ship sails on a meridian either north or south, she makes no departure, and her distance and difference of latitude are the same.

53. If a ship sails due east or west, she makes no difference of latitude, and her departure and distance are the same; but,

54. *Fig.* 82. If a ship sails from any point A to any other point C, neither lying in the same meridian with A, nor in the same parallel of latitude; draw AB, the meridian of the place sailed from, and from C let fall the perpendicular CB; then AC is the distance sailed; AB the difference of latitude of the places A and C, and BC is the departure or distance of the meridians of A and C; also, CAB is the angle of course, and ACB the complement of the course.

55. The difference of latitude and departure are always the legs of a right angled triangle, and the distance sailed is the hypothenuse. The departure is always opposite to the angle of course, and the difference of latitude is opposite to the complement of the course.

56. When the course is 45°, or 4 points, the difference of latitude is equal to the departure.

57. When the course is less than 45°, or four points, the difference of latitude is greater than the departure; but when the course is greater than 45° or four points, the departure is greater than the difference of latitude.

58. When a figure representing a ship's course is to be constructed, it must first be considered whether the

ship

ship sails northward or southward; and whether she goes to the eastward or westward of the place she departed from; that the lines of the figure may be drawn accordingly.

Let the upper part of the paper, or whatever the figure is drawn on, represent the north, then the lower part will be south, the right hand east, and the left hand west.

Draw a north and south line to represent the meridian of the place sailed from; then if the course is to the southward, mark the upper end of the line for the place sailed from; but if the course is northward, mark the lower end of the line for that place.

The line representing the distance sailed, must be drawn through the point sailed from; and the parts given must be taken from a scale, and the figure compleated according to the conditions of the question.

A TABLE

A Table

Of the Proportions for Solving the Cases of Plain Sailing : *in the Nautical Terms.*

Cases	*Given.*	*Sought.*	*Solutions.*
1.	Courſe and Diſtance	Diff. lat. and Depart.	R : coſ. courſe : : diſt. : diff. lat. R : ſine courſe : : diſt. : departure
2.	Diſtance and Diff. lat.	Courſe and Depart.	Diſt. : diff. lat. : : R : coſ. courſe R : ſine courſe : : diſt. : departure
3.	Courſe and Diff. lat.	Diſt. and Depart.	Coſ. Cour.: R : : diff. lat. : diſtance R: tan. cour. : : diff. lat. : departure
4	Courſe and Depart.	Diſt. and Diff. lat.	Sine cour. : R : : depart. : diſtance. Tan. cour.: R : : depart. : diff. lat.
5.	Diſtance and Depart.	Courſe and Diff. lat.	Diſt. : depart. : : R : Sine courſe R : coſ. courſe : diſt. : diff. lat.
6	Diff. lat. and Depart	Courſe and diſtance.	Diff. lat. : depart. : : R : tan. courſe Sine cour. : R : : depart. : diſtance.

Hence, to ſolve the ſix caſes of plain ſailing, is no more than to ſolve as many right angled plain triangles.

PLAIN SAILING, *Fig.* 82.

CASE 1. Given the courſe and diſtance ſailed, to find the difference of latitude and departure.

EXAM. 1. A ſhip, from latitude 12° 30′ N. has ſailed N. W. by N. 147 miles; what latitude hath ſhe come to, and what departure hath ſhe made?

The courſe, being three points, is 33° 45′,
To find the difference of latitude:
As Rad. : Cos courſe :: Diſt. : diff. Lat. = 122.2 *m.*
To find the departure,
As Rad. : Sine courſe :: Diſt. : departure = 81.7.

To latitude ſailed from	12° 30′ N.
Add diff. of latitude	2° 2′
Sum is the latitude come to	14° 32′ N.

EXAM. 2. A ſhip from latitude 20° 20′ N. has ſailed 265 miles N. E. by E. ½ E.; required the latitude come to, and the departure?

Anſ. Latitude 22° 25′ N. departure 233.7 miles.

Caſe 2. Given the diſtance ſailed, and difference of latitude, to find the courſe and departure.

EXAM. 1. A ſhip from latitude 15° 20′ S. ſails 320 miles between S. and W. and then is in latitude 19° 25′ S.; required her courſe and departure?

Anſ. Courſe 40° 3′ S. Weſtward, departure 206 miles.

2. A ſhip from latitude 20° 49′ N. ſails 268 miles between S. and E. and then is in latitude 17° 1′ N.; required her courſe and departure?

Anſ. Courſe S. 31° 43′ W. departure 141.

CASE

CASE 3. Given the difference of latitude and course, to find the distance and departure?

EXAM. 1. A ship from latitude 14° 17′ S. sails S. E. ¾ E. until she is in latitude 19° 10′ S.; required the distance sailed, and departure?

Ans. Distance 491.8 *m.* departure 395 *m.*

2. A ship from latitude 19° 39′ N. sails W. N. W. until she is in latitude 23° 30′ N.; required her distance and departure?

Ans. Distance 603.6 departure 557.6.

CASE 4. Given the course and departure, to find the distance sailed, and difference of latitude.

EXAM. 1. A ship from latitude 16° 16′ S. sails S. W. by W. ½ W. until she has made 499 miles of Westing; required the distance sailed, and the latitude come to?

Ans. Dist. 565.8 *m.* and lat. come to is 20° 43′ S.

2. A ship from latitude 27° 37′ N. sails S. E. by S. ½ E. until she has made 256 miles of Easting; required her distance sailed, and the latitude come to?

Ans. Distance 429.7, latitude 21° 52′ N.

CASE 5. Given the distance and departure, to find the course and difference of latitude.

EXAM. 1. A ship from latitude 1° 30′ N. sails 294 miles between S. and E. and then has made 186 miles of Easting; what course did she steer, and what latitude hath she come to?

Ans. Course 39° 14′; or S. E. by S. ½ E. and she is in latitude 2° 18′ South.

2. A ſhip from latitude 19′ S. ſails 396 miles between W. and N. and thereby made 219 miles of departure; required her courſe and latitude come to?

Anſ. Courſe N. 34° 51′ W. latitude 2° 22′ N.

Case 6. Given the difference of latitude and departure, to find the courſe and diſtance ſailed.

Exam. 1. A ſhip from latitude 23° 28′ N. has ſailed between N. and W. until ſhe came to latitude 27° 1′ N. and made 199 miles of Weſting; required her courſe and diſtance? *Anſ.* Courſe N. 43° 3′ W. diſt. 291.5 *m.*

2. A ſhip from latitude 48° 30′ N. is arrived in latitude 49° 54′ N. and is got 54 miles to the weſtward of her departed meridian; what courſe did ſhe ſteer, and what diſtance hath ſhe ſailed?

Anſ. Courſe 32° 44′, or N. W. by N. diſt. 99.86 *m.*

Of the Table *of difference of Latitude and Departure.*

This table conſiſts of numbers expreſſing the ſides of right angled triangles, the diſtance being the hypothenuſe, the difference of latitude and departure the two legs, and the courſe and its complement are the acute angles. Any two of theſe being given, (except the two acute angles) the other parts of the triangle may be found by inſpection, if within the extent of the table.

The diſtances 1, 2, 3, &c. at top and bottom of each page may be accounted 10, 20, 30, or 100, 200, 300, &c. if the numbers under the titles of difference of latitude and departure are increaſed in the ſame proportion.

Hence, if the diſtance conſiſts of two ſignificant figures, the difference of latitude and departure are to be taken out at twice, and, if of three figures, at thrice.

1. Given

1. Given the courſe and diſtance failed, to find the difference of latitude and departure.

Find the courſe on the left hand ſide of the page, if it be leſs than 45°, or 4 points; and on the right hand ſide, if it be greater than 45°; and even with the courſe, below or above the diſtance, you have the difference of latitude and departure.

EXAM. 1. A ſhip ſails S. S. W. ¼ W. 50 miles, required her difference of latitude and departure?

Courſe.	Diſt.	Diff. Lat.	Depart.
2¼ p.	50	42.85	25.75 the anſwer.

2. A ſhip ſails E. N. E. ¼ E. 68 miles; required her difference of latitude and departure?

Courſe.	Diſt.	Diff. Lat.	Depart.
6¼	60	20.21	56.49
	8	3.06	7.39
	68	23.27	63.88 the anſwer.

3. A ſhip ſails 148 miles on a courſe 49° S. eaſtward, required her difference of latitude and departure.

Courſe.	Diſt.	Diff. Lat.	Departure.
49°	100	65.60	75.47
	40	26.24	30.18
	8	5.24	6.03
	148	97.08	111.68 the anſwer.

2. The difference of latitude and departure being given, to find the courſe and diſtance.

Find the difference of latitude and departure together in their columns, or the neareſt numbers to them; and the

the course is on the same line at the side of the page, and the distance at the top or bottom.

When the difference of latitude and departure cannot be found nearly in the table, take some part of each, such as $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{12}$; and these parts being found, the course is even with them on the side of the page, and the same part of the distance on the head or foot of the page, as was taken of the difference of latitude and departure.

EXAM. 1. Given the difference of latitude 59 miles S. and the departure 68 miles W.; required the course and distance?

Ans. The course is 49° S. Westward, or S. W. $\frac{1}{2}$ W. and the distance 90 miles.

In this example, the difference of latitude and departure are found, nearly, in the column marked with 9 of distance, and even with 49° on the side of the page.

2. Given the difference of latitude 30 miles N. and the departure 18 miles E.; required the course and distance?

Here the difference of latitude and departure are not to be found together in the table; but, dividing each by 5, the quotients 6 and 3.6 are found under 7 of distance, and even with 31°; therefore, the distance is $7 \times 5 = 35$ miles, and the course is 31° N. Eastward.

3. Given the difference of latitude 86.2 miles S. and the departure 42 miles E.; required the course and distance?

In this example, divide both 86.2 and 42. by 12, the quotients 7.18 and 3.5 are found together under the distance 8, and even with the course 26°; therefore the course is 26° S. Eastward, and the distance is $8 \times 12 = 96$ miles.

A table

A table of this extent will do pretty well when the course is even degrees, or points and quarter points of the compass; but, when the course is not exactly found in the table, there will be some error in the answer.

To work a traverse, or to reduce a compound course to a single one.

Find the difference of latitude and departure for each single course, and place them properly in columns, marked N. S. E. W.; find the sum of each column, and the difference of the columns N. and S. will be the difference of latitude; also the difference of the columns E. and W. will be the departure which the ship has made in the whole traverse; and, from these, the direct course and distance may be found.

Exam. 1. A ship from latitude 39° 30′ N. meeting with contrary winds, sailed S. E. by S. 67 *m.* S. E. by E. 53 *m.* W. S. W. 45 *m.* N. E. by N. 74 *m.* W. 57 *m.* and S. by E. 83 *m.*; required her difference of latitude, departure, course, and distance from the place she sailed from, and the latitude come to? *Fig.* 83.

Courses.	*Dist.*	*N.*	*S.*	*E.*	*W.*
S. E. by S.	67		55.7	37.2	
S. E by E.	53		29.4	44.1	
W. S. W.	45		17.2		41.6
N. E. by N.	74	61.5		41.1	
W.	57				57.
S. by E.	83		81.4	16.2	
					98.6
		61.5	183.7	138.6	
			61.5	98.6	
			122.2	40.0	

By these different courses, the ship hath got 122.2 *m.*

to

to the south, and 40 *m.* to the east of the place sailed from. Hence the course is 18° 7′ south-eastward, and the distance 128.6 *m.* The latitude come to is 37° 28′ N.

2. Yesterday noon we were in latitude 3° 18′ S. and since then, we have plied on the following courses, viz. N. N. E. 22 *m.* N. by W. 30 *m.* N. E. by E. 40 *m.* E. S. E. 25 *m.* S. S. W. 18 *m.* N. W. by N. ½ W. 50 *m.* N. E. ½ E. 42 *m.* W. by S. ½ W. 45 *m.* S. W. by S. 20 *m.* E. by N. ¼ E. 62 *m.*; required our present latitude and departure, with the course and distance made good?

Ans. The ship is in latitude 1° 39′ S. departure 58.4 *m.* course 30° 32′ northeastward, or N. N. E. ¾ E. and distance 115 miles.

3. A ship in latitude 45° N. is bound for a port in latitude 50° 57′ N. bearing N. W. by W. but, by contrary winds is forced to steer the following courses, viz. N. N. W. 35 leagues, N. E. by N. 42 leagues, W. N. W. 38 leagues, W. 20 leagues, N. W. by N. 23 leagues, N. 12 leagues, E. N. E. 39 leagues, S. W. by W. 48 leagues, N. by E. 31 leagues, N. N. W. ½ W. 38 leagues, and W. by N. 45 leagues; required the course and distance made good, as also the course and distance from the ship to the port she is bound for?

Ans. The course made is 34° 10′ northwestward, distance 210.7 *m.* and the course from the ship to the port is 47° 38′ southwestward, and distance 82 miles.

PARALLEL SAILING.

The distance of two meridians on the equator being given, to find their distance in any parallel of latitude, and *vice versa.* *Fig.* 84.

Let

Let EP represent a quadrant of the meridian, E a point on the equator, P the pole, and C the centre of the earth, and L any point on the meridian between the equator and the pole, whose latitude is LE, and its complement LP. Draw LB parallel to CE; then, if the quadrant CPE revolve round the axis CP, the point E will describe the equator, and the point L a parallel of latitude. These circles are, in proportion to each other as their *radii* CE, and BL. But wholes are as their similar parts; therefore, any part of the equator is to a similar part of the parallel of latitude described by the point L, as CE is to BL; that is, as the radius is to the co-sine of the latitude. Hence, as radius is to the co-sine of any latitude, so is the minutes difference of longitude, or the distance of the two meridians on the equator, to their distance on the parallel of latitude.

CASE 1. Given the latitude and difference of longitude, to find the distance on the parallel.

EXAM. 1. A ship in latitude 32° N. sails due W. until her difference of longitude is 384 miles; required the distance sailed?

As Rad. : Cos. lat. 32° : : *m.* diff. long. 384 : dist. 325.6.

2. How many miles E. or W. in the latitude of 56° are equal to 1° of longitude?

Ans. 33.5 *m.*

CASE 2. Given the distance sailed in any parallel of latitude, to find the difference of longitude.

Exam. A ship from latitude 53° 36′ N. longitude 10° 18′ E. sails due W. 236 *m.*; required her present longitude?

Ans. 3° 40′ E

Case 3. Given the difference of longitude, and the distance sailed, to find the latitude of the parallel sailed on.

Exam. In what latitude does 384 miles of meridional distance answer to 500 miles difference of longitude?

Ans. 39° 49′.

Case 4. Having the distance of two places in a given parallel, to find the distance of two other places lying under the same meridians with the former in another given parallel of latitude.

Exam. Two ships in latitude 32° 20′ N. distant from each other 256 miles, sail directly N. until they are in latitude 44° 30′ N.; how far are they then asunder?

Ans. 216 *m.*

MIDDLE LATITUDE SAILING.

Since the distance of any two meridians varies with the latitude, when a ship sails any distance on an oblique rhumb, and thereby gets into a different latitude; what is called her departure in plain sailing, is not the distance between the meridian of the place sailed from, and the meridian of the ship in her present latitude. To correct this error, the distance of the two meridians in the middle parallel between the latitude sailed from, and that come to, is taken for the true departure; and hence, this correction is called *middle latitude sailing*; which, although

though not perfectly accurate, answers pretty well for keeping an account of the ship's difference of longitude in small distances, such as one day's sailing.

The computations depend on the following rules:

1. Take the half sum of the two latitudes for the middle latitude; and then, by parallel sailing, as co-sine of middle latitude is to the radius, so is the departure to the difference of longitude. Or,

2. Because Cos. mid. lat. : Rad. : : Dep. : Diff. long; therefore,

Cos. mid. lat. × Diff. long. = Rad. × Depart.; but,

Diff. lat. : Dep. : : Rad. : Tan. course; therefore,

Diff. lat. × Tan. course = Cos. mid. lat. × diff. long. Hence, as Cos. mid. lat. is to Tan. course, so is diff. lat. to diff. long.

Case 1. Given the latitudes and longitudes of two places, to find their bearing and distance.

Exam. 1. What is the course and distance from Cape Clear, in Ireland, in latitude $51°$ $18'$ N. longitude $9°$ $50'$ W. to the island of St. Mary, one of the Azores, in latitude $37°$ N. and longitude $22°$ $56'$ W.?

Diff. lat. $14°$ $18 = 858$ *m.* sum of lats. $= 88°$ $18'$. mid. lat. $= 44°$ $9'$. Diff. long. $= 13°$ $6' = 786$ *m.*

Then, as Diff. lat. : Diff. long. : : Cos. mid. lat. : Tan. course $= 33°$ $19'$, or S. W. by S. nearly.

As Cos. course : Rad. : : Diff. lat. : Dist. $= 1026.7$ miles.

 2. What

2. What is the course and distance from Leith, in latitude 56° N. longitude 3° W. to Bergen, in latitude 60° N. and longitude 6° E.

Ans. The course is 50° northeastward, or N. E. ½ E. nearly, and distance is 373.3 miles.

CASE 2. One latitude, course, and distance sailed being given, to find the other latitude and difference of longitude.

EXAM. A ship from the Lizard in latitude 50° N. and longitude 5° 14′ W. sails 150 miles on a course 50° 6′ south-westward; required her latitude and longitude?

Ans. Latitude 48° 24′, longitude 8° 10′ W.

As Rad. : Cos. course : : Dist. : Diff. lat. = 96.2 *m.* = 1° 36′; therefore, the latitude of the ship is 48° 24′, and the middle latitude is 49° 12′. As Cos. mid. lat. : Tan. course : : Diff. lat. : Diff. long. 176 *m.* or 2° 56′. In the same manner, examples may be wrought in all the cases.

2. A ship from latitude 55° 57′ N. longitude 6° 30′ W. sails N. W. by W. 212 miles; required her longitude and latitude?

Ans. Longitude 11° 53′ W. latitude 57° 55′ N.

CASE 3. Given the distance and difference of latitude, to find the course and difference of longitude.

EXAM. 1. A ship from latitude 48° 30′ N. sails 194 miles between N. and W. and then is in latitude 50° N. required her course and difference of longitude?

Ans. Course 62° 22′, or N. W. by W. ½ W. difference of longitude 4° 23′ W.

2. A

2. A ſhip from latitude 47° 27′ N. longitude 15° W. ſails 424 miles between N. and E. and then is in latitude 51° 59′ N.; required her courſe and longitude?

Anſ. Courſe 50° 6′, or N. E. ½ E. longitude 6° 37′ W.

CASE 4. Given the courſe and difference of latitude, to find the diſtance and difference of longitude.

EXAM. 1. A ſhip from a port in latitude 49° 19′ N. and longitude 12° 14′ W. ſails S. S. E. ½ E. until ſhe is in latitude 45° 49′ N.; required the diſtance ſailed, and the longitude of the ſhip?

Anſ. Diſtance 238 *m.* longitude 9° 28′ W.

2. A ſhip from latitude 45° 30′ N. and longitude 17° 30′ W. ſails N. W. by W. ¼ W. until ſhe is in latitude 49° 32′ N.; required her diſtance and longitude?

Anſ. Diſtance 470.7 *m.* longitude 27° 28′ W.

CASE 5. The courſe and departure being given, to find the diſtance ſailed, with the difference of latitude, and of longitude.

EXAM. A ſhip from latitude 50° 20′ N. and longitude 4° 12′ E. ſails E. N. E. until her departure is 119 miles: Required the diſtance ſailed, with the latitude and longitude of the ſhip?

Anſ. Diſtance 128.8 *m.* latitude 51° 9′ N. and longitude 7° 20′ E.

CASE 6. The diſtance ſailed and departure being given, to find the courſe, the difference of latitude and difference of longitude.

EXAM.

EXAM. A ſhip from latitude 58° 20′ N. and longitude 4° 12′ W. ſails 256 miles between S. and E. and then has made 128 miles of eaſting: Required her courſe, latitude and longitude?

Anſ. Courſe S. 30° E. latitude 54° 40′, long. 0° 20′ W.

CASE 7. The difference of latitude and departure being given, to find the courſe, the diſtance ſailed, and difference of longitude.

EXAM. A ſhip from latitude 42° 30′ S. and longitude 43° 12′ W. ſails between S. and W. until ſhe is in latitude 49° 2′ S. and has made 212 miles of weſting: Required her courſe, diſtance and longitude?

Anſ. Courſe S. 28° 24′ W. diſtance 445.7 and longitude 48° 16′ W.

Queſtions in middle latitude may be ſolved by the Table of difference of latitude and departure.

CASE 1. With the complement of the middle latitude found among the courſes, and the difference of longitude among the diſtances, find the departure in its column.

2. With the difference of latitude and departure found together, you have the courſe and diſtance as in plain ſailing.

EXAM. Let the difference of latitude = 858 *m.* the difference of longitude = 786 *m.* the middle latitude = 44° 9′, and its complement = 46° nearly, as in example 1.

Courſe.	Diff. Long.	Depart.
46°	700	503.54
	80	57.54
	6	4.31
	786	565.39

The

The difference of latitude being 858, and the departure 565.39; if these be divided by 128, the quotients 6.7031 and 4.4171 are nearly found even with the course 33° under the distance 8; therefore, the course is 33°, and distance 128 × 8 = 1024 miles.

CASE 2. With the course found among the degrees, and the distance in its column, find the difference of latitude and departure, as in plain sailing.

2. Find the comp. of the middle latitude among the courses, and the departure, or some part of it, in its column; the corresponding distance is the difference of longitude, or a part of it.

EXAM. A ship from latitude 50° N. sails 150 *m.* on a course 50° South-westward; required her difference of latitude and of longitude?

To the course 50°, and distance 150 *m.* the difference of latitude and departure are 96.4 and 114.9 *m.*; therefore, the ship is in 48° 24′ N. The middle latitude is 49° 12′, and its comp. near 41°. Divide the departure 114.9 by 22, the quotient is 5.222; which is found even with 41° below 8; therefore, the difference of longitude is 8 × 22 = 176 *m.* W.

To work a TRAVERSE *by Middle Latitude.*

1. Find the difference of latitude and departure for each course, as in plain sailing.

2. Make two columns marked E. and W.; find the difference of longitude made on each course, and place them in their proper columns; the difference of their sums is the difference of longitude made in the whole traverse.

EXAM.

EXAM. A ſhip from latitude 50° N. and longitude 6° 14′ W. ſails, 1ſt, W. S. W. 26 *m.* 2d, S. E. by E. 30 *m.* 3d, S. W. by W. ½ W. 36 *m.* 4th, W. 64 *m.* 5th, S. W. ½ W. 25 *m.* 6th, S. E. by S. 37 *m.* 7th, S. by W ¼ W. 40 *m.*; required the latitude and longitude of the ſhip, with the direct courſe and diſtance made good?

Courſes.	D.	N.	S.	E.	W.	Lats.	M.Lat.	E.	W.
						50° 0			
W S W	26		10.0		24.0	49 50	49° 55′		37.27
S E by E	30		16.7	24.9		49 33	49 41	38.48	
S W by W half W	36		17.0		31.8	49 16	49 24		48.86
W	64				64.0	49 16	49 16		98.07
S W half W	25		15.9		19.3	49 0	49 8		29.49
S E by S	37		30.8	20.6		48 29	48 45	31.24	
S by W 1-4th W.	40		38.8		9.7	47 50	48 9		14.53
			129.2	45.5	148.8			69.72	228.22
					45.5				69.72

Depart. 103.3 Diff. Long. 158.50

or 2° 38′; hence the ſhip is in longitude 8° 52′ W.

For the Courſe and Diſtance.

Departed latitude	= 50° 0′ N
Difference of latitude 129 *m.*	= 2 9 S
Preſent latitude	47 51 N
Middle latitude	48 55

1. As diff. lat.	= 129.2	2. As ſine cour.	38° 52′
Is to diff. long.	158.5	Is to radius,	
So is Cos. mid. lat.	48° 55′	So is depart.	103.3
To tang. courſe	38 52	To diſtance	= 164.6
or S. W. by S. ½ W.			

This is the true way of working a traverſe by middle latitude; but many ſeamen work the ſame way as in plain

plain sailing; and, having found the course and distance made, and the latitude come to, they find the middle latitude; and then,

As co-sine of middle lat. is to tangent of course,
So is difference of latitude to difference of longitude.

Or, by the Traverse Table,

Find the complement of the middle latitude among the courses, and the departure in its column, then the distance answering thereto is the difference of longitude: And this method is correct enough for any run that can be made in one day in latitudes less than 54°; but, in higher latitudes, it is necessary to find the difference of longitude to each course, as in this example.

MERCATOR's SAILING.

This method of solving nautical questions is a correction of the errors of the plain chart, proposed by Mr. GERARD MERCATOR, a *Fleming*, who published the first chart of this kind about the year 1550. Its true principles were discovered, and demonstrated by Mr. EDWARD WRIGHT, about the year 1590.

In Mercator's chart, the meridians are parallel right lines; but to correct the errors arising from thence, the degrees of latitude are enlarged in the same proportion as the degrees of longitude are.

Thus, Northing and Southing, is every where proportional to Easting and Westing; and the bearings of places from each other are the same on this chart as on the globe itself. The length of any degree of latitude in this projection, is determined by the following theorem.

The natural length of any part of the meridian, such as 1′, is to its length on Mercator's chart, as radius is to the secant of the latitude of that part of the meridian. *Fig*. 85.

Let EP be a quadrant of the meridian, E a point on the equator, P the pole, CE the radius of the equator, LB the co-sine of the latitude of the point L, and CS the secant of that latitude. Because the triangles CEA, CSE are similar, CA : CL : : CE : CS ; that is, as Cos. lat. : Rad. : : Rad. : Sec. lat. ; but as Cos. lat. is to Rad. so is any part of that parallel of latitude to a similar part of the equator ; therefore, as radius is to sec. lat. so is any part of the parallel to a similar part of the equator. But the equator is equal to the meridian, as being both great circles ; and, in Mercator's chart, every parallel of latitude is equal to the equator ; consequently, the natural length of any part of a parallel of latitude is to its length on Mercator's chart, as radius is to the secant of the latitude ; and therefore, to enlarge the several parts of the meridian in the same proportion, the natural length of any part of the meridian must be to its length on Mercator's chart, as radius is to the secant of the latitude of that part of the meridian.

The lengths of the several parts of the enlarged meridian being calculated and ranged in order in a table, are called *meridional parts*.

Mr. *Edward Wright*, who calculated the first table, made the meridional parts for 1 minute equal to the secant of 1 minute ; for two minutes, equal to the sum of the secants of 1 and 2 minutes ; and, for 3 minutes, equal to the sum of the secants of 1, 2, and 3 minutes ; and so on, by a constant addition of secants.

There-

There is in most books of Navagation, a table of meridional parts, from which the meridional parts answering to any latitude may be had, and consequently the meridional difference of latitude by subtraction or addition, according as the latitudes are of the same or of contrary names.

Let A and C. *Fig.* 86. represent any two places lying in different latitudes and longitudes, properly laid down on Mercator's chart. Let AD be the meridian of the place A; draw AC, and from C let fall the perpendicular CD; set the proper difference of latitude from A to B, and draw BE parallel to DC, then AE is the true distance of the places A and C. AC the enlarged distance, AD the enlarged difference of latitude, DC the difference of longitude, and BE the departure.

CASE 1. Given the latitudes and longitudes of two places, to find their bearing and distance.

EXAM. Required the course and distance from Caithness-point, in the north of Scotland, in latitude 58° 46′ N. longitude 3° 17′ W. to New York in North America in latitude 41° 5′ N. longitude 74° 15′ West?

The proper diff. lat. is 17° 41′ = 1061 *m.* and the diff. long. is 70° 58′ = 4258 *m.*

The meridional parts answering	58° 46′	= 4382.1
Ditto	41° 5′	= 2708.3
Meridional difference of latitude		= 1673.8

1. As Merid. diff. lat. : Diff. long. :: Rad. : Tan. course = 68° 32′, or W. S. W.

2. As Cos. course : Rad. :: Prop. diff. lat. : Dist. = 2899.2 *m.*

Exam. 2. What is the courſe and diſtance from Leith, in latitude 56° N. longitude 3° W. to Bergen, in Norway, in latitude 60° N. longitude 6° E.?

Anſ. Courſe N. 50° E. Diſtance 373.3 miles.

Case 2. Given the latitude and longitude of the place ſailed from, with the courſe and diſtance ſailed, to find the latitude and longitude of the ſhip.

Exam. A ſhip takes her departure from a place in latitude 51° 18′ N. longitude 9° 50′ W. and ſteers S. 33° 8′ W. until ſhe has run 1024 miles; required her preſent latitude and longitude?

Anſ. Latitude 37° 1′ N. longitude 22° 55′ W.

2. A ſhip from latitude 56° N. longitude 3° W. ſails N. E. by N. 373.; required her preſent latitude and longitude?

Anſ. Latitude 61° 10′ N. longitude 3° 38′ E.

Case 3. The courſe and difference of latitude being given, to find the diſtance ſailed, and difference of longitude.

Exam. 1. A ſhip from latitude 56° 4′ N. and longitude 3° W. ſails N. N. E. $\frac{1}{4}$ E. until ſhe is in latitude 60° 29′ N.; required the diſtance ſailed, and the longitude come to?

Anſ. The diſtance is 293.1 miles, and the ſhip hath come to 0° 59′ E. longitude.

2. A ſhip from latitude 52° 20′ N. and longitude 6° 24′ W. ſails W. S. W. $\frac{1}{4}$ W. until ſhe is in latitude 47° 49′ N.; what diſtance hath ſhe ſailed, and what longitude hath ſhe come to?

Anſ. Diſtance ſailed is 804.5 miles, and the longitude come to is 26° 5′ W.

Case

CASE 4. Given the difference of latitude and distance, to find the course and difference of longitude.

EXAM. 1. A ship from latitude 53° 14′ N. and longitude 2° E. sails 256 miles between N. and E. and then is in latitude 56° N.; required her course and present longitude;

Ans. Course 49° 35′ Northeastward, or N. E. ½ E. longitude 7° 37′ E.

2. A ship from latitude 40° 25′ S. and longitude 40° 25′ W. sails 413 miles between N. and E. and is then in latitude 36° 19′ S.; what was her course, and what longitude hath she come to?

Ans. Course 53° 27′, or N. E. ¾ E. and the ship has come to longitude 33° 22′ W.

CASE 5. The latitude sailed from, the course and difference of longitude being given; to find the distance sailed and the latitude of the ship.

EXAM. A ship from latitude 45° 31′ N. and longitude 1° 24′ W. sails W. N. W. ¼ W. until she is in 10° 25′ W. longitude; required the distance sailed, and the latitude of the ship?

Ans. Distance 392 miles, and the ship is in latitude 47° 43′ N.

CASE 6. The latitude and longitude of the place sailed from, with the course and departure being given, to find the distance sailed, with the latitude and longitude of the ship.

EXAM. A ship from latitude 46° 24′ S. and longitude 4° 24′ E. sails E. N. E. ¾ N. until she has made 256 miles of easting; required the distance sailed, with the latitude and longitude of the ship?

Ans.

Ans. Distance 298.4 *m.* Latitude 43° 51′ Long. 10° 26′ E.

CASE 7. The difference of latitude and departure being given, to find the course, distance and difference of longitude.

EXAM. A ship from latitude 60° 24′ N. and longitude 43° W. sails between S. and E. until she is in latitude 56° 30′ N. and has made 226 miles of departure; required her course, distance and longitude?

Ans. Course S. E. nearly. Distance 325.4 *m.* and the longitude of the ship 35° 47′ W.

A Traverse by MERCATOR's *Sailing.*

Suppose a ship from latitude 68° 38′ N. and longitude 8° 40′ E. has sailed the following courses, *viz.* N. E. by N. 63 *m.* N. E. 38 *m.* N. N. E. 56 *m.* N. 30 *m.* N. W. by N. 25 *m.* N. N. W. ½ W. 36 *m.* N. by E. 40 *m.* N. E. by E. ½ E. 72 *m.* S. E. 50 *m.* and E. N. E. 65 *m.*; required the ship's latitude and longitude, with her course and distance from the place where she has come to, unto the North Cape in latitude 71° 27′ N. and longitude 26° 30′ E.?

Courses.

Courses.	D	N	S	E	W	Lats.		Merid. diff.	Longitudes. E	Longitudes. W
						68	38			
N E by N	63	52.4		35.0		69	30	145	96.88	
N E	38	26.9		26.9		69	57	78	78.	
N N E	56	51.7		21.4		70	49	155	64 2	
North	30	30				71	19	93		
N W by N	25	20.8			13.9	71	40	66		44 1
N N W ½ W	36	31.8			17.	72	12	103		55.
N by E	40	39.2		7.8		72	51	130	25.86	
N E by E ½ E	72	33.9		63.5		73	25	117	218 8	
S E	50		35.4	35.4		72	49	124	12	
E N E	65	24.9		60 1		73	14	86	207.2	
		311.6	35.4	250.1	30.9				814 94	99.1
		35.4		30.9					99.1	
		276.2		219.2				Diff. Long.	715.8	

or 11° 56′ E
Departed Long. 8 40 E

Ship's Long. 20 36

For the course and distance from the ship to the north Cape,

		mer. pts.
Ship's lat.	73° 14′	6583
Cape's lat.	71 27	6230
	1 47 = 107′	353 = merid. diff. lat.
Cape's longitude	26° 30′	
Ship's longitude	20 36	
	5 54	
	60	
	354	

The meridional difference latitude being nearly equal to the difference longitude, the course is 4 points or S. E. and the distance from the ship to the Cape is 152 miles nearly.

When

When a table of meridional parts is not at hand, the defect may be supplied by the logarithmic tangents. *See Robertson's Navigation, 2d edition, page* 532.

1. To find the meridional parts answering to any given latitude.

To half of the given latitude add 45°; find the logarithm tangent of the sum, reject the index, and of the seven remaining figures, point off three on the right hand for decimals; then divide by 1.2633114, the quotient is the meridional parts required.

Note. 1.2633114 is the natural tangent of 51° 38′ 9″.

EXAM. Required the meridional parts answering to 60° of latitude? *Ans.* 4527.36.

The half of 60° is 30°
To which add 45°

Sum 75° whose logarithmic tangent is 10.5719475. This without its index, and pointed by the rule is 5719.475, and then divide it by 1.2633114.

1.2633114)5719.4750(4527.36 the answer.

2. Required the meridional parts answering to latitude 55° 24′? *Ans.* 4010.019.

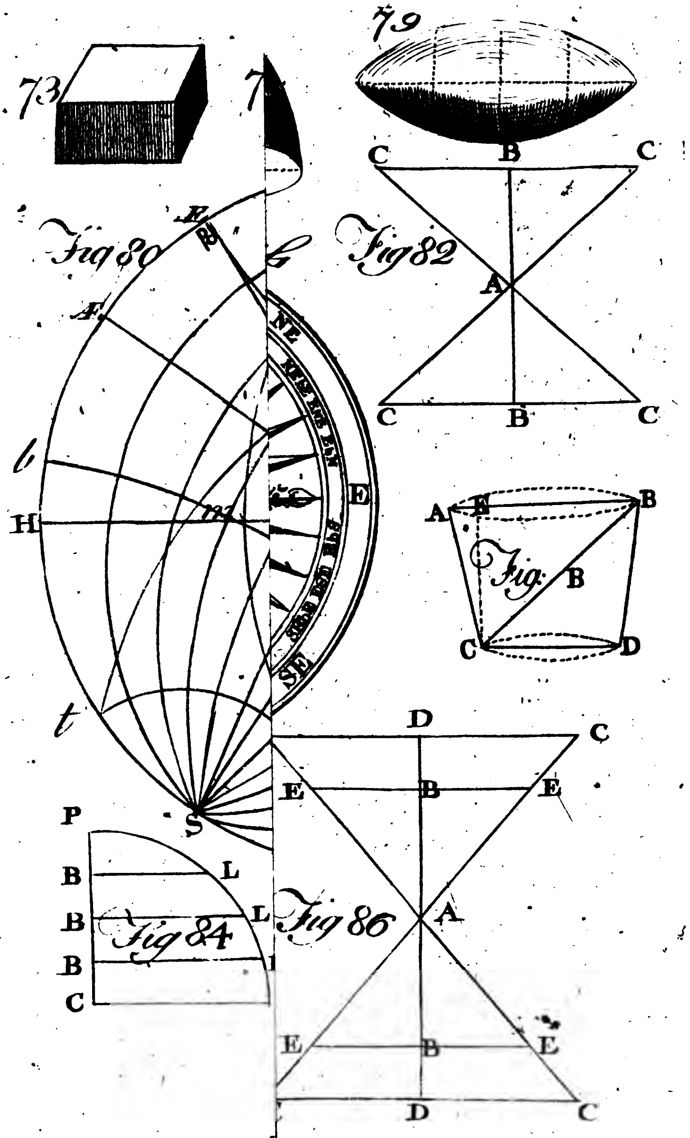
The half of 55° 24′ is 27° 42′
To which add 45°

72° 42′ log. tan. 10.5065903

1.2633114)5065.9030(4010.019.

2. To find the meridional difference of latitude between any two places, their latitudes being given.

When


73
79
C B C
Fig 82
A
C B C
Fig 80
H
b
t
S
E
NE
SE
A E B
Fig.
B
C D
P
B L
B L
Fig 84
B
C
D C
E B E
Fig 86
A
E B E
D C

When the given places are both in north, or both in south latitude,

Take the difference of the logarithm tangents of the half complements of the latitudes; account four figures of it next the index to be integers, and then divide it by 1.2633114, the quotient is the meridional difference of latitude.

Exam. 1. Required the meridional difference of latitude between 13° 30′ and 50° North latitudes?

Ans. 2656.87.

				Log. Tan.
The half comp. of 13° 30′	is	38° 15′		9.8967116
The half comp. of 50 00	is	20 00		9.5610659
				3356.457

1.2633114)3356.4570(2656.87

2. Required the meridional difference of latitude between the Lizard, in latitude 49° 57′ N. and Cape Verd, in latitude 14° 43′ North?

Ans. 2576.93.

			Log. Tan.
Half complement of 14° 43′	is	37° 38′½	9.8872024
Half complement of 49° 57′	is	20° 1′½	9.5616551
			3255.473

1.2633114)3255.4730(2576.93

When one place is in North latitude and the other in South, add 90° to one of the latitudes, and take half of the sum for the half complement of that latitude; take half the complement of the other latitude, and divide the

 difference

difference of the logarithm tangents of these two by 1.2633114, the quotient is the meridional difference of latitude. Or,

Take the half complements of the latitudes as before; take the sum of the logarithm co-tangents of these, rejecting the indices, and divide it by 1.2633114.

EXAM. 1. Required the meridional difference of latitude between 50° 54′ N. and 19° 24′ S. latitude?

Ans. 4746.18.

	50° 54′				
add	90 00				
	140 54	half of this is	70	27′	10.4496481
		half comp. of 19° 24′ is	35	18	9.8500575
				Difference	5995.906

1.2633114)5995.9060(4746.18

2. Required the meridional difference of latitude, between the Lizard, in latitude 49° 57′ N. and the island of St. Helena, in latitude 15° 45′ South?

Ans. 4426.93.

		Co-tang. without index.
Lizard's half comp. lat. is	20° 1′ 1/2	4383448
St. Helena's half comp. lat. is	37 7 1/2	1209154
		5592.602

1.2633114)5592.6020(4426.93

By these rules, the meridional parts in any latitude, or the meridional difference of latitude between any two places may be found, and all questions in Mercator's sailing answered in the usual way; but the same thing may be done by the logarithmic tangents alone, without the meridional parts. For it is demonstrated in the Philosophical

Philosophical Transactions, N°. 219, That the meridional line on *Mercator*'s *Chart* is a scale of the logarithm tangents of the half complements of the latitudes. That these logarithm tangents of Mr. *Briggs*' form, are a scale of the differences of longitude upon the rhumb, which makes an angle of 51° 38′ 9″ with the meridian; and, that the differences of longitude on different rhumbs, are to one another as the tangents of the angles which these rhumbs make with the meridian. Hence,

As the difference of the logarithm tangents of the half complements of the latitudes of any two places,
Is to the difference of longitude between these places,
So is the tangent of 51° 38′ 9″ (Log. 10.1015104)
To the tangent of the course.

By a right application of this rule, all the cases in *Mercator*'s *sailing* may be solved.

EXAMPLES.

CASE I. What is the course and distance from Caithness-point, in latitude 58° 46′ N. and longitude 3° 17′ W. to New York, in latitude 41° 5′ N. longitude 74° 15′ W.?

Latitudes.		Log. Tang.
41° 5′	half complement is 24° 27′½	9.6578665
58 46	half complement is 15 37	9.4464107
	Difference	2114.558

The difference of longitude is 4258 min.

As diff. of log. tan. of half co-lats.	2114.558	3.3252196
Is to diff. of long. in minutes	4258.	3.6292057
So is the tangent of	51° 38′ 9″	10.1015104
To the tangent of course	68° 32′	13.7307161
or W. S. W. 1° 2′ westerly		10.4054965

The course being known, the distance may be found as before.

CASE 2. A ship from latitude 50° N. and longitude 6° 15′ W. sails 250 miles, on a course 50° 6′ Southwestward; required the latitude and longitude of the ship?

The difference of latitude, and consequently the present latitude, is found as formerly; in this example.

Departed latitude	50° 00′	N.
Difference of latitude	2 40	S.
Present latitude	47 20	

		Log. Tan.
The half comp. of 47° 20′ is 21° 20′		9.5916812
Ditto of 50 00 is 20 00		9.5610659
		306.153
As tangent of	51° 38′ 9″	10.1015104
Is to tangent of course	50 6	10.0777263
So is the diff. of tang.	306.153	2.4859386
To diff. of longitude	289.83	12.5636649
or nearly 290′=4° 50′ W.		2.4621545
Departed long. 6 15 W.		
Present long. 11 5 W.		

CASE 3. A ship sails from latitude 49° 55′ N. to a place in latitude 13° 10′ N. on a course 50° Southwestward; required her difference of longitude?

Ans. 53° 1′ ½ W.

		Log. Tan.
The half comp. of 13° 10′ is 38° 25′		9.8993082
Ditto 49 55 is 20 2½		9.5620477
	difference	3372.605

As

As tang. of	50° 38′ 9″	10.1015104
Is to tang. course	50°	10.0761865
So is diff. of log. tang.	3372.605	3.5279648
To the diff. of long.	3181.5	13.6041513
or 53° 1′ ½ Westward.		3.5026409

CASE 4. A ship from latitude 32° 10′ N. and longitude 15° 12′ W. sails 3280 miles between South and West, and then is in latitude 20° 19′ South; required her course and longitude?

The difference of latitude is 52° 29′ = 3149 miles.

As Dist. : Diff. lat. : : Rad. : Cos. course = 16° 15′ Southwestward.

Lat. 32° 10′ N.		
add 90 00		
2)122 10		Log. Tan.
61° 5′		10.2577391
½ comp. of 20° 19′ is 34° 50′ ½		9.8426698
	Difference	4150.693

As tang.	51° 38′ 9″	10.1015104
Is to tang. course	16° 15′	9.4645990
So is diff. tang.	4150.693	3.6181206
To diff. of longitude	957.65	13.0827196
or	15° 57′ W.	2.9812092
Departed long.	15 12 W.	
Present long.	31° 9′ W.	

A TABLE

A Table of the Refraction of the Heavenly Bodies, *to be subtracted from the observed Altitude.*

Height of the eye above the sea in feet.	Dip of the horizon in minutes.	Altit.	Refrac.	Altit.	Refrac.	Altit.	Refr.	Altit.	Refr.
Feet.	M. S.	D.	M. S.	D.	M. S.	D.	M.S.	D.	M.S.
3	1 39	0	33 0	11	4 47	27	1 51	43	1 1
5	2 8	0½	28 22	12	4 23	28	1 47	44	0 59
7	2 31	1	24 29	13	3 57	29	1 42	45	0 57
9	2 52	1½	19 25	14	3 45	30	1 38	46	0 55
12	3 18	2	18 35	15	3 30	31	1 35	47	0 53
15	3 42	2½	16 26	16	3 17	32	1 31	48	0 51
18	4 3	3	14 36	17	3 4	33	1 28	49	0 49
21	4 22	3½	13 6	18	2 54	34	1 24	50	0 48
25	4 46	4	11 51	19	2 44	35	1 21	55	0 40
30	5 14	4½	10 48	20	2 35	36	1 18	60	0 33
35	5 39	5	9 54	21	2 27	37	1 16	65	0 26
40	6 2	6	8 28	22	2 20	38	1 13	70	0 21
50	6 44	7	7 20	23	2 14	39	1 10	75	0 15
60	7 23	8	6 29	24	2 7	40	1 8	80	0 10
70	7 59	9	5 48	25	2 2	41	1 5	85	0 5
80	8 32	10	5 15	26	1 56	42	1 3	90	0 0

To find the Latitude of a Ship at Sea.

Observe the meridian altitude of the sun's upper or lower limb by a quadrant.

Correct the observed altitude of the sun's limb, by subtracting the dip of the horizon and the refraction from it; and adding the semidiameter, if the lower limb was observed, or subtracting it, if the upper limb was observed; the result is the true altitude of the sun's center.

Subtract the altitude of the sun's center from 90°, the remainder is the zenith distance, which is north, if the zenith is to the north of the sun; or south, if the zenith is to the south of the sun.

Take the sun's declination out of the Nautical Almanack, or any good Table, for the time and place, noting whether it be north or south; and then,

If the zenith distance and declination are both north, or both south, add them together; but if the one be north and the other south, subtract the less from the greater, and the sum or difference will be the latitude, of the same name with the greater.

EXAMPLES.

1. April 14th, 1790, the meridian altitude of the sun's lower limb was 40° 20′; the zenith being north of the sun; and the altitude of the observer's eye, 20 feet above the surface of the sea. What was the Latitude?

Altitude

Altitude of ſun's lower limb	40°	20′	0″
dip, ſubtract		4	16
	40	15	44
refraction, ſubtract		1	6
	40	14	38
ſun's ſemidiameter, add		15	58
true altitude ſun's center	40	30	36
	90	0	0
zenith diſtance	49	29	24 N.
ſun's declination	9	34	40 N.
Latitude	59	4	4 N.

2. May 26th, 1790, being at ſea in longitude 67° 30′ E. by account; the meridian altitude of the ſun's lower limb was 26° 12′, the zenith being to the ſouth of the ſun, and the obſerver's eye 18 feet above the ſurface of the water: required the latitude of the ſhip?

	h.	*m.*			
Time at the ſhip	24	0 past noon of the 25th			
Longitude 67° 30′ E.	4	30			
time at London	19	30			
Sun's declination May 25th	21°	1′	57″ N.	daily diff. 10′ 57″	
increaſe in 19 *h.* 30 *m.* add		8	54		
ſun's declination at the ſhip	21	10	51 N.		

Altitude

Altitude sun's lower limb	26°	12′	0″
dip on 18 feet, subtract		4	3
	26	7	57
refraction, subtract		1	56
	26	6	1
Sun's semidiameter, add		15	48
Altd. of the sun's center	26	21	49
	90	0	0
zenith distance	63	38	11 S.
declination	21	10	51 N.
Latitude of the ship	42	27	20 South.

When the latitude is to be found from the meridian altitude of a fixed star,

Subtract the dip and refraction from the observed altitude, and then find the latitude from the zenith distance and declination of the star, as before.

3. April 15th, 1790, the meridian altitude of the star Syrius was observed, 53°. 35′; the zenith being to the north of the star, and the height of the observer's eye 22 feet above the surface of the sea: What was the latitude?

Observed altitude		53° 35′ 0″
dip	4′ 28″	
refraction	1 28	
		— 5 56
true altitude of the star		53 29 4
		90
zenith distance		36 30 56 N.
declination		16 26 21 S.
Latitude		20 4 35 N.

If the meridian altitude of a circum-polar star be observed, when it is below the pole; or the meridian altitude of the sun at midnight, in any place where it does not set: Correct the altitude as above directed, and then, to the altitude of the star, or center of the sun, add the complement of its declination, the sum is the latitude of the place; of the same name with the declination.

4. June 12th, 1786, in a high northern latitude, 23° W. longitude, the eye being 20 feet above the horizon; the meridian altitude of the sun's lower limb, below the pole, was 8° 15′; required the latitude?

	h.	*m.*
Ship's time	12	0 *p. m.*
Long. 23° W.	1	32
London time	13	32 *p. m.*

Sun's decl. June 12th	23° 11′ 54″ N.	
increase in 13 *h.* 32 *m.*	1 57	
Sun's declin. at ship	23 13 51 N.	

Altitude

Altitude sun's lower limb		8°	15′	0″
dip	4′ 16″			
refraction	6 19	—	10	35
Altitude sun's L. L.		8	4	25
Semidiameter			15	47
Altitude sun's center		8	20	12
Co-declination		66	46	9
Latitude		75	6	21 North.

It has been found by observation, that the magnetic needle does not point due north, except in some few places; and that its direction is different at different times in the same place.

The distance between the true north and the direction of the magnetic needle, is called the *variation of the compass*, which, when the needle points to the east of the true north, is called *East variation*; but, when it points west of the true north, it is called *West variation*.

In order to find the variation of the compass, the two following problems are necessary, viz.

1. The latitude of the place, and the declination of any celestial object being given, (that declination being less than the complement of the latitude), to find its amplitude.

RULE. As the co-sine of the latitude
Is to the sine of the declination,
So is the radius
To the sine of the amplitude.

Exam. 1. In latitude 56° N. the sun's declination being 20° 12′ N.; required the sun's amplitude?

As co-sine latitude	56°	0′	9.7475617
Is to sine declination	20	12	9.5381943
So is radius S.	90	0	10.0000000
To sine amplitude	38	8	9.7906326

That is, the sun rises E. 38° 8′ N. or sets W. 38° 8′ N. according as the calculation was made for the morning or evening.

2. In latitude 10° 12′ N. the sun's declination being 23° 12′ N.; required the sun's amplitude?

Ans. E. 23° 36′ N.

Note. In the torrid zone, the amplitude and declination are nearly equal.

3. On the 20th of January, 1779, in the latitude of 34° 15′ S.; what was the sun's amplitude at rising?

In questions of this kind, the sun's declination must be found accurately for the time and meridian. In this example, if the calculation is made for the morning, the declination is 20° 14′ S. and his amplitude E. 24° 44′ S.; but, if for the evening, the declination is 20° 7′ S. and the amplitude E. 24° 35′ S.

4. In the latitude of 56° N.; required the amplitude of the star *Arcturus*, its declination being 20° 22′ N.?

Ans. E. 38° 29′ N.

2. The latitude of the place, the sun's declination and altitude being given, to find his azimuth.

Add together the complement of the latitude, the complement of the sun's altitude, and his distance from the

the elevated pole; take half the sum, from which subtract the sun's distance from the elevated pole.

Take the arithmetical complement of the co-sine of the latitude, and of the co-sine of the sun's altitude; take also the sine of the half sum, and the sine of the difference between the half sum and the sun's distance from the elevated pole; the half sum of these four logarithms is the co-sine of half the sun's azimuth from the north in north latitude, and from the south in south latitude.

Note. When the latitude and declination are both north or both south, subtract the declination from 90°; but, when the one is north and the other south, add the declination to 90°; the difference, or sum, is the sun's distance from the elevated pole.

EXAM. I. In the latitude of 51° 32′ N. the sun's altitude was observed to be 46° 20′, when his declination was 23° 28′ N.; what was the sun's azimuth?

Ans. 113° 3′ from the North.

Comp. latitude	38° 28′	Arith. comp.	0.2061683
Comp. altitude	43 40	Arith. comp.	0.1608604
Sun's dist. from the pole	66 32		
Sum	148 40		
½ sum	74 20	Sine	9.9835582
Subtract	66 32		
Difference	7 48	Sine	9.1326297
			2)19.4832166
Co-sine of	56° 31′ 29″		9.7416083

Which doubled, is = 113 2 58 or 113° 3′ nearly.

That

That is, the ſun's azimuth is 113° 3′ from the north, and conſequently 66° 57′ from the ſouth; and, if the obſervation was made in the forenoon, the ſun bears E. S. E. nearly; but W. S. W. if it was made in the afternoon.

To work an AZIMUTH *by Gunter's Scale.*

Find the ſum, half ſum, and difference as above, and then,

As R. : Cos. lat. :: Cos. alt. : a. 4th ſine.

As the 4th ſine : Sine of the ½ ſum :: the ſine of the diff. : a. 5th ſine;

againſt which, on the line of verſed ſines is the azimuth from the north.

In this example, ſet one foot of the compaſſes to ſine 90°, and extend the other to ſine 38° 28′; with this extent, ſet one foot to ſine 43° 40′, and the other will fall on ſine 25° 26′; keep the foot there, and extend the other to ſine 74° 20′ (the half ſum;) with this extent ſet one foot to ſine 7° 48′, and the other will fall on ſine 17° 43′ againſt which, on the line of verſed ſines is 113° 3′, the ſun's azimuth.

2. In latitude 39° 50′ N. when the ſun's declination was 13° 25′ N. his altitude was 50°; required the ſun's azimuth?

Anſ. 121° 36′ from the north; found as in the laſt example.

3. In the latitude of 51° 32′ N. the ſun's declination being 19° 39′ S. and his altitude 15° 30′; what is the ſun's azimuth?

To 90° 0'
add 19 39

109 39 sun's distance from the pole.

Comp. latitude	38° 28'	Arith. comp. 0.2061683	
Comp. altitude	74 30	Arith. comp. 0.0160895	
Sun's dist. from the pole	109 39		
Sum	222 37		
½ sum	111 18½	Sine	9.9692473
Difference	1 39½	Sine	8.4603922
			2)18.6518973
		Co-sine of 77° 47'	9.3259486

which being doubled, is 155° 34', the sun's azimuth from the North.

By Gunter's scale, the extent from radius to the sine of 38° 28' will reach from the sine of 74° 30' to the sine of 36° 50', and the extent from 36° 50' to the sine of 111° 18' ½ (68° 41' ½), will reach from sine 1° 39' ½ to sine 2° 34', against which, on the line of versed sines, is 155° 34', the azimuth.

4. In latitude 56° S. the sun's declination was 23° 28' South, and his altitude 44°; required the sun's azimuth?

Ans. 116° 13' from the south, and 63° 47' from the north.

The sun's azimuth being found, the hour of the day when the observation was made may be found thus,

As,

As the sine of the sun's distance from the elevated pole,
Is to the sine of the sun's azimuth;
So is the sine complement of the sun's altitude,
To the sine of the angle at the pole;
which, in time, is the hour from noon.

In the last example, to find the time when the sun's altitude was observed.

As sine of	66°	32′	0″
Is to sine of	116	13	0
So is the sine of	46	0	0
To the sine of	44	42	39

which, reduced to time, at the rate of 15° to 1 hour, is 2 hours, 58 minutes, 50 seconds from noon; which shows, that the observation was made either at 9 hours, 1 minute, 10 seconds in the forenoon, or at 2 hours, 58 minutes, 50 seconds in the afternoon.

Or, the same things being given as for finding the azimuth, the hour of the day may be found by the following Rule:

Add the complement of the latitude, the sun's distance from the elevated pole, and the complement of the sun's altitude, and take half of the sum; from which subtract the complement of the sun's altitude, and then;

To the arithmetical complements of the co-sine of the latitude, and of the sine of the sun's distance from the elevated pole, add the logarithm sine of the half sum, and the logarithm sine of the difference between the half sum and the complement of the sun's altitude: the half sum of these four logarithms is the co-sine of half the hour angle, which being doubled and reduced to time, gives

Fig. 1.

Fig. 2.

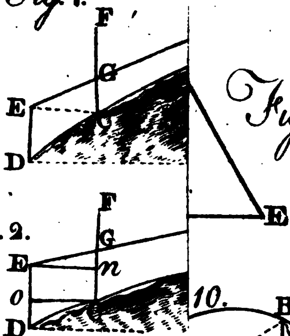


Fig. 5.

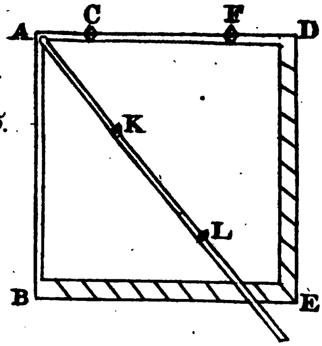


Fig. 10.

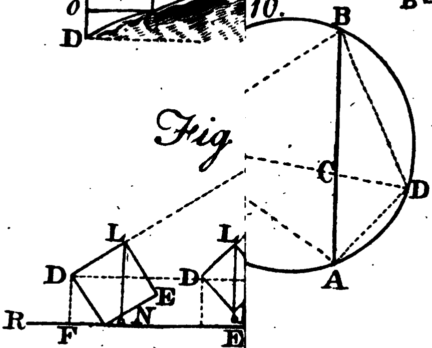


Fig. 11.

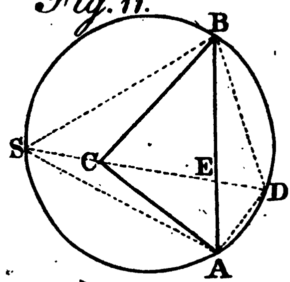


Fig. 12

Fig. 15.

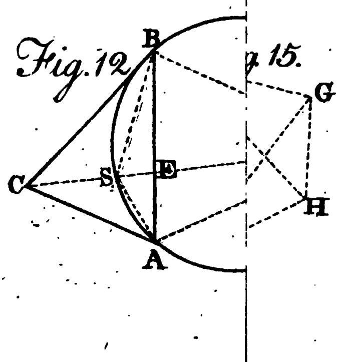
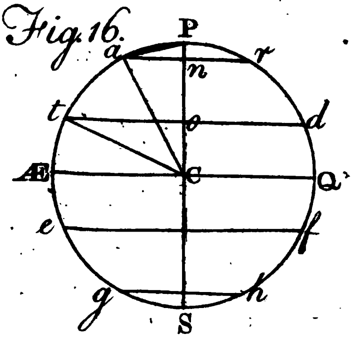


Fig. 16.



gives the apparent time from noon, when the obſervation was made.

EXAM. In latitude 39° 50′ N. the ſun's declination being 13° 25′ N. and the true altitude of his center 50°: What was the hour from noon?

Anſ. 2 hours 17 m. from apparent noon.

Comp. lat.	50° 10′		arith. com.		0.1146891
polar diſt.	76 35		ditto		0.0120175
Comp. alt.	40 0				
Sum	166 45				
half ſum	83 22 30		ſine		9.9970903
difference	43 22 30		ſine		9.8368116
					2)19.9606085

co-ſine of 17° 7′ 34″ = 9.9803042

which doubled is 34° 15′ 8″ the hour angle, and 34° 15′ 8″ reduced to time, is 2 hours 17 m. from apparent noon.

The altitude of any fixed ſtar being known by obſervation, to find the apparent time.

Subtract the dip of the horizon, and the refraction, from the obſerved altitude of the ſtar; and take its right aſcenſion and declination from any good Table; and then, with the complement of the latitude, the ſtar's polar diſtance, and the complement of its altitude, find the hour angle, (as in the laſt problem) which reduce into

 time;

time; and to this time apply the star's right ascension in time; that is, add it to the hour angle, if the star be west of the meridian, or subtract it from the hour angle, if the star be east of the meridian; the sum, or difference, is the right ascension of the mid-heaven; from which subtract the sun's right ascension in time, the remainder is the apparent time, nearly; then say, as 24 hours is to the daily difference of the sun's right ascension, so is the time past noon to a fourth, which being subtracted from the former time, the remainder is the true apparent time.

EXAM. April 14th, 1790, in latitude 48° 56′ N. longitude 66° W. the altitude of Aldebaran was observed 22° 24′ 30″; the star being west of the meridian, and the height of the observer's eye above the surface of the sea being 21 feet; what was the apparent time, when that observation was made?

Dip on 21 feet	4′	22″
Refraction	2	18
Subtract	6	40
Observed altitude 22°	24	30

true alt. of the star 22 17 50; Declin. N. 16° 4′ 24″

polar distance 73 55 36

Comp-

Comp. latitude	41° 4′	arith. comp.	0.1824765
polar distance	73 55 36	arith. comp.	0.0173182
Comp. altitude	67 42 10		
Sum	182 41 46		
half sum	91 20 53	Sine	9.9998802
difference	23 38 43	Sine	9.6032235
			2)19.8028984
Co-sine of	37° 9′ 26″		9.9014492

doubled is = 74 18 52 = the hour angle.

	h. m. s.		h. m. s.
which in time is	4 57 15	April 14th,	
Star's R. A.	4 23 53	Sun's R. A.	1 31 28
		Long. 66° W.	+41
R. A. of mid heav.	9 21 8		
Sun's R. A.	1 32 9	Sun's R. A. at ship	1 32 9
		As 24 h. 3′ 41″ :: 7 h.	
	7 48 59	49 m. : 1′ 12″	
Subtract	— 1 12		

Ans. Apparent time 7 47 47 P. M.

To find the time of apparent noon, and thereby regulate a watch or clock, by equal altitudes of the sun.

 With

With any good inſtrument obſerve the ſun's altitude any time before noon, and mark the time ſhewn by the watch; and in the afternoon obſerve when the ſun has the ſame altitude, and mark the time ſhown by the watch: the middle point between the two obſervations is the time of apparent noon, or mid-day.

Note 1. The altitude of the ſun's lower, or upper limb, or both, may be obſerved, and from either of theſe, the altitude of the center may be found by adding or ſubtracting the ſun's ſemidiameter at the time.

2. If the obſervations are taken near noon, on any day from the 20th of May to the 20th of July; or from the 20th of November to the 19th of January; the time of apparent noon will be found without any conſiderable error; but if the obſervations be made on any day within two months of either equinox, the variation of the ſun's declination during the interval between the obſervations will cauſe a ſmall error; which in any habitable part of the earth will not be ſo much as half a minute; and if the obſervations are made within about an hour of noon, the error will be very ſmall.

For correcting this error, there is a table in ROBERTSON's Navigation, Vol. I. p. 328.

When equal altitudes are taken at ſea in order to find the time of apparent noon; the ſhip ſhould ly by during the interval, and then, add the times ſhown by the watch and 12 hours; the half ſum of theſe is the apparent time ſhown by the watch when the ſun was on the meridian of that place; and hence the watch may be rectified to apparent time.

EXAM.

EXAM. 1. May 20th, 1790, at 8 hours 40 *m.* forenoon, and 3 hours 16 *m.* afternoon, by a watch, the sun had equal altitudes; required the time of noon and going of the watch?

		h.	m.	
add together	{	8	40	A. M.
		3	16	P. M.
		12	0	
		2)23	56	
Time of app. noon		11	58	
The watch noon		12	00	
Watch too fast			2	minutes.

EXAM. 2. July 1, 1790, at 8 hours 10 *m.* 58 *s.* forenoon, and at 3 hours 58 *m.* 34 *s.* afternoon, by a watch, the sun had equal altitudes; required the going of the watch?

	h.	m.	s.
	8	10	58
	3	58	34
	12	0	0
	2)24	9	32
Time of appar. noon	12	4	46
The watch noon	12	0	0
The watch too slow		4	46

To

To observe the Sun's Amplitude *with an* Azimuth Compass.

Move the box about till the four cardinal points of the card coincide with the four lines in the ſides of the compaſs box; keep the box ſteady in this poſition, and turn the index until the center of the ſun, at its riſing or ſetting, is ſeen through both the ſights; then the degrees intercepted between the north point of the compaſs and the end of the index pointing to the ſun, will be the Magnetic Amplitude ſought.

For an Azimuth.

Place the box and compaſs as before, and hold it ſteady; turn the index until the ſights being directed to the center of the ſun, the ſhadow of the thread joining the ſights fall directly along the middle line of the index; and then the degrees intercepted between the north point of the card and that end of the index next the ſun, ſhall give the Magnetic Azimuth of the ſun.

The azimuth of any ſtar is found in the ſame manner, only you are to view the ſtar through the ſights.

Having the true and magnetic amplitude, or azimuth, of any celeſtial object, to find the variation of the compaſs.

If the true and magnetic amplitude, or azimuth, be the ſame, then there is no variation, and the needle points due north; but, if they are not the ſame, let both be reckoned from the north, eaſtward when the obſervation is made in the forenoon, and weſtward in the afternoon; and then the difference between the true and magnetic

netic amplitude, or azimuth, is the variation of the compass.

To know whether the VARIATION *be* EAST *or* WEST.

If the amplitude be taken at sun-rising, or the azimuth in the forenoon, and if the magnetical amplitude, or azimuth, be farther from the north than the true, the variation is westward; but, if it be nearer to the north, the variation is eastward.

If the observation be made in the afternoon, or at sun-setting, and if the magnetic amplitude, or azimuth, be farther from the north than the true, the variation is eastward; but, if it be nearer to the north, the variation is westerly.

EXAM. 1. Suppose at sun-setting his amplitude, by the compass, is 7° 10′ from West towards North, and his true amplitude is W. 25° 15′ N.; required the variation, and which way?

Magnetic amplitude	N. 82°. 50′ W.
True amplitude	N. 64 45 W.
Variation	18 5 E.

2. Suppose at sun-rising the amplitude, by the compass, is E. 16° 12′ N. and his true amplitude E. 30° 10′ N.; required the variation, and which way?

Magnetic amplitude	N. 73° 48′ E.
True amplitude	N. 59 50 E.
Variation	13 58 W.

3. Sup-

3. Suppose the sun's magnetic azimuth is N. 124° 30′ E. and his true azimuth is N. 113° 3′ E.; what was the variation of the compass, and which way?

Magnetic azimuth	N.	124° 30′	E.
True azimuth	N.	113 3	E.
Variation		11 27	W.

To know whether the variation is E. or W. the common rule is, let the observer's face be turned towards the sun, and if the true amplitude, or azimuth, be to the right hand of the magnetic, the variation is East; but, if the true be to the left hand of the magnetic, the variation is West.

Having the course set, and the variation of the compass, to find the true course made.

If the variation is W. call the N. W. quarter the 1st, the S W. the 2d, the S. E. the 3d, and the N. E. the 4th; but, if the variation is East, call the N. E. quarter the 1st, the S. E the 2d, the S. W the 3d, and the N. W. the 4th; then, if the course be in the 1st or 3d quarter, add the variation to the given course set; but, if it be in the 2d or 4th quarters, subtract; and the sum, or difference, shall be the course, corrected by the variation.

Exam. 1. Suppose the variation is $1\frac{1}{4}$ points W. it is required to correct the following courses steered by the compass, viz. N. W. $\frac{1}{4}$ W. S. S. W. S. E. by E. $\frac{1}{2}$ E. and N. E. by E.

Courses

Courses N. W. $\frac{1}{4}$ W. = $4\frac{1}{4}$ Pt.	S. S. W. = 2 Pt.
Variation add $1\frac{1}{4}$	Var. sub. $1\frac{1}{4}$
Cor. course N. W. by W. $\frac{1}{2}$ W. = $5\frac{1}{2}$	S. $\frac{3}{4}$ W. = $0\frac{3}{4}$

Courses S. E. by E. $\frac{1}{2}$ E. = $5\frac{1}{2}$ Pt.	N. E. by E. = 5 Pt.
Variation add $1\frac{1}{4}$	Var. sub. $1\frac{1}{4}$
Cor. course E. S. E. $\frac{3}{4}$ E. = $6\frac{3}{4}$	N. E. by N. $\frac{3}{4}$ E. = $3\frac{3}{4}$

3. Suppose the variation is 21° 30′ E. it is required to correct the following courses steered by the compass.

Course set N. E. by N. = 33° 45′	S. S. E. = 22° 30′
Variation add 21 30	Var. sub. 21 30
Correct course N. 55° 15′ E.	Cor. course S. 1° 00′ E.

Course set W. S. W. = 67° 30′	N. W. by W. = 56° 15′
Variation add 21 30	Var sub. 21 30
Correct course S. 89° 00′ W.	Cor. course N. 34° 45′ W.

SECT. VII.

GUNNERY.

Definitions *and* Principles.

1. THE path of every projectile depends on two different forces, namely, the impellent force, whereby the motion is first begun, and which, acting alone, would carry the body forward in a straight line; and the force of gravity, by which the projectile during the whole time of its flight is urged downwards toward the center of the earth; and because both forces act continually whilst the body is in motion, its path is a curve line lying between the directions of both. It has been proved by many authors, that the path of a projectile *in vacuo* is a curve line called a *parabola*; but in air, the resistance it meets with changes the form of its path.

Most writers on this subject have thought that the action of the air on such projectiles as cannon-balls, was too small to be taken notice of; and therefore have supposed that the path of a ball in the air was truly, or very nearly, the same, as *in vacuo*.

But, although it is now known, that the resistance of the air to the motions of large and heavy bodies, such as bombs and cannon-balls, is much greater than has been commonly represented, and therefore must affect both the magnitude and figure of their path: Yet, if the amplitude

amplitude of the projection anſwering to any given elevation be firſt found by experiment, (which is always ſuppoſed in gunnery), the amplitude, in all other caſes, where the elevations and velocities do not differ very much from the firſt, may be determined to a ſufficient degree of exactneſs, by the rules ariſing from the parabolic hypotheſis; becauſe, in all ſuch caſes, the effects of the air's reſiſtance will be nearly as the amplitudes; and, were they accurately ſo, the proportions of the amplitudes at different elevations would then be the ſame as *in vacuo*. Hence, the common rules given by the writers on gunnery, although not mathematically true, may be ſafely admitted in practice.

2. The point where the projectile begins to move, is called the *point of projection*.

3. The impetus of a piece is the height to which, with its proper charge of powder, it could make the ball aſcend, when fired off in a direction perpendicular to the horizon; or the perpendicular height from which the ball muſt fall, to acquire the velocity it hath at the point of projection.

4. The elevation is the angle contained between the axis of the gun and a right line drawn from the point of projection in the horizontal level.

5. The amplitude, random, or range of a projectile, is the diſtance between the point of projection and the place where it impinges on the ground; Or, it is the horizontal diſtance which the projectile paſſeth over in its flight.

6. Different elevations of the ſame piece give different amplitudes; for if, when a piece is fired off, its axis be

either in the plane of the horizon, or perpendicular to it, there will be no amplitude; because, in the first case, the ball, by the action of gravity, will meet the ground immediately on leaving the mouth of the gun; and, in the last, it will fall down on the point of projection. At equal distances from either of these directions, such as, at an elevation of 25 or 65 degrees, the amplitudes will be equal. And,

When the elevation is 45 degrees, the amplitude will be the greatest possible.

7. The greatest amplitude is double to the impetus of the piece; and, when the elevation is 45 degrees, the greatest altitude of the ball is one-fourth part of the amplitude.

8. If a body be projected oblique to the horizon, it will fall there again in the same oblique direction, and with the same velocity with which it was projected.

9. The time which a heavy body, projected at an elevation of 45°, will continue in the air, before it arrives at the horizon, will be equal to the time that the same body would take to descend by the force of gravity, through a space equal to its amplitude, or horizontal range.

10. In most problems in gunnery, it is supposed that the gunner makes an experiment on every gun he has the care of, with its ordinary charge of powder, at an elevation of 45°, in order to find the greatest amplitude of the piece, and then half of this is the impetus.

The r les made u'e of in solving the following problems, are demonstrated b the writers on projectiles.

Of

Of Projections made on the Plane of the Horizon.

PROB. 1. The impetus of the piece and the distance of the object aimed at being known, to find the elevation, so as to strike the object.

RULE. The horizontal ranges of equal bodies projected with the same velocity at different elevations, are to one another as the sines of twice the angles of elevation.

EXAM. Required the elevation necessary to strike an object on the horizon at the distance of 5170 yards, the impetus of the piece being 3375 yards.

As twice the impetus, or greatest amplitude 6750,
Is to the given distance of the object 5170;
So is the radius or sine of twice 45°,
To the sine of twice the elevation = 50°.
Hence, the lower elevation is 25°, and the higher 65°;
For these are equally distant from 45°.

PROB. 2. The angle of elevation, and the amplitude being given, to find the greatest altitude of the ball.

RULE. As radius is to the tangent of the angle of elevation, so is one fourth part of the correspondent amplitude to the greatest altitude of the ball.

EXAM. If a piece fired off at an elevation of 25° throws the ball to the distance of 5170 yards; what is the greatest altitude of the ball?

Ans. 602.8 yards. One fourth part of 5170 is 1292.5.

As Rad. : Tan. elev. = 25° :: 1292.5 : 602.8.

EXAM.

EXAM. 2. If the impetus of a piece be 4000 feet; what is the proper elevation to hit an object at the distance of 4200 feet? and what is the greatest altitude of the ball?

Ans. The lower elevation is 15° 50′, and the greatest altitude of the ball is 298 feet.

PROB. 3. The impetus of a piece, and its elevation being given, to find the amplitude.

EXAM. 1. Suppose the impetus of the piece to be 3375 yards; how far will the same piece, with an equal charge of powder, range the shot, at an elevation of 25°?

Ans. 5170 yards.

As radius or sine of twice 45°,
Is to the sine of twice the given elevation 50°;
So is twice the impetus, 6750 yards,
To the amplitude required = 5170 yards.

EXAM. 2. Let the impetus be 4000 feet, and the given elevation 30° 16′; required the amplitude?

Ans. 6965 feet.

PROB. 4. The angle of elevation, and the distance of an object aimed at being given, to find the impetus, so as to strike the object.

EXAM. 1. Let the angle of elevation be 32° 12′, and the distance of the object aimed at be 6500 feet; required the impetus of the piece? *Ans.* 3604 feet.

As the sine of twice the angle of elevation, 64° 24′,
Is to radius or sine of twice 45°;
So is the given amplitude 6500,
To twice the impetus required 7208
Whence 3604 feet is the impetus.

EXAM.

Exam. 2. Let the angle of elevation be 30° 16′, and the distance of the object or amplitude = 6965 feet; what is the impetus of the piece?

Ans. 4000 feet.

Hence, the impetus of a piece may be found by firing it off at any elevation, and measuring the horizontal range of the ball.

When the elevation is 15°, the amplitude is equal to the impetus; because the sine of twice 15 or of 30° is equal to half of the radius.

Prob. 5. The amplitude of a piece at any elevation being given, to find the amplitude at any other elevation; and the contrary.

Exam. 1. If a piece, fired off at an elevation of 25° 12′, throws the ball to the distance of 5250 feet; how far will the same piece, with an equal charge of powder, throw the ball when fired off at an elevation of 36° 15′?

Ans. 6498 feet.

As the sine of twice the first elevation	50° 24′
Is to the sine of twice the second	72° 30′
So is the first amplitude	5250,
To the amplitude required	6498

Exam. 2. If a piece fired off at an elevation of 37° 28′ throws the ball to the distance of 2956; at what elevation must the same piece be fired off to throw the ball to the distance of 2550 feet?

Ans. 28° 12′.

Prob. 6. The elevation of the piece, and the greatest altitude of the ball being given, to find its greatest altitude at any other elevation.

Rule.

RULE. The heights to which equal bodies projected with equal velocities rise in the air, are, in proportion to one another, as the versed sines of twice the angles of elevation.

EXAM. If the greatest altitude of a ball when fired off at an elevation of 24° 5′ be 180 yards; at what elevation will the greatest altitude be 400 yards?

As the first altitude 180,
Is to the second 400;
So is the versed sine of twice the first elevation 48° 10′;
To the versed sine of twice the elevation required 74° 56′.
Half of which 37° 28′ is the answer.

This proportion may be wrought on Gunter's scale, thus: The extent of the compasses from 180 to 400, on the line of numbers, will reach from the versed sine of 48° 10′ to 105° 4′, the supplement of which 74° 56′ is the double of the elevation.

PROB. 7. The elevation and amplitude being given, to find the time of the flight, or of the ball's continuance in the air.

RULE. As the radius is to the tangent of the elevation, so is the given amplitude in feet to the square of four times the number of seconds required.

EXAM. Let the elevation be 32°, and the amplitude 5280; required the time of the flight?

Ans. $14\frac{1}{3}$ seconds.

As Rad. : Tan. 32° :: 5280 : 3300, the square root of which is 57.44; and one fourth of 57.44 is 14.36, or $14\frac{1}{3}$ nearly.

But

But when the elevation is 45°, (which is commonly the case in throwing bombs), then one fourth part of the square root of the distance in feet, is the number of seconds which the projectile continues in the air. The knowledge of this is necessary in adjusting the fusee.

Of PROJECTIONS *when the Object aimed at is above or below the level of the Piece.*

PROB. 8. The horizontal distance, and the angle of elevation or depression of an object being given, together with the impetus, to find the elevation of the piece, so as to hit the object.

RULE. As the radius is to the tangent of the elevation or depression of the object, so is twice the impetus to a fourth number; which add to, or subtract from, the given horizontal distance, according as the object is elevated or depressed. Then say,

As twice the impetus is to the sum, or remainder, so is the co-sine of the given elevation or depression, to the co-sine of an angle, which added to and subtracted from, the angle included between the object and the zenith, gives the doubles of the complements of two different elevations whereon the ball will hit the object.

EXAM. Let the horizontal distance of the object be 5600 feet, and its elevation 8° 15′, and let the impetus of the piece be 4000 feet; required the elevation?

1. As Rad. : Tan. 8° 15′ :: 8000 : 1160, which added to 5600, gives 6760.

2. As 8000 : 6760 :: Cos. 8° 15′ : Cos. 33° 15′.

Add and ſubtract 33° 15′ to and from 81° 45′, and you have 115° and 48° 30′, the halves of which are 57° 30′ and 24° 15′, whoſe complements 32° 30′, and 65° 45′ are the two elevations required.

The rules relating to oblique projections are ſeldom attended to in actual ſervice.

In all ſhooting on aſcents and deſcents, it is beſt to find the angle between the object and the zenith, and take the complement of half that angle for the elevation, and find by trial what charge will reach the object; for, on this elevation, a leſs charge of powder will do the buſineſs than on any other: And this method is commonly practiſed by gunners.

PROB. 9. The amplitude of projection with a given quantity of powder being known, to find the quantity of powder neceſſary to hit an object at a given diſtance, the elevation remaining the ſame.

RULE. When equal bodies are projected with different velocities, at the ſame elevation, the horizontal ranges are in proportion to one another as the weights of the charges of powder, nearly.

EXAM. If a 24 pounder, elevated to 45°, 16lb. of powder will throw the ball to the diſtance of 6750 yards; how much powder will throw the ſame ball 5170 yards, at the ſame elevation?

Anſ. $12\frac{1}{4}$ lb.

As the firſt horizontal range 6750,
Is to the ſecond 5170;
So is the weight of the given charge 16 lb.
To the weight of the charge required 12.26 lb.

Note. This rule cannot be accurate.

Cannons

Cannons are either made of iron or braſs, of different lengths and bores, and named from the weights of the balls ſuited to their capacities. Thus, a great gun whoſe ball weighs 12 lb. is called a 12 pounder; and one whoſe ball weighs 24 lb. is called a 24 pounder, &c.

The charges of powder, according to the Engliſh uſage, are about the weight of the ball for proof; half its weight for ſervice, and one-fourth of its weight for ſalutes.

The French allow two-thirds of the weight of the ball for ſervice, and twice as much for proof.

Large braſs guns are allowed a greater charge of powder than iron ones of the ſame bore.

At ſea, after five or ſix rounds in warm ſervice, the quantity of powder is diminiſhed every charge, until it come to about one-third of the weight of the ball.

Of the COMPUTATION *of* SHOT.

Cannon-balls and ſhells are uſually piled up in a pyramidal, or priſmatic form; the baſe being either an equilateral triangle, a ſquare, or a rectangle. In the triangle and ſquare, the pile is a pyramid, finiſhed by a ſingle ball at the top; but, when the baſe is a rectangle, the pile is finiſhed by a ſingle row of balls.

In pyramids of balls, the number of balls on each ſide of the baſe decreaſes from the baſe upwards by 1; that is, if the number of balls in one ſide of the baſe be 20, the number in the tire next above that will be 19, and in

the next 18, &c. until all the sides of the pyramid meet at the top in a single ball. Therefore, if the base be a triangle, the several tires of balls will be triangular numbers, which, reckoned from the top, are

1. 3. 6. 10. 15. 21. 28. 36. 45. 55. 66. 78. &c.

But, if the base be a square, the tires of balls will be square numbers, which, beginning at the top, are

1. 4. 9. 16. 25. 36. 49. 64. 81. 100. 121. 144. 169. &c.

1. To find the sum of a series of triangular numbers beginning at 1, the number of terms being given.

Put n = the number of the terms, and then $\frac{n \times \overline{n+1} \times \overline{n+2}}{6}$ is the sum of the series.

Exam. What is the sum of 8 terms of the series of triangular numbers?

Here $n = 8$, and $\frac{8 \times 9 \times 10}{6} = 120$, is the answer.

2. To find the sum of a series of square numbers beginning with 1, the number of terms being given?

Put n = the number of terms, then $\frac{n \times \overline{n+1} \times \overline{2n+1}}{6}$ is the sum of the series.

Exam. It is required to find the sum of 10 terms of the series of square numbers?

Here $n = 10$, and $\frac{10 \times 11 \times 21}{6} = 385$, is the answer.

3. To find the number of shot in a triangular pile.

Count the number of balls in one side of the base; to that number add 1, and to the same number add 2; then multiply

multiply these three numbers, one by the other, and the product by the third, and divide the product by 6; the quotient is the number of shot in the pile.

Exam. 1. How many balls are there in a finished triangular pile, one side of the base consisting of 24 balls?

By adding 1 and 2 to 24, the three numbers are 24. 25. 26. and $\frac{24 \times 25 \times 26}{6} = 2600$ balls, the answer.

2. Required the number of balls in a finished triangular pile, one side of the lowest tire being 30 balls?

Ans. 4960.

Note. In triangular and square pyramidal piles, the number of horizontal tires, or the number of balls from top to bottom, is always equal to the number in one side of the base, or bottom tire.

4. To find the number of shot in a square pyramidal pile.

Count the number of balls in one side of the base; to the number add 1, and to its double add 1; then multiply these three numbers into each other continually, and divide the product by 6; the quotient is the number of balls in the pile.

Exam. It is required to find the number of shot in a square pile, each side of its base consisting of 20 balls?

By adding 1 to 20, and 1 to the double of 20, the three numbers are 20, 21, and 41, and $\frac{20 \times 21 \times 41}{6} =$ 2870 balls, the answer.

5. In a broken pyramidal pile, either triangular or square.

Compute

Compute the number of balls in the whole pile according to its base, and then, by the number in one side of the uppermost tire, compute the number in the part that is wanting, the difference of these two is the number of balls in the broken pile.

Exam. 1. It is required to find the number of balls in a broken triangular pile, one side of its base being 30 balls, and one side of the uppermost tire being 15?

Ans. 4280.

In the whole pile there is $\frac{30 \times 31 \times 32}{6} = 4960$

In the part wanting there is $\frac{15 \times 16 \times 17}{6} = 680$

Difference 4280

2. How many balls are there in a broken square pile, the number in one side of the base being 40, and the number in a side of the uppermost tire being 20?

Ans. 19270.

In the whole pile there is $\frac{40 \times 41 \times 81}{6} = 22140$

In the part wanting there is $\frac{20 \times 21 \times 41}{6} = 2870$

Difference 19270

In rectangular piles, each horizontal tire is a rectangle, the uppermost being one row of balls; and the number of horizontal tires is equal to the number in the breadth of the base.

6. To

6. To find the number of shot in an oblong pile.

Compute the number of shot in a square pile, each side of its base being the breadth of the rectangular base. Then take the difference between the length and breadth of the rectangular base; multiply the breadth + 1 by half of the breadth, and multiply the product by the said difference, the last product is the number of balls in the prismatic pile; and the sum of the two is the number of balls in the whole pile.

Note. Every oblong pile consists of a square pyramid, (each side of its base being the breadth of the oblong pile) and the triangular prism.

EXAM. There is a compleat oblong pile of 15 tires, the number of balls in the top row being 32; it is required to find the number of shot in this pile.

Ans. 4960.

Here, the difference between the length and breadth of the base is 31, and the breadth of the base is 15.

A square pile, the side of whose base is 15, is $\frac{16 \times 31 \times 15}{6} = 1240$. The breadth of the base + 1 is 16, and the half breadth is $7\frac{1}{2}$, and $16 \times 7\frac{1}{2} \times 31 = 3720$. Lastly, $1240 + 3720 = 4960$.

7. To find the number of shot in a broken oblong pile.

To twice the length, and to twice the breadth of the uppermost tire, add the number of tires — 1, and multiply the two sums together; also multiply the number of tires + 1, by the same number — 1, and add one third of this product to the former: Then one fourth part of the

the sum, multiplied by the number of tires, shall give the number of shot in the oblong broken pile.

EXAM. Suppose there is a broken oblong pile of shot, the length of the uppermost tire being 25, and its breadth 16 balls, and the number of tires 11; it is required to find the number of shot in the pile?

Here twice the length is 50, and twice the breadth is 32, and the number of tires — 1 is 10.

Then $60 \times 42 = 2520$ is the first product, also $12 \times 10 = 120$ is the second product, one third of which is 40, and $2520+40=2560$.

Lastly, $\frac{2560 \times 11}{4} = 640 \times 11 = 7040$ balls, the answer.

SECT. VIII.

THE NATURE AND USE OF LOGARITHMS.

IF the natural numbers are considered as terms of an infinite series of proportionals, beginning at unity, and either increasing or decreasing to infinity, the logarithm of any number is its distance from unity in that series. Or,

Logarithms are artificial numbers, so related to each other, and to the natural numbers, that the sum of any two logarithms is the logarithm of the product of their two natural numbers; and the difference of any two logarithms is the logarithm of the quotient of their two natural numbers. Logarithms are a series of numbers in arithmetical progression corresponding to other numbers

in

in geometrical progression; and their nature and properties are to be derived from the known properties of these progressions.

If the common difference of any series of numbers in arithmetical progression, whose first term is 0, be represented by d; and the common ratio of a series of numbers in geometrical progression, whose first term is 1, be represented by r; then shall the co-efficients of d in the arithmetical progression be equal to the indices, or exponents, of r in the geometrical progression.

Examples.

Arith. Prog.	0	d	$2d$	$3d$	$4d$	$5d$	$6d$	$7d$	$8d$,		&c.
Geom. Prog.	$1:$	$r:$	$r^2:$	$r^3:$	$r^4:$	$r^5:$	$r^6:$	$r^7:$	r^8,		&c.

1. If $d=1$, and $r=3$, the above progressions will be,

Arith.	0	1	2	3	4	5	6	7	8	9
Geom.	1 :	3 :	9 :	27 :	81 :	243 :	729 :	2187 :	6561 :	19683

2. If $d=3$, and $r=3$, then,

Arith.	0	3	6	9	12	15	18	21	24	27
Geom.	1 :	3 :	9 :	27 :	81 :	243 :	729 :	2187 :	6561 :	19683

3. If $d=1$, and $r=10$, then,

Arith.	0	1	2	3	4	5	6
Geom.	1 :	10 :	100 :	1000 :	10000 :	100000 :	1000000

In these progressions, it is manifest,

1. That, when $d=1$, every term in the arithmetical series expresses the distance of its correspondent term of the geometrical series from unity, or 1, agreeable to our definition of logarithms.

 2. That

2. That if any two or more terms of the arithmetical series be added, and their correspondent terms of the geometrical series be multiplied into each other, the sum and product shall be corresponding terms.

EXAMPLES.

1. In the general series, the sum of $2d$ and $3d$ is $5d$, and the product of their corresponding terms, viz. $r^2 \times r^3 = r^5$; which is the term corresponding to $5d$.

2. In the progression where $d=1$ and $r=3$, we have $2+3=5$, and $9 \times 27=243$, the term corresponding to 5.

3. The sum of 2, 3, and 4, is 9; and $9 \times 27 \times 81 = 19683$, the term corresponding to 9.

4. In the series where $d=3$, and $r=3$. The sum of 6 and 18 is 24; and the product of $9 \times 729=6561$, the term corresponding to 24. And the same holds true every where.

3. If any term in the arithmetical series be doubled or tripled, and its corresponding term in the geometrical series be squared, or cubed, the results will be corresponding terms.

EXAM. In the series where $d=1$ and $r=3$. The double of 3 is 6, and the square of its corresponding term 27 729, which corresponds to 6.

2. The triple of 3 is 9, and the cube of 27 is 19683; which is the term corresponding to 9.

4. If any term in the arithmetical series be subtracted from another, and their corresponding terms in the geometric series be divided one by the other, the difference and quotient shall be correspondent terms.

EXAM.

EXAM. The difference between 2 and 5 is 3; and, if 243 be divided by 9, the quotient is 27, which corresponds to 3.

5. If any term in the arithmetical series be divided by 2 or 3, and the square or cube root of the corresponding term be extracted, the quotient and root shall be corresponding terms.

EXAM. 1. The half of 8 is 4, and the square root of 6561 is 81, which corresponds to 4.

2. The third part of 9 is 3, and the cube root of 19683 is 27, which is the correspondent term to 3.

Hence, it is evident, that the terms of the arithmetical progression are logarithms of their corresponding terms of the geometrical progression. And since d and r may represent any number whatsoever; if $d=1$, and $r=10$, (as in the last progression), the arithmetical progression will be the common logarithms now in use. But these are only the logarithms of the numbers 10, 100, 1000, &c.; and, in order to find out the logarithms of the intermediate numbers 2, 3, 4, &c. it may be observed,

1. In any geometrical progression, if between any two adjacent terms, any number of mean proportionals be interpolated, a series of terms will be produced, which are also in geometrical progression.

EXAM. Let the proposed geometrical progression be

$$1 \qquad r^4 \qquad r^8 \qquad r^{12}$$

Place one mean between every two terms, and the series shall be

$$1 \qquad r^2 \qquad r^4 \qquad r^6 \qquad r^8 \qquad r^{10} \qquad r^{12}$$

And if three means be placed between every two terms, the series shall be

$$1 : r : r^2 : r^3 : r^4 : r^5 : r^6 : r^7 : r^8 : r^9 : r^{10} : r^{11} : r^{12}$$

2. In like manner, if between the terms of an arithmetical progression, there be interpolated any number of arithmetical means, there will be produced a series in arithmetical progression.

EXAM. In the arithmetical progression

0 : 4 : 8 : 12 : 16 : 20

Place one mean between every two terms, and the progression will be

0 : 2 : 4 : 6 : 8 : 10 : 12 : 14 : 16 : 18 : 20

Place three means between every two terms, and the series shall be

0:1:2:3:4:5:6:7:8:9:10:11:12:13:14:15:16:17:18:19:20

And if one mean be placed between every two terms of this series, we shall have

0 : $\frac{1}{2}$: 1 : $1\frac{1}{2}$: 2 : $2\frac{1}{2}$: 3 : $3\frac{1}{2}$: 4 : $4\frac{1}{2}$: 5 : $5\frac{1}{2}$: 6, &c.

Therefore, the number of terms in any progression may be increased to infinity, by interpolating means between every two adjacent terms. Hence,

When any system of logarithms is proposed for the natural geometrical series 1, 10, 100, 1000, 10000, &c. if a great number of mean proportionals be interpolated between the adjacent terms, and correspondent arithmetical means be interpolated between their corresponding logarithms, the logarithms of the intermediate natural numbers will be had. For example,

A mean proportional between 1 and 10 is 3.16227766, and its logarithm is .5000000, viz. the half sum of 0 and 1, and a mean proportional between 3.16227766, and 10 is 5.6234132, and its logarithm is .7500000, viz. the half sum of 1 and .5; and, if we calculate mean proportionals,

tionals, and their correſponding logarithms continually between 10 and the laſt found mean, until we find a mean proportional greater than 9; and then calculate mean proportionals between that mean and the next leſs than 9; the 25th mean will be ſo nearly equal to 9, that it will not want above $\frac{1}{10000000}$ of an unit; and, by finding an arithmetical mean anſwering to every geometrical mean, the 25th will be 0.9542425, which is the common logarithm of the number 9.

In this manner, the firſt tables of logarithms were calculated; but this method being very tedious, ſhorter methods have been invented by means of infinite ſerieſes, whereby logarithms may be made, or the truth of any logarithm examined, with leſs trouble.

The noble inventor of logarithms made choice of a ſyſtem different from that in preſent uſe; but, ſoon after he had publiſhed his logarithms, he propoſed another more commodious form, and communicated his invention to Mr. HENRY BRIGGS, Profeſſor of Geometry at Oxford, who, upon Lord NAPIER's deceaſe, calculated and publiſhed tables of logarithms of this kind; and hence thoſe logarithms that were firſt publiſhed are called *Napier's logarithms*, and the other, which are now in common uſe, are called *Briggs's logarithms*, although both kinds were Lord NAPIER's invention. A ſpecimen of each kind follows:

Numbers.

Numbers.	Napier's Logarithms.	Numbers.	Briggs's Logarithms.
1	.0000000	1	.0000000
10	2.3025850	10	1.0000000
100	4.6051700	100	2.0000000
1000	6.9077550	1000	3.0000000
10000	9.2103400	10000	4.0000000
100000	11.5129250	100000	5.0000000

These two systems of logarithms are so related to each other, that *Briggs*'s logarithms may be said to be derived from *Napier*'s; and, in order to calculate *Briggs*'s logarithms, it is necessary, first of all, to calculate *Napier*'s logarithm of 10, for which purpose, the following series has been invented.

If any number be represented by $\frac{1+e}{1-e}$, *Napier*'s logarithm of that number will be

$2\times e+\frac{e^3}{3}+\frac{e^5}{5}+\frac{e^7}{7}+\frac{e^9}{9}+\frac{e^{11}}{11}+\frac{e^{13}}{13}$, &c. But, in applying this series to practice, it will be proper to begin with a small number; for the less e is taken, the more swiftly will the series converge.

Exam. Let it be required to find *Napier*'s logarithm of 2?

$\frac{1+e}{1-e}=2$ $1+e=2-2e$ ∴ $3e=1$ $e=\frac{1}{3}$ and $ee=\frac{1}{9}$

e=

$e =$	.3333333333	$e =$	.3333333333
e^3	370370370	$\frac{1}{3}e^3$	.0123456790
e^5	41152263	$\frac{1}{5}e^5$	8230452
e^7	4572473	$\frac{1}{7}e^7$	653210
e^9	508052	$\frac{1}{9}e^9$	56450
e^{11}	56450	$\frac{1}{11}e^{11}$	5131
e^{13}	6272	$\frac{1}{13}e^{13}$	482
e^{15}	696	$\frac{1}{15}e^{15}$	46
e^{17}	77	$\frac{1}{17}e^{17}$	4
			.3465735898
			× 2

Napier's logarithm of 2= .6931471796

And, because the cube of 2 is 8, multiply the logarithm of 2 by 3, the product is the logarithm of 8=2.0794415388.

And, because $8 \times \frac{5}{4} = 10$, calculate the logarithm of $\frac{5}{4}$, and add it to the logarithm of 8, the sum shall be the logarithm of 10.

$\frac{1+e}{1-e} = \frac{5}{4}$ $\quad 4+4e=5-5e \therefore 9e=1$ $\quad e=\frac{1}{9}$, and $ee=\frac{1}{81}$

$e =$	.1111111111	$e =$	.1111111111
e^3	13717421	$\frac{1}{3}e^3$	4572473
e^5	169350	$\frac{1}{5}e^5$	33870
e^7	2090	$\frac{1}{7}e^7$	298
e^9	25	$\frac{1}{9}e^9$	2
			.1115717754
			× 2

Napier's logarithm of $\frac{5}{4}$ = .2231435508
To which add the log. of 8 2.0794415388

And *Napier's* log. of 10 is =2.3025850896

In

In calculating the logarithm of 2, $e=\frac{1}{3}$, and therefore the first step is to write down the decimal of $\frac{1}{3}=.333$, &c. To get e^3, I multiply e by $e^2=\frac{1}{9}$: that is, I divide .333, &c. by 9, and write the quotient .037037, &c. under the value of e. For e^5, I multiply e^3 by e^2; that is, I divide .037037, &c. by 9, and so on, dividing every power of e by 9, I get e^7, e^9, e^{11}, &c.; then I divide each power by its index, and the sum of the quotients is the half of *Napier's* logarithm of 2. In the same manner, the logarithm of $\frac{5}{4}$, or of any other number, is found.

Napier's log. of 10 comes out = 2.302585096; but if in the first step of the calculation, e had been taken nearer to the truth, and made to consist of 17 places of decimals, and all the rest of the operation carried on accordingly, *Napier's* logarithm of 10 would have been =2.302585092994045, and its reciprocal (that is 1 divided by it) =.4342944819032 51.

A system of logarithms being given, and one or more of another system, any logarithm of that other system may be found thus:

As any logarithm of the given system is to its correspondent logarithm in the other system, so is any other logarithm in the given system to its correspondent logarithm in the other system.

EXAM. *Napier's* logarithm of 10 being 2.3025850, and his logarithm of 2 = .6931471, also *Briggs's* logarithm of 10 being 1.0000000; it is required to find *Briggs's* logarithm of 2?

As

As *Napier's* logarithm of 10 = 2.3025850
Is to *Briggs's* logarithm of 10 = 1.0000000
So is *Napier's* logarithm of 2 = .6931471
To *Briggs's* logarithm of 2 = .3010299

And because the two first terms of this proportion are constant, and the second term is 1, if any of *Napier's* logarithms be divided by 2.3025850, or multiplied by its reciprocal, viz. .4342944, the quotient, or product, will be *Briggs's* logarithm of the same number. Therefore,

To calculate *Briggs's* logarithms directly, we must multiply the series $2 \times e + \frac{e^3}{3} + \frac{e^5}{5} + \frac{e^7}{7}$, &c. by .43429, &c.; or, multiply $e \times \frac{e^3}{3} + \frac{e^5}{5} + \frac{e^7}{7}$ &c. by the double of .43429, &c. viz. .8685889. Put $m = .8685889$, &c. and then the series expressing *Briggs's* logarithm of $\frac{1+e}{1-e}$ will be $me + \frac{me^3}{3} + \frac{me^5}{5} + \frac{me^7}{7} + \frac{me^9}{9} + \frac{me^{11}}{11} + \frac{me^{13}}{13}$ &c. Or,

$$m \times \overline{e + \frac{e^3}{3} + \frac{e^5}{5} + \frac{e^7}{7} + \frac{e^9}{9} + \frac{e^{11}}{11} + \frac{e^{13}}{13} + \frac{e^{15}}{15}}.$$

EXAM. 1. Let it be required to find *Briggs's* logarithm of 2?

$\frac{1+e}{1-e} = 2$ $\quad 1+e = 2-2e \therefore 3e = 1 \quad e = \frac{1}{3}$, and $e^2 = \frac{1}{9}$.

$m = .8685889638$

me	$= .2895296546$	me	$= .2895296546$
me^3	32199616	$\frac{1}{3}me^3$	107233205
me^5	35744401	$\frac{1}{5}me^5$	7148880
me^7	3971600	$\frac{1}{7}me^7$	567371
me^9	441288	$\frac{1}{9}me^9$	49032
me^{11}	49032	$\frac{1}{11}me^{11}$	4457
me^{13}	5448	$\frac{1}{13}me^{13}$	419
me^{15}	605	$\frac{1}{15}me^{15}$	40
me^{17}	67	$\frac{1}{17}me^{17}$	4

Briggs's logarithm of $2 = .3010299954$

2. To find *Briggs*'s logarithm of 3; because $2 \times 1\frac{1}{2} = 3$, therefore calculate the logarithm of $\frac{3}{2}$, and add it to the logarithm of 2, the sum is the logarithm of 3.

$\frac{1+e}{1-e} = \frac{3}{2}$, $2+2e = 3-3e \because 5e = 1$ $e = \frac{1}{5}$, and $e^2 = \frac{1}{25}$.

$m = .8685889638$

me	$= .1737177927$	me	$= .1737177927$
me^3	69487117	$\frac{1}{3}me^3$	23162372
me^5	2779484	$\frac{1}{5}me^5$	555897
me^7	111179	$\frac{1}{7}me^7$	15882
me^9	4447	$\frac{1}{9}me^9$	494
me^{11}	177	$\frac{1}{11}me^{11}$	16

Briggs's logarithm of $\frac{3}{2}$	$= .1760912588$
To which add the log. of 2	$.3010299954$
The sum is the log. of 3	$= .4771212542$

3. To

3. To find the logarithm of 4; because the square of 2 is 4.

Multiply the logarithm of 2		= .3010299954
By		2
The product is the log. of 4		= .6020599908

4. To find the logarith. of 5; because 10 divided by 2 quotes 5.

From the logarithm of	10	= 1.0000000000
Subtract the logarithm of	2	.3010299954
The remainder is the log. of	5	.6989700046

5. To find the logarithm of 6; because $3\times2=6$.

To the logarithm of	3	= .4771212542
Add the logarithm of	2	.3010299954
The sum is the logarithm of	6	.7781512496

6. To find the logarithm of 7; because $6\times\frac{7}{6}=7$, therefore calculate the logarithm of $\frac{7}{6}$, and add it to the logarithm of 6, the sum is the logarithm of 7.

$\frac{1+e}{1-e}=\frac{7}{6}$ $6+6e=7-7e$ ∴ $13e=1$ ∴ $e=\frac{1}{13}$, and $e^2=\frac{1}{169}$

m = .8685889638

me =	.0668145356	me =	.0668145356
me^3	3953522	$\frac{1}{3}me^3$	1317840
me^5	.23393	$\frac{1}{5}me^5$	4678
me^7	138	$\frac{1}{7}me^7$	19

Logarithm of	$\frac{7}{6}$	= .0669467893
Add the logarithm of	6	.7781512496
The sum is the log. of	7	= .8450980389

 7. To

7. To find the logarithm of 8; because the cube of 2 is 8.

Multiply the logarithm of	2	=	.3010299954
By			3
The product is the log. of	8	=	.9030899862

8. To find the logarithm of 9; because the square of 3 is 9.

Multiply the logarithm of	3	=	.4771212542
By			2
The product is the log. of	9	=	.9542425084

9. To find the logarithm of 11; because $10 \times \frac{11}{10} = 11$, find the logarithm of $\frac{11}{10}$, and add it to the logarithm of 10, the sum is the logarithm of 11.

$\frac{1+e}{1-e} = \frac{11}{10}$ $\quad 10+10e = 11-11e \quad \because \quad 21e = 1 \quad e = \frac{1}{21}$, and $e^2 = \frac{1}{441}$.

$m = .8685889638$

$me =$ .0413613792		$me =$	.0413613792
me^3 937899		$\frac{1}{3}me^3$	312633
me^5 2126		$\frac{1}{5}me^5$	425

Logarithm of	$\frac{11}{10}$	=	.0413926850
Add the logarithm of	10		1.0000000000
The sum is the logarithm of 11			1.0413926850

10. To

10. To find the logarithm of 12; because $6 \times 2 = 12$.

To the logarithm of	6	=	.7781512496
Add the logarithm of	2		.3010299954
The sum is the logarithm of	12	=	1.0791812450

11. To find the logarithm of 13; because $12 \times \frac{13}{12} = 13$; find the logarithm of $\frac{13}{12}$, and add it to the logarithm of 12, the sum is the logarithm of 13.

$\frac{1+e}{1-e} = \frac{13}{12}$ $\quad 12 + 12e = 13 - 13e \quad \therefore \quad 25e = 1 \quad e = \frac{1}{25}$, and

$e^2 = \frac{1}{625}$

$m = .8685889638$

me =	.0347435585	me =	.0347435585
me^3	555896	$\frac{1}{3}me^3$	185298
me^5	889	$\frac{1}{5}me^5$.	178

Logarithm of	$\frac{13}{12}$	=	.0347621061
Add the logarithm of	12		1.0791812450
The sum is the logarithm of	13	=	1.1139433511

12. To find the logarithm of 14; because $7 \times 2 = 14$.

To the logarithm of	7	=	.8450980389
Add the logarithm of	2		.3010299954
The sum is the logarithm of	14	=	1.1461280343

13. To

13. To find the logarithm of 15; because $5 \times 3 = 15$.

To the logarithm of	5	=	.6989700046
Add the logarithm of	3		.4771212542
The sum is the logarithm of	15	=	1.1760912588

14. To find the logarithm of 16; because the square of 4 is 16.

Multiply the logarithm of	4	=	.6020599908
By			2
The product is the log. of 16		=	1.2041199816

15. To find the logarithm of 17; because $16 \times \frac{17}{16} = 17$, calculate the logarithm of $\frac{17}{16}$, and add it to the logarithm of 16, the sum is the logarithm of 17.

$\frac{1+e}{1-e} = \frac{17}{16}$ $\quad 16 + 16e = 17 - 17e \therefore 33e = 1 \quad e = \frac{1}{33}$, and

$e^2 = \frac{1}{1089}$.

$m = .8685889638$

$me =$	.0263208776	$me =$	.0263208776
me^3	241697	$\frac{1}{3}me^3$	80565
me^5	221	$\frac{1}{5}me^5$	44

Logarithm of	$\frac{17}{16}$	=	.0263289385
Add the logarithm of	16		1.2041199816
The sum is the logarithm of	17	=	1.2304489201

16. To

16. To find the logarithm of 18; because $9 \times 2 = 18$.

To the logarithm of	$9 =$	.9542425084
Add the logarithm of	2 -	.3010299954
The sum is the logarithm of 18		1.2552725038

17. To find the logarithm of 19; because $\frac{19}{18} \times 18 = 19$, calculate the logarithm of $\frac{19}{18}$, and add it to the logarithm of 18, the sum is the logarithm of 19.

$\frac{1+e}{1-e} = \frac{19}{18}$ $\quad 18 + 18e = 19 - 19e \because 37e = 1 \quad e = \frac{1}{37}$, and

$e^2 = \frac{1}{1369}$.

$m = .8685889638$

$me =$ .0234753774		$me =$ .0234753774	
me^3	171478	$\frac{1}{3}me^3$	57159
me^5	125	$\frac{1}{5}me^5$	25

Logarithm of	$\frac{19}{18}$	$=$ .0234810958
Add the logarithm of	18	1.2552725038
The sum is the logarithm of 19		$=$ 1.2787535996

By these calculations, we have obtained the logarithms of the first 20 numbers true to eight places of figures, which are as follows:

No.

N°.	Logarithms.	N°.	Logarithms.
1	.00000000	11	1.04139268
2	.30102999	12	1.07918124
3	.47712125	13	1.11394335
4	.60205999	14	1.14612803
5	.69897000	15	1.17609125
6	.77815125	16	1.20411998
7	.84509803	17	1.23044892
8	.90308998	18	1.25527250
9	.95424250	19	1.27875360
10	1.00000000	20	1.30102999

In the same manner, the whole table of logarithms may be made to any extent.

1. It may be observed, that this work is most tedious at the beginning, and becomes easier as we advance; for, fewer terms of the series give the logarithm of $\frac{13}{12}$ than of $\frac{3}{2}$; and, in numbers above 100 and below 1000, the first term will give the logarithm true to seven places of figures; and, in any number above 1000, the first term of the series will give the logarithm true to eight or nine places.

2. When the logarithm of any number is known, the logarithms of all its multiples by 10, 100, 1000, &c. are also known; for the logarithm of 2 is the same with the logarithm of 20, 200, 2000, 20000, or 200000, &c. except the indices which are to be prefixed according to the number of figures in the natural number. And the same is true of any other number.

3. The logarithms of all composite numbers are found by addition or subtraction, as may be seen in the above examples.

examples. It is only in calculating the logarithms of prime numbers that the ſeries muſt be applied.

The logarithms of prime numbers may alſo be found by either of the two following rules:

1. The logarithms of the numbers next below and above any prime number being given, to find the logarithm of that prime number.

Let a and b repreſent the adjacent numbers, whoſe logarithms are known; $p=$ the number whoſe logarithm is ſought; $y=ab+\frac{a^2+2ab+b^2}{4}=2ab+1$; and $n=$ .43429448190, &c. Then the logarithm of the ratio of the geometrical mean between a and b, viz. $\surd ab$, to the arithmetical mean, viz. p, ſhall be $=\frac{n}{y}+\frac{n}{3y^3}+\frac{n}{5y^5}+\frac{n}{7y^7}$, &c.

In numbers between 20 and 100, the firſt term of this ſeries will give the logarithm true to 9 places of figures; and, in numbers above 100, the firſt term will give the logarithm true to 12 or 14 places of figures. Therefore, ſince the firſt term of the ſeries will produce any logarithm for common uſe, we have this practical rule for finding the logarithm of any prime number above 20.

Multiply the adjacent numbers, one by the other, and add 1 to double of the product. Divide .43429448190 by the ſum, and add the quotient to the half ſum of the logarithms of the adjacent numbers, the ſum is the logarithm required.

Exam. Let it be required to find the logarithm of 23?

$a=22$, $b=24$, and $2ab+1=1057=y$
$1057).434294481 90(.00041087462$

The log. of 22	=	1.34242268082
Ditto of 24	=	1.38021124171
		2)2.72263392253 sum
Half sum	=	1.36131696126
To which add		.00041087462
Log. of 23	=	1.36172783588 too little by 2 in the 10th place.

2. The logarithm of the ratio of the geometrical mean to the arithmetical mean may also be found thus:

Let y represent the difference between the logarithms of the adjacent numbers, and z the number whose logarithm is sought. Then $y \times \frac{1}{4z}+\frac{1}{24z^3}+\frac{7}{360z^5}$ is the logarithm of the ratio of the geometrical to the arithmetical mean.

This series does not converge so quickly as the former, and therefore not so proper for small numbers. But, if the prime number is between 100 and 1000, the first term of the series will give the logarithm true to 9 places of figures; and, in numbers above 1000, to 12 or 14 places: Wherefore, to find the logarithm of any prime number above 100, divide the difference of the logarithms of the adjacent numbers by four times that prime

prime number, and add the quotient to the half sum of the adjacent logarithms, the sum is the logarithm required.

Exam. Let it be required to find the logarithm of 101?

The log. of 100	=	2.0000000000
Ditto of 102	=	2.0086001717
Sum	=	4.0086001717
Half sum	=	2.0043000858
Diff.	=	.0086001717

2=101 × 4=404).0086001717(.0000212875
Add 2.0043000858

Logarithm of 101 = 2.0043213733
True to the 10th place.

The first of these rules was invented by Dr. *Halley*, the other by Dr. *John Keil*.

The use of logarithms is to shorten the operations of multiplication, division, raising powers, and extracting roots, in common arithmetic.

To answer this end, it is necessary to have the logarithms of all numbers, ready to be used as occasion requires. But because, in the common problems where logarithms are most useful, the numbers seldom exceed five or six figures, tables of logarithms have been made for all numbers under 10 or 100 thousand only, from which the logarithm of any number, and the number

anſwering to any logarithm, may be had true to ſix or ſeven figures, according to the extent of the tables.

Since the logarithm of 1 is 0, and of 10 is 1, the logarithms of numbers between 1 and 10 are greater than 0, and leſs than 1; that is, they are decimal fractions. *Vide p.* 281, *&c.*

And, ſince the logarithm of 10 is 1, and of 100 is 2, the logarithms of numbers between 10 and 100 will be greater than 1, and leſs than 2, or mixed numbers, having 1 in the place of integers, and the reſt decimal fractions.

The integral part of a logarithm is called its *index*, or *characteriſtic*; becauſe it denotes the number of figures in the natural number anſwering to the logarithm.

The index of a logarithm is always an unit leſs than the number of figures in the integer number, whoſe logarithm it is. Thus, the index of the logarithm of a number conſiſting of one figure is 0, of 2 figures is 1, of 3 figures 2, and 4 figures, 3, &c. as in theſe examples.

Numb.	Logarithms.	Numb.	Logarithms.
7	0.8450980	325675	5.5127845
17	1.2304489	4897255	6.6899527
139	2.1430148	92675000	7.9669626
5262	3.7211508	687540000	8.8372980
56990	4.7557987	4997560000	9.6987580

Therefore, the indices of logarithms (being known by the number of figures in their natural numbers) need not be printed in the tables.

When

When a logarithm is propofed, its index fhows how many integer figures the correfponding number confifts of, being always one figure more than there are units in the index of the logarithm.

Numbers which confift of the fame fignificant figures, have the fractional parts of their logarithms always the fame, and differ only in their indices, as in the following

EXAMPLES.

Numb.	Logarithms.
47500	4,6766936
4750	3,6766936
475	2,6766936
47·5	1,6766936
4·75	0,6766936
·475	9,6766936
·0475	8,6766936

The index of the logarithm of a decimal fraction is negative; thus the index of .475 is properly —1, of .0475 —2, and of .00475 —3; but, to avoid negative numbers in addition and fubtraction, their complements to 10 or 100 are made ufe of; and, inftead of —1, —2, —3, the indices are made 9, 8, 7, or 99, 98, 97.

The following table contains the logarithms of numbers from 1 to 10000; their fractional parts confifting of 7 figures, from which the logarithm of any number may be had from 1 to 10000000 by the following rules:

1. To find the logarithm of any number below 1000.

Find the number near the beginning of the table in the column marked N°. at the head; and next it on the right

right hand is the fractional part of the logarithm, to which prefix its proper index.

	Numb.	Logarith.
Examples.	19	1.2787536
	364	2.5611014
	795	2.9003671

2. To find the logarithm of any number consisting of four figures.

Find the first three figures of the number in the column marked N°. and the fourth or last figure on the head of the page; below which, opposite to the first three figures, you have the four or five last figures of the logarithm; to which prefix the common figures in the second column marked with 0, or 5, and the index 3.

	Numb.	Logarith.
Examples.	1974	3.2953471
	3785	3.5780659
	4977	3.6969676
	6748	3.8291751

N. B. The common figures in the second column answer to those in every column after it, in the same and following lines, until the figures in the following column begins with 0, and then you are to take the common figures next below, as in the fourth example above.

3. To find the logarithm of any number consisting of five, six, or seven figures.

Find the logarithm of the first four figures as before, to which prefix the index according to the number of figures

figures in the given number; and then take the difference between the logarithm found, and the next greater in the table; which multiply by the remaining figure or figures in the given number; and strike off the number of figures from the product by which the difference was multiplied. Add the remaining part of the product to the logarithm before found; the sum is the logarithm required.

EXAM. 1. What is the logarithm of 25768?

The logarithm of 25760 is 4.4109459

Difference	1685	add	1348
Multiply by	8		
	1348|0	Log. of 25768=	4.4110807

2. What is the logarithm of 498797?

The logarithm of 498700 is 5.6978374

		add	845
Difference	872		
Multiply	97	Log. of 498797=	5.6979239
	6104		
	7848		
	845|84		

3. What is the logarithm of 5678979?

Ans. 6.7542703.

4. To find the logarithm of a proper fraction, subtract the logarithm of the denominator from the logarithm of the numerator, the remainder is the logarithm of the fraction.

EXAM.

EXAM. What is the logarithm of $\frac{179}{479}$?

From the log. of 179 (add 10 to its index)	12.2528530
Subtract the log. of 479	2.6803355
Ans.	9.5725175

To find the logarithm of a mixt number, such as $1\frac{1}{2}$, $2\frac{1}{4}$, $23\frac{11}{47}$, &c.

Reduce the mixt number to an improper fraction, and subtract the logarithm of the denominator from the logarithm of the numerator; the remainder is the answer.

EXAM. 1. What is the logarithm of $1\frac{1}{2}$?

$1\frac{1}{2} = \frac{3}{2}$	From the log. of 3	.4771213
	Subtr. the log. of 2 —	.3010300
	There remains the log. of $1\frac{1}{2}$ =	.1760913

2. What is the logarithm of $2\frac{1}{4}$?

$2\frac{1}{4} = \frac{9}{4}$	log. of 9	.9542425
	ditto of 4	.6020600
	log. of $2\frac{1}{4}$	.3521825

3. Required the logarithm of $23\frac{11}{47}$?

$23\frac{11}{47} = \frac{1092}{47}$.	log. of 1092	3.0382226
	ditto of 47 —	1.6720979
	log. of $23\frac{11}{47}$ =	1.3661247

4. What is the logarithm of $197\frac{5}{19}$?

$197\frac{5}{19} = \frac{3748}{19}$	log. of 3748	3.5737996
	ditto of 19 —	1.2787536
	log. of $197\frac{5}{19}$ =	2.2950460

5. To

5. To find the logarithm of a decimal fraction, proceed as if it were an integer, and prefix the proper index.

N°.	Logarithms.	N°.	Logarithms.
.5	9.6989700	.075	8.8750613
.25	9.3979400	.0015	7.1760913
.125	9.0969100	.000325	6.5118834
.4795	9.6807886	.00007925	5.8989993

6. To find the number anſwering to any given logarithm.

Look for the fractional part of the given logarithm in the different columns until you find it, either exactly, or the next leſs. And, in a line with the logarithm found, in the column marked N°. you have three figures of the number ſought, and on the top of the page you have one figure more, which write down, and point it according to the index of the given logarithm.

If the logarithm is not found exactly in the tables, and more than four figures are required, ſubtract the logarithm found in the table from the given one, and divide the remainder by the difference between the logarithm found and the next greater; the quotient joined to the four figures already found, gives the number anſwering to the logarithm.

Examples.

To find the number anſwering to the logarithm 2.7342957. Seek in the ſecond column for the three figures next the index (.734), and, having found them, look among the columns on the right hand for the laſt four figures, and in that under 3, you find 2396; there-

fore the number is 5423; but, because the index of the logarithm is 2, the number answering hath only three integer figures, and the last is a decimal, viz. 542.3.

If more figures are desired, divide the difference between 2957 and 2396, viz. 561, by 801, the difference of the logarithms (annexing 0 to 561), the quotient 7, joined to the former, makes the answer 542.37.

Required the numbers answering to these logarithms.

0.7369425	1.2786536
1.2345678	2.9003671
2.6549764	3.7654321
3.7496543	4.6849776
5.9769423	1.2497894

LOGARITHMICAL ARITHMETIC.

7. To perform multiplication by logarithms, add the logarithm of the multiplier to the logarithm of the multiplicand; the sum is the logarithm of the product.

EXAMPLES.

1. Multiply	379	2.5786392
By	47	1.6720979
Product	17813	4.2507371

2. Multiply 79.8 by 2.79.

8. To perform division by logarithms.

Subtract the logarithm of the divisor from the logarithm of the dividend, the remainder is the logarithm of the quotient.

EXAM.

Exam. 1. Divide	25768	4.4110807
By	364	2.5611014
Quotient	70.7912	1.8499793

2. Divide 476954 by 89.5.

9. To find a fourth proportional to three given numbers, or to work the rule of three by logarithms:

Add the logarithms of the second and third terms, and subtract the logarithm of the first term from the sum; the remainder is the logarithm of the fourth term, or answer.

Exam. If 497 yards cost L. 287; what will 389 yards cost? *Ans.* L. 224 : 12 : 8.

As 497	2.6963564
Is to 287	2.4578819
So is 389	2.5899496
	5.0478315
	2.6963564
To L. 224.633	2.3514751

N. B. Instead of subtracting the logarithm of the first term, some chuse to add its arithmetical complement.

10. To find the arithmetical complement of a logarithm; begin at the left hand, and write down what each figure wants of 9, and what the last significant figure wants of 10; so the arith. comp. of 2.6963564 is 7.3036436.

11. To raise a number to any power, by logarithms.

Multiply the logarithm of the given number by the index of the power; the product is the logarithm of the power required.

EXAM. Required the third power or cube of 81?

81 its log. is	1.9084850
Multiply by	3
Anʃ. 531441	5.7254550

What is the 3d power of $37\frac{11}{16}$? *Anʃ.* 53529.34

$37\frac{11}{16}=\frac{603}{16}$	603	2.7803173
	16	—1.2041200
	Log. of $\frac{603}{16}$ =	1.5761973
	Multiply by	3
		4.7285919

The 3d power of $37\frac{11}{16}$ is 53529.34

Note. In large numbers, the logarithms give the anſwer true only to ſeven figures; and even the 7th figure is ſometimes doubtful, as in this example, where it is too little by 1.

2. Required the fourth power of 1.05?

1.05 its logarithm is	0.0211893
Multiply by	4
Anʃ. 1.215506	0.0847572

It is required to find the 7th power of 1.05? *Anʃ.* 1.4071.

1.05 its logarithm is	0.0211893
Multiply by	7
	0.1483251

The 7th power of 1.05 = 1.4071.

3. Required the ſixth power of .05?

Its logarithm is	8.6989700
Multiply by	6
Anʃ. .000000015625	52.1938200

Here

Here the tens are thrown away from the index 52, and the 2 remaining ſhews that the firſt ſignificant figure muſt be in the eighth place from unity, and therefore ſeven cyphers muſt be prefixed.

12. To extract roots by logarithms.

Divide the logarithm of the given number by the index of the power; the quotient is the logarithm of the root; that is, divide by 2 for the ſquare root, by 3 for the cube root, and by 4 for the biquadrate, &c.

1. What is the ſquare root of 2? *Anſ* 1.414213.

The logarithm of 2 = 0.3010300

$\frac{1}{2}$ = 0.1505150

The ſquare root is 1.414213.

2. Required the ſquare root of 19110?

Anſ. 138.2389.

19110 its log. is 4.2812607

$\frac{1}{2}$ = 2.1406303

The ſquare root is 138.2389.

3. What is the ſquare root of $99\frac{11}{12}$? *Anſ.* 9.99583.

$99\frac{11}{12} = \frac{1199}{12}$ 1199 log. 3.0788192

12 log. 1.0791812

2)1.9996380

0.9998190

The ſquare root of $99\frac{11}{12}$ is 9.99583.

To find the ſide of a ſquare which ſhall contain a given ſquare any number of times.

To the logarithm of the given ſquare add the logarithm of the number of times the ſquare ſought is to contain

contain the given one, and divide the ſum by 2, the quotient is the logarithm of the ſide of the ſquare required.

EXAM. Suppoſe there is a ſquare which contains an acre, it is required to find the ſide of a ſquare which ſhall contain 12 acres? *Anſ.* 1095.445 links.

1 Acre is 100000 ſquare links.	Log.	5.0000000
	add the log. of 12	1.0791812
		2)6.0791812
1095.445 links.		3.0395906

To find the ſide of a ſquare which ſhall contain an area equal to a given rectangle.

Take the logarithms of the length and breadth of the given rectangle; and half their ſum is the logarithm of the ſide of the ſquare.

EXAM. Let there be a right angled parallelogram, its length 600 feet, and breadth 430; it is required to find the ſide of a ſquare which ſhall contain the ſame area?

Anſ. 532.728 feet.

Length	660	log.	2.8195439
Breadth	430		2.6334685
			2)5.4530124
ſide of the ſquare 532.728 F.			2.7265062

To find the ſide of a ſquare, equal to a triangle; the baſe and perpendicular height of the triangle being given in numbers.

To the logarithm of the baſe add the logarithm of half the perpendicular, and divide the product by 2, the quotient is the logarithm of the ſide of the ſquare.

EXAM.

EXAM. Let the base of a triangle be one mile, and the perpendicular height of it three quarters of a mile; it is required to find the side of a square which shall contain the same area as the triangle? *Ans.* 3233.326 feet.

	Feet.	
1 mile =	5280 log.	3.7226339
½ the perpend. ⅜ =	1980	3.2966652
		2)7.0192991
	Feet	
Side of the sqr.	3233.326	3.5096495

To find the side of a square equal to a trapezium, its diagonal and sides being given.

The diagonal divides the trapezium into two triangles; therefore, calculate the area of each triangle by its three sides, the sum of these is the area of the trapezium in numbers: The half of the logarithm of the area is the logarithm of the side of the square, equal to the trapezium.

EXAM. Let the side AB=46, BC=37, CD=75, and DA=63 feet, also the diagonal AC=73 feet: required the side of a square which shall contain the same area as the trapezium? *Ans.* 53.136 feet.

For the area of the triangle ACD.

1st, 73	105.5	105.5	105.5		105.5	2.0232525
75	73.	75.	63.		32.5	1.5118834
63					30.5	1.4842998
	32.5	30.5	42.5		42.5	1.6283889
2)211						
						2)6.6478246
105.5						
					2108.2	3.3239123

For

For the area of the triangle ACB.

2d,	73	78	78	78	78	1.8920946
	46	73	46	37	41	1.6127839
	37	—	—	—	32	1.5051500
		5	32	41	5	0.6989700
	2)156					2)5.7089985
	78				715.3	2.8544992

3d, Sq^r. feet.
triangle ACD = 2108.2
ACB = 715.3

Sum = 2823.5 log. 3.4507878 (÷ 2)

Anſ. 53.136 F. 1.7253939

In the ſame manner, the ſide of a ſquare may be found, whereof the area is equal to the area of any given rectilineal figure.

The diameter of a circle being given, to find the ſide of a ſquare of the ſame area.

Multiply the logarithm of the given diameter by 2, and to the product add the logarithm of .7854, the ſum is the logarithm of the area of the circle; and its half is the logarithm of the ſide of the ſquare.

Exam. Let the diameter of a circle be 112.837 feet, it is required to find the ſide of a ſquare of equal area with the circle? *Anſ.* 99.9992 feet.

Diam. 112.837. its log. 2.0524516
Mult. 2
4.1049032
.7854 log. 9.8950909
2)3.9999941 log. area of the circle
1.9999970
Side of the ſquare 99.9992 F.

What

1. What is the cube root of 208? *Ans.* 5.92499.

208 its log.	2.3180633
The root 5.92499	$\frac{1}{3}$ 0.7726877

2. Required the cube root of 11976? *Ans.* 22.879.

11976 log.	4.0783117
The root 22.879	$\frac{1}{3}$ 1.3594372

3. What is the cube root of 312908547069? *Ans.* 6789.

To find the logarithm of this large number, take the four first figures on the left hand 3129, and find their logarithm; then take the difference between this logarithm and the next greater in the table (1387): Multiply the remaining figures of the given number 08547069 by 1387: strike off eight figures from the product; add the remaining figures to the logarithm formerly found, and the sum is the logarithm of 312908547069.

312900000000 log.	11.4954056	08547069
For 08547069 add	118	1387
312908547069 log.	11.4954174	59829483
		68376552
Cube root 6789	$\frac{1}{3}$ 3.8318058	25641207
		8547069
		118.54784703

4. Extract the cube root of $53529\frac{481}{1280} = \frac{68517601}{1280}$? *Ans.* 37.6875.

From the log. of	68517601 =	7.8358022
Subtr. the log. of	1280 =	3.1072100
		3)4.7285922
The cube root 37.6875		1.5761974

5. Extract

5. Extract the cube root, and the biquadrate root of $\frac{15}{768}$?

15 log.	11.1760913	4)38.2907301
768 log.	2.8853612	.3738 9.5726825
	3)28.2907301	biquadrate root.
Cube root .2693	9.4302433	

In this example. From the logarithm of the numerator 15, (10 being added to its index,) subtract the logarithm of the denominator 768; the remainder 8.2907301 is the logarithm of the given fraction: then prefix 2 to the index of this logarithm, and divide the whole 28.2907301 by 3, the quotient 9.4302433 is the logarithm of the cube root. And for the biquadrate root, prefix 3 to the index, which makes it 38.2907301 and divide the whole by 4, the quotient is the logarithm of the biquadrate root.

N. B. In extracting the roots of proper fractions by logarithms, prefix to the index of the logarithm a number less by 1 than the index of the root, and then divide by the index of the root.

6. Suppose there are three cubic vessels, their sides being 11.5 17.25 and 20.2 inches: It is required to find the side of a cubic vessel which shall contain as much as all the three?

11.5 log.	1.0606978	17.25 log.	1.2367891
	3		3
1520.874 =	3.1820934	5132.952	3.7103673

20.2 log.	1.3053514
	3
8242.409	3.9160542

1520.874

	1520.874		
	5132.952		
	8242.409		
Sum of the cubes	14896.235	log.	4.1730765
	Ans. 24.605		$\frac{1}{3}$ 1.3910255

Logarithms are of great use in the solution of problems which require tedious calculations, such as,

In the mensuration of surfaces.

When the four sides and the diagonal of any quadrilateral field are measured, to find its area; such as, in *Q.* 12, *p.* 143, here are two triangles, and in each of them the three sides are given to find its area; (the rule for performing this by logarithms, is given in *p.* 119;) and the sum of these areas is the area of the field.

EXAM. 1. What is the area of the quadrilateral field ABCD, its diagonal BD=3096 links, the side AB=1976, the side BC=1760 links, CD=2016 links and DA=2340 links? *Fig.* 58.

1. For the area of the triangle ABD.

2340	3706	3706	3706
1976	2340	1976	3096
3096			
	1366	1730	610
2)7412			
3706			

2. For the area of the triangle CBD.

3096	3436	3436	3436
2016	3096	2016	1760
1760			
	340	1420	1676
2)6872			
3436			

	Logarithms.		Logarithms.
3706	3.5689054	3436	3.5360532
1366	3.1354507	1676	3.2242740
1730	3.2380461	1420	3.1522883
610	2.7853298	340	2.5314789
	2)12.7277320		2)12.4440944
Sq. links.		Sq. links.	
2311351	6.3638660	1667428	6.2220472

1667428
———— *A. R. Fall.*
3978779 = 39 3 6 the anſwer.

1. To find the area of any regular polygon by logarithms; it ſide and the perpendicular falling from the center on it, being given. Multiply the length of the given ſide by the number of ſides, and take half of the product, and then, to the logarithm of half the ſum of the ſides, add the logarithm of the perpendicular, the ſum is the logarithm of the area of the polygon.

EXAM. What is the area of a heptagon, its ſide being 21 yards 2 feet, and the perpendicular falling from the center on that ſide = 67.486 feet?

Anſ. 1705 ſquare yards, 8 feet 8 inches.

	Yd.	*F.*
Side	21	2
	3	
	65 feet.	
	7	
Sum of the ſide	455	
half ſum	227.5	

Half

Half ſum of the ſides 227.5 log. 2.3569814
perpendicular 67.486 1.8292137

Square feet 15353.06 4.1861951
or 1705 yards 8 feet 8 inches.

2. The diameter of a circle being given to find its area by logarithms.

Multiply the logarithm of the diameter by 2, and to the product, add the logarithm of .7854, the ſum is the logarithm of the area.

Exam. Let the diameter of a circle be 1760 yards, what is its area? *Anſ.* 2432855 ſquare yards.

1760 log. 3.2455127
2

6.4910254
.7854 log. 9.8950909

6.3861163

Area ſqr. yds. 2432855.

Menſuration of Solids by Logarithms.

1. The ſide of a cube being given, to find its ſolid content, find the logarithm of the given ſide, and multiply it by 3, the product is the logarithm of the ſolid content.

Exam. 1. The ſide of a cube is 19 feet, what is its ſolid content? *Anſ.* 6859 cubic feet.

19 its log. 1.2787536
3

3.8362608

cubic feet 6859.

2. What

2. What is the ſolid content of a cube, its ſide being 9 feet 1 inch? *Anſ.* 749 cubic feet 757 inches.

F.	*I.*		
9	1 = 109 inches.	log.	2.0374265
			3
cubic inches 1295028.8			6.1122795
	Subtract 1728	log.	3.2375437
			2.8747358

cubic feet 749.438
or 749 feet 757 inches.

3. The length, breadth, and thickneſs of any ſolid body having equal rectangular baſes being given, to find its ſolid content by logarithms.

Reduce the given dimenſions to the ſame denomination; take their logarithms from the table and add them, the ſum is the logarithm of the ſolid content.

Exam. Let the length of a log of wood be 17 feet 6 inches, the breadth 2 feet 3 inches, and the thickneſs 1 foot 9 inches? What is its ſolid content?

Anſ. 68 cubic feet 1566 inches.

	Feet.	
Length	17.5	1.2430380
Breadth	2.25	0.3521825
Thickneſs	1.75	0.2430380

68.9062 feet = 1.8382585
or 68 cubic feet 1566 inches.

4. To find the ſolid content of a priſm, its baſes and length being given, by logarithms.

Calculate the logarithm of the area of one of its baſes, by ſome of the former rules; to which add the logarithm of

of the given length, the ſum is the logarithm of the ſolid content: If it be in inches ſubtract the logarithm of 1728 from it, and the remainder is the logarithm of the ſolid content in feet.

EXAM. What is the ſolid content of a priſm, whoſe baſes are regular pentagons, one ſide of each pentagon being 2 feet $9\frac{1}{2}$ inches, and the perpendicular falling from the centre on that ſide 23.05 inches: alſo the length of the priſm 6 feet 8 inches?

$$\begin{array}{rl}
 & F. \quad I. \\
\text{Side} = & 2 \quad 9\tfrac{1}{2} \\
 & 12 \\
\hline
 & 33.5 \\
 & 5 \\
\hline
 & 2)167.5 \\
\hline
\tfrac{1}{2} \text{ ſum of the ſides} & 83.75
\end{array}$$

Half ſum, ſides of the baſe	83.75	1.9229848
Perpendicular	23.05	1.3626709
Log. area of the baſe in inches		3.2856557
Length in inches	80	+ 1.9030900
Log. of the ſolid cont. in inches		5.1887457
Subtr. log. of	1728	—3.2375437
Log. of ſolid content, feet,	89.372 =	1.9512020

or 89 cubic feet 643 inches.

5. To find the ſolid content of a pyramid, its baſe and height being given, by logarithms.

Calculate the ſolid content of a priſm of the ſame baſe and altitude, and one third part of it is ſolidity of the pyramid.

EXAM.

Exam. What is the solid content of a pyramid, the side of its square base being 9 feet 6 inches, and its height 25 feet 7 inches? *Ans.* 769.632 cubic feet.

$25\frac{7}{12}$	9.5 log.	0.9777236
12		2
307	Log. area of the base	1.9554472
$\frac{307}{12}$ log. 1.4079572	Log. of the height	1.4079572
	3)2308.896	3.3634044

Solidity 769.632 Feet.

Note. The rules for calculating the solid contents of the Cylinder and Cone, are the same with those for the prism and pyramid.

6. What is the solid content of a Cylinder, the diameter of the base being 5 feet 6 inches, and its length 21 feet 10 inches? *Ans.* 518.724 cubic feet.

F. *I.*	Diam^r. of the base 5.5 log.	0.7403627
21—10		2
12		1.4807254
$\frac{262}{12}$ = log. 1.3391201	.7854 log.	9.8950909
	length $\frac{262}{12}$ log.	1.3391201
	Solid content 518.724 F.	2.7149364

7. What is the solid content of a Cone, the diameter of its base being 36 inches, and its axis, or height, 48 inches?

Ans. 9 cubic feet 734 inches.

Diam.

Diam. 36 log.	1.5563025
	× 2
	3.1126050
7.854 log.	9.8950909
axis 48 log.	1.6812412
3)48858.15 inches,	4.6889371

Solid content 16286.05 inches,
or 9 cubic feet 734 inches.

8. The length of a round tree being given in feet, and its mean girt in inches, to find its solid content, by logarithms.

Multiply the logarithm of the mean girt by 2, and to the product add the logarithms of .0795775 and of the length, and from the sum subtract the logarithm of 144; the remainder is the logarithm of the solid content of the tree in cubic feet.

EXAM. What is the solid content of a round tree, 25 feet 9 inches long, and its mean girt $57\frac{1}{2}$ inches: what is its solid content? *Ans.* 47.0479 cubic feet.

Mean girt 57.5 log.	1.7596678
	× 2
	3.5193356
.0795775 log.	8.9007902
length 25.75	1.4107772
	3.8309030
144 log.	−2.1583625
47.0479 feet.	1.6725405

S f 2. The

2. The length of a round tree is 45 feet 6 inches, and its mean girt is $61\frac{1}{4}$ inches; what is its solid content?

Ans. 94.33 cubic feet.

Mean girt 61.25	log.	1.7871061
		× 2
		3.5742122
.0795775	log.	8.9007902
length 45.5		1.6580114
		4.1330138
144	log.	−2.1583625
94.33		1.9746513

9. To find the solid content of a tetrahedron by logarithms, the side of one triangular face being given.

Multiply the logarithm of the given side by 3, and to the product add half the logarithm of 2, and from the sum subtract the logarithm of 12, the remainder is the logarithm of the solid content.

EXAM. Let the side of one of the containing triangles be 12: required the solid content of the tetrahedron?

Ans. 203.646 cubic inches, or feet, according to the name of the given side.

12 its log.	1.0791812
	× 3
	3.2375436
$\frac{1}{2}$ log. of 2	+ 0.1505150
	3.3880586
log. of 12	−1.0791812
solid cont. 203.646	2.3088774

2. Let

2. Let the side of one of the triangles be 20 inches: required the solid content of the tetrahedron?

Ans. 942.8 cubic inches.

20 log.		1,3010300
		× 3
		3.9030900
½ log. of 2 =		0.1505150
		4.0536050
log. of 12	—	1.0791812
942.8		2.9744238

10. To find the solid content of an octahedron by logarithms; the side of one triangular face being given.

Multiply the logarithm of the given side by 3, and from the product subtract the logarithm of 3; add half the logarithm of 2 to the remainder; the sum is the logarithm of the solid content.

Exam. Let the side of one of the containing triangles be 12 inches; what is the solid content of the octahedron? *Ans.* 814.586 cubic inches.

side 12 log.		1.0791812
		× 3
		3.2375436
Subt^r. the log. 3		0.4771213
		2.7604223
½ log. of 2	+	0.1505150
Solid cont. 814.586.		2.9109373

11. To find the solid content of a dodecahedron by logarithms; the area of one of the containing penta-

 gons,

gons, and the diameter, or height of the ſolid being given.

To the logarithm of the area of the pentagonal face, add the logarithm of twice the diameter; the ſum is the logarithm of the ſolid content.

Exam. Let the area of one of the pentagons be 247.7487 ſquare inches, and the diameter of the ſolid 26.72424 inches, what is the ſolid content of the dodecahedron; *Anſ.* 13241.78 cubic inches.

given area 247.7487	log.	2.3940113
twice the diam^r. 53.44848		1.7279352
		4.1219465

Solid cont. 13241.78 inches, or 7 cubic feet, 1145 cubic inches.

12. To find the ſolid content of an icoſahedron by logarithms; the ſide of one of the containing triangles, and the diameter of the ſolid being given.

To twice the logarithm of the given ſide add half the logarithm of 3, and from the ſum ſubtract the logarithm of 4; the remainder is the area of one of the containing triangles; to which add the logarithm of the diameter and the logarithm of 20, and from the ſum ſubtract the logarithm of 6; the remainder is the logarithm of the ſolid content of the icoſahedron.

Exam. What is the ſolid content of an icoſahedron; its diameter being 18.138264 inches, and the ſide of each containing triangle 12 inches?

Anſ. 3769.96 cubic inches.

Given

Given side 12, its log.	1.0791812
	× 2
	2.1583624
half the log. of 3 +	0.2385606
	2.3969230
Subtr. the log. 4	0.6020600
	1.7948630
18.138264 log.	1.2585932
20. log.	1.3010300
	4.3544862
6 log. subtr.	0.7781513
Solid content 3769.96	3.5763349

Of Logarithm Sines and Tangents.

The tables of sines and tangents has the 90° of the quadrant disposed thus. At the head of the two first pages, there is 0 degrees, and in the left hand columns are the minutes of a degree. At the head of the next two pages, there is 1°, and so on to 44° on the head of the two last pages, at the foot of which is 45, from whence they proceed gradually backward unto the first two, where there is 89°; and on each page, the right hand columns contain the minutes of a degree increasing upwards.

13. To find the logarithm sine and tangent of any number of degrees and minutes below 45°. Look for the degrees on the head of the page, and the minutes in the left hand column; and, opposite to the minutes, you have their sines and tangents in their respective columns.

Sines

			Sines.	Tan.
EXAMPLES.	19°	0′	9.5126419	9.5369718
	19	24	9.5213488	9.5467346
	19	56	9.5326608	9.5594914

14. To find the logarithm ſine and tangent of any number of degrees and minutes above 45°.

Look for the degrees at the foot of the page, and the minutes in the right hand column, and oppoſite to the minutes you have the ſine and tangent, each in its proper column, marked at the foot with ſine or tangent.

			Sines.	Tan.
EXAMPLES.	53°	00′	9.9023486	10.1228856
	53	23	9.9045230	10.1289428
	53	49	9.9069446	10.1358197
	53	57	9.9076820	10.1379422

15. To find the ſine or tangent of a number of degrees, minutes, and ſeconds.

Find the ſine or tangent of the degrees as before; then find the difference between the ſine or tangent found and the next greater, and ſay, as 60 is to that difference, ſo is the number of ſeconds to the part to be added for the ſeconds.

EXAM. Require the ſine of 19° 24′ 36″?

The ſine of 19° 24′ is 9.5213488, which ſubtract from 9.5217074, the ſine of 19° 25′; the diff. is 3586.

Then, as 60″ : 3586 :: 36″ : 2151, which add to 9.5213488, and the ſum 9.5215639 is the ſine of 19° 24′ 36″. Proceed in the ſame manner when a tangent is wanted.

16. To find the co-ſine or co-tangent of any number of degrees and minutes.

Look

Look for the fine or tangent of the given number of degrees and minutes, and next it in the column of the fame name you have the co-fine or co-tangent required.

Thus the co-fine of 15° 24′ is 9.9841200, which is the fine of 74° 36′, and the co-tangent of 73° 36′ is 9.4400363, or the tangent of 15° 24′.

17. To find the number of degrees and minutes anfwering to a given logarithm fine.

Seek the given logarithm fine in the columns of fines; and, if you find it, or the next lefs in the firft column on the left hand, you have the degrees on the head of the page, and the minutes on the left, counting downwards. But, if you find it in the fecond column, you have the degrees at the foot of the page, and the minutes on the right hand, counting upwards. Proceed in the fame manner with tangents.

EXAMPLES.

Sines.	Tangents.
9.6843280 = 28° 54′	9.8764564 = 36° 57′
9.9532976 = 63 54	10.1097654 = 52 9

18. When the given logarithm fine is not found exactly in the table, and it is required to find feconds, fubtract the logarithm fine found in the table from the given one, alfo fubtract the fame from the next greater in the table, and fay,

As the difference of the two tabular fines is to 60″, fo is the difference between the given one, and the next lefs than it, found in the table, to the feconds required.

EXAM-

EXAM. Required the degrees, minutes, and seconds answering to the log. sine 9·9533476

Next less in the tables is 63° 54′ 9·9532897

Difference 579

Next greater	9·9533516
Subtract	9·9532897
	619

As 619 : 60″ :: 579 : 56″

Therefore the answer is 63° 54′ 56″.

19. To find the natural sine or tangent of any arc by these tables.

Find the logarithm, sine, or tangent of the given arc in the table; cancel its index, and find the number answering to it in the table of logarithms to seven figures; it is the answer.

20. To find the log. secant of any arc. Because the radius is a mean proportional between the co-sine of an arc and its secant; therefore, subtract the co-sine of the given arc from 20,0000000, the remainder is the logarithm secant.

21. To find the logarithm versed sine of any arc. Multiply the logarithm sine of half the arc by 2, add .3010300 to the product, and subtract the radius from the sum; the remainder is the logarithm versed sine of the arc.

THE END.

A

TABLE

OF THE

LOGARITHMS

OF ALL NUMBERS,

From 1 to 10,000.

N°	Log.	N°	Log.	N°	Log.	N°	Log.
0		22	3424227	44	6434527	66	8195439
1	0000000	23	3617278	45	6532125	67	8260748
2	3010300	24	3802112	46	6627578	68	8325089
3	4771213	25	3979400	47	6720979	69	8388491
4	6020600	26	4149733	48	6812412	70	8450980
5	6989700	27	4313638	49	6901961	71	8512583
6	7781513	28	4471580	50	6989700	72	8573325
7	8450980	29	4623980	51	7075702	73	8633229
8	9030900	30	4771213	52	7160033	74	8692317
9	9542425	31	4913617	53	7242759	75	8750613
10	0000000	32	5051500	54	7323938	76	8808136
11	0413927	33	5185139	55	7403627	77	8864907
12	0791812	34	5314789	56	7481880	78	8920946
13	1139434	35	5440680	57	7558749	79	8976271
14	1461280	36	5563025	58	7634280	80	9030900
15	1760913	37	5682017	59	7708520	81	9084850
16	2041200	38	5797836	60	7781513	82	9138139
17	2304489	39	5910646	61	7853298	83	9190781
18	2552725	40	6020600	62	7923917	84	9242793
19	2787536	41	6127839	63	7993405	85	9294189
20	3010300	42	6232493	64	8061800	86	9344985
21	3222193	43	6334685	65	8129134	87	9395193

N°	Log.	N°	Log.	N°	Log.	N°	Log.
88	9444827	126	1003705	164	2148438	202	3053514
89	9493900	127	1038037	165	2174839	203	3074960
90	9542425	128	1072100	166	2201081	204	3096302
91	9590414	129	1105897	167	2227165	205	3117539
92	9637878	130	1139434	168	2253093	206	3138672
93	9684829	131	1172713	169	2278867	207	3159703
94	9731279	132	1205739	170	2304489	208	3180633
95	9777236	133	1238516	171	2329961	209	3201463
96	9822712	134	1271048	172	2355284	210	3222193
97	9867717	135	1303338	173	2380461	211	3242825
98	9912261	136	1335389	174	2405492	222	3263359
99	9956352	137	1367206	175	2430380	213	3283796
100	0000000	138	1398791	176	2455127	214	3304138
101	0043214	139	1430148	177	2479733	215	3324385
102	0086002	140	1461280	178	2504200	216	3344538
103	0128372	141	1492191	179	2528530	217	3364597
104	0170333	142	1522883	180	2552725	218	3384565
105	0211893	143	1553360	181	2576786	219	3404441
106	0253059	144	1583625	182	2600714	220	3424227
107	0293838	145	1613680	183	2624511	221	3443923
108	0334238	146	1643529	184	2648178	222	3463530
109	0374265	147	1673173	185	2671717	223	3483049
110	0413927	148	1702617	186	2695129	224	3502480
111	0453230	149	1731863	187	2718416	225	3521825
112	0492180	150	1760913	188	2741578	226	3541084
113	0530784	151	1789769	189	2764618	227	3560259
114	0569049	151	1818436	190	2787536	228	3579348
115	0606978	153	1846914	191	2810334	229	3598355
116	0644580	154	1875207	192	2833012	230	3617278
117	0681859	155	1903317	193	2855573	231	3636120
118	0718820	156	1931246	194	2878017	232	3654880
119	0755470	157	1958997	195	2900346	233	3673559
120	0791812	158	1986571	196	2922561	234	3692159
121	0827854	159	2013971	197	2944662	235	3710679
122	0863598	160	2041200	198	2966652	236	3729120
123	0899051	161	2068259	199	2988531	237	3747483
124	0934217	162	2095150	200	3010300	238	3765770
125	0969100	163	2121876	201	3031961	239	3783979

N°	Log.	N°	Log.	N°	Log.	N°	Log.
240	3802112	278	4440448	316	4996871	354	5490033
241	3820170	279	4456042	317	5010593	355	5502284
242	3838154	280	4471580	318	5024271	356	5514500
243	3856063	281	4487063	319	5037907	357	5526682
244	3873898	282	4502491	320	5051500	358	5538830
245	3891661	283	4517864	321	5065050	359	5550944
246	3909351	284	4533183	322	5078559	360	5563025
247	3926970	285	4548449	323	5092025	361	5575072
248	3944517	286	4563660	324	5105450	362	5587086
249	3961993	287	4578819	325	5118834	363	5599066
250	3979400	288	4593925	326	5132176	364	5611014
251	3996737	289	4608978	327	5145478	365	5622929
252	4014005	290	4623980	328	5158738	366	5634811
253	4031205	291	4638930	329	5171959	367	5646661
254	4048337	292	4653829	330	5185139	368	5658478
255	4065402	293	4668676	331	5198280	369	5670264
256	4082400	294	4683473	332	5211381	370	5682017
257	4099331	295	4698220	333	5224442	371	5693739
258	4116197	296	4712917	334	5237465	372	5705429
259	4132998	297	4727564	335	5250448	373	5717088
260	4149733	298	4742163	336	5263393	374	5728716
261	4166405	299	4756712	337	5276299	375	5740313
262	4183013	300	4771213	338	5289167	376	5751878
263	4199557	301	4785665	339	5301997	377	5763414
264	4216039	302	4800069	340	5314789	378	5774918
265	4232459	303	4814426	341	5327544	379	5786392
266	4248816	304	4828736	342	5340261	380	5797836
267	4265113	305	4842998	343	5352941	381	5809250
268	4281348	306	4857214	344	5365584	382	5820634
269	4297523	307	4871384	345	5378191	383	5831988
270	4313638	308	488550[illegible]	346	5390761	384	5843312
271	4329693	309	4899585	347	5403295	385	5854607
272	4345689	310	4913617	348	5415792	386	5865873
273	4361626	311	4927604	349	5428254	387	5877110
274	4377506	312	4941546	350	5440680	388	5888317
275	4393327	313	4955443	351	5453071	389	5899496
276	4409091	314	4969296	352	5465427	390	5910646
277	4424798	315	4983106	353	5477747	391	5921768

N°	Log.	N°	Log.	N°	Log.	N°	Log.
392	5932861	430	6334685	468	6702459	506	7041505
393	5943926	431	6344773	469	6711728	507	7050080
394	5954962	432	6354837	470	6720979	508	7058637
395	5965971	433	6364879	471	6730209	509	7067178
396	5976952	434	6374897	472	6739420	510	7075702
397	5987905	435	6384893	473	6748611	511	7084209
398	5998831	436	6394865	474	6757783	512	7092700
399	6009729	437	6404814	475	6766936	513	7101174
400	6020600	438	6414741	476	6776070	514	7109631
401	6031444	439	6424645	477	6785184	515	7118072
402	6042261	440	6434527	478	6794279	516	7126497
403	6053050	441	6444386	479	6803355	517	7134905
404	6063814	442	6454223	480	6812412	518	7143298
405	6074550	443	6464037	481	6821451	519	7151674
406	6085260	444	6473830	482	6830470	520	7160033
407	6095944	445	6483600	483	6839471	521	7168377
408	6106602	446	6493349	484	6848454	522	7176705
409	6117233	447	6503075	485	6857417	523	7185017
410	6127839	448	6512780	486	6866363	524	7193313
411	6138418	449	6522463	487	6875290	525	7201593
412	6148972	450	6532125	488	6884198	526	7209857
413	6159501	451	6541765	489	6893089	527	7218106
414	6170003	452	6551384	490	6901961	528	7226339
415	6180481	453	6560982	491	6910815	529	7234557
416	6190933	454	6570559	492	6919651	530	7242759
417	6201361	455	6580114	493	6928469	531	7250945
418	6211763	456	6589648	494	6937269	532	7259116
419	6222140	457	6599162	495	6946052	533	7267272
420	6232493	458	6608655	496	6954814	534	7275413
421	6242821	459	6618127	497	6963564	535	7283538
422	6253125	460	6627578	498	6972293	536	7291648
423	6263404	461	6637009	499	6981005	537	7299743
424	6273659	462	6646420	500	6989700	538	7307823
425	6283889	463	6655810	501	6998377	539	7315888
426	6294096	464	6665180	502	7007037	540	7323983
427	6304279	465	6674530	503	7015680	541	7331973
428	6314438	466	6683859	504	7024305	542	7339993
429	6324573	467	6693169	505	7032914	543	7347998

N°	Log.	N°	Log.	N°	Log.	N°	Log.
544	7355989	582	7649230	620	7923917	658	8182259
545	7363965	583	7656685	621	7930916	659	8188854
546	7371926	584	7664128	622	7937904	660	8195439
547	7379873	585	7671559	623	7944880	661	8202015
548	7387806	586	7678976	624	7951846	662	8208589
549	7395723	587	7686381	625	7958800	663	8215135
550	7403627	588	7693773	626	7965743	664	8221681
551	7411516	589	7701153	627	7972675	665	8228216
552	7419391	590	7708520	628	7979596	666	8234742
553	7427251	591	7715875	629	7986506	667	8241258
554	7435098	592	7723217	630	7993405	668	8247765
555	7442930	593	7730547	631	8000294	669	8254261
556	7450748	594	7737864	632	8007171	670	8260748
557	7458552	595	7745170	633	8014037	671	8267225
558	7466342	596	7752463	634	8020893	672	8273693
559	7474118	597	7759743	635	8027737	673	8280151
560	7481880	598	7767012	636	8034571	674	8286599
561	7489629	599	7774268	637	8041394	675	8293038
562	7497363	600	7781513	638	8048207	676	8299467
563	7505084	601	7788745	639	8055009	677	8305887
564	7512791	602	7795965	640	8061800	678	8312297
565	7520484	603	7803173	641	8068580	679	8318698
566	7528164	604	7810369	642	8075350	680	8325089
567	7535831	605	7817554	643	8082110	681	8331471
568	7543483	606	7824726	644	8088859	682	8337844
569	7551123	607	7831887	645	8095597	683	8344207
570	7558749	608	7839036	646	8102325	684	8350561
571	7566361	609	7846173	647	8109043	685	8356906
572	7573960	610	7853298	648	8115750	686	8363241
573	7581546	611	7860412	649	8122447	687	8369567
574	7589119	612	7867514	650	8129134	688	8375884
575	7596678	613	7874605	651	8135810	689	8382192
576	7604225	614	7881684	652	8142476	690	8388491
577	7611758	615	7888751	653	8149132	691	8394780
578	7619278	616	7895807	654	8155777	692	8401061
579	7626786	617	7902852	655	8162413	693	8407332
580	7634280	618	7909885	656	8169038	694	8413595
581	7641761	619	7916906	657	8175654	695	8419848

N°	Log.	N°	Log.	N°	Log.	N°	Log.
696	8426092	734	8656961	772	8876173	810	9084850
697	8432328	735	8662873	773	8881795	811	9090209
698	8438554	736	8668778	774	8887410	812	9095560
699	8444772	737	8674675	775	8893017	813	9100905
700	8450980	738	8680564	776	8898617	814	9106244
701	8457180	739	8686444	777	8904210	815	9111576
702	8463371	740	8692317	778	8909796	816	9116901
703	8469553	741	8698182	779	8915375	817	9122221
704	8475727	742	8704039	780	8920946	818	9127533
705	8481891	743	8709888	781	8926510	819	9132839
706	8488047	744	8715729	782	8932068	820	9138139
707	8494194	745	8721563	783	8937618	821	9143432
708	8500333	746	8727388	784	8943161	822	9148718
709	8506462	747	8733206	785	8948696	823	9153998
710	8512583	748	8739016	786	8954225	824	9159272
711	8518696	749	8744818	787	8959747	825	9164539
712	8524800	750	8750613	788	8965262	826	9169800
713	8530895	751	8756399	789	8970770	827	9175055
714	8536982	752	8762178	790	8976271	828	9180303
715	8543060	753	8767950	791	8981765	829	9185545
716	8549130	754	8773713	792	8987252	830	9190781
717	8555192	755	8779470	793	8992732	831	9196010
718	8561244	756	8785218	794	8998205	832	9201233
719	8567289	757	8790959	795	9003671	833	9206450
720	8573325	758	8796692	796	9009131	834	9211661
721	8579353	759	8802418	797	9014583	835	9216865
722	8585372	760	8808136	798	9020029	836	9222063
723	8591383	761	8813847	799	9025468	837	9227255
724	8597386	762	8819550	800	9030900	838	9232440
725	8603380	763	8825245	801	9036325	839	9237620
726	8609366	764	8830934	802	9041744	840	9242793
727	8615344	765	8836614	803	9047155	841	9247960
728	8621314	766	8842288	804	9052560	842	9253121
729	8627275	767	8847954	805	9057959	843	9258276
730	8633229	768	8853612	806	9063350	844	9263424
731	8639174	769	8859263	807	9068735	845	9268567
732	8645111	770	8864907	808	9074114	846	9273704
733	8651040	771	8870544	809	9079485	847	9278834

N°	Log.	N°	Log.	N°	Log.	N°	Log.
848	9283959	886	7474337	924	9656720	962	9831751
849	9289077	887	9479236	925	9661417	963	9836263
850	9294189	888	9484130	926	9666110	964	9840770
851	9299296	889	9489018	927	9670797	965	9845273
852	9304396	890	9493900	928	9675480	966	9849771
853	9309490	891	9498777	929	9680157	967	9854265
854	9314579	892	9503649	930	9684829	968	9858754
855	9319661	893	9508515	931	9689497	969	9863238
856	9324738	894	9513375	932	9694159	970	9867717
857	9329808	895	9518230	933	9698816	971	9872192
858	9334873	896	9523080	934	9703469	972	9876663
859	9339932	897	9527924	935	9708116	973	9881128
860	9344985	898	9532763	936	9712758	974	9885590
861	9350032	899	9537597	937	9717396	975	9890046
862	9355073	900	9542425	938	9722028	976	9894498
863	9360108	901	9547248	939	9726656	977	9898946
864	9365137	902	9552065	940	9731279	978	9903389
865	9370161	903	9556878	941	9735896	979	9907827
866	9375179	904	9561684	942	9740509	980	9912261
867	9380191	905	9566486	943	9745117	981	9916690
868	9385197	906	9571282	944	9749720	982	9921115
869	9390198	907	9576073	945	9754318	983	9925535
870	9395193	908	9580858	946	9758911	984	9929951
871	9400182	909	9585639	947	9763500	985	9934362
872	9405165	910	9590414	948	9768083	986	9938769
873	9410142	911	9595184	949	9772662	987	9943172
874	9415114	912	9599948	950	9777236	988	9947569
875	9420081	913	9604708	951	9781805	989	9951963
876	9425041	914	9609462	952	9786369	990	9956352
877	9429996	915	9614211	853	9790929	991	9960737
878	9434945	916	9618955	954	9795484	992	9965117
879	9439889	917	9623693	955	9800034	993	9969492
880	9444827	918	9628427	956	9804579	994	9973864
881	9449759	919	9633155	957	9809119	995	9978231
882	9454686	920	9637878	958	9813655	996	9982593
883	9459607	921	9642596	959	9818186	997	9986952
884	9464523	922	9647309	960	9822712	998	9991305
885	9469433	923	9652017	961	9827234	999	9995655

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101	43214	47512	51805	56094	60380
102	86002	90257	94509	98756	03000
103	0128372	32587	36797	41003	45205
104	70333	74507	78677	82843	87005
105	0211893	16027	20157	24284	28406
106	53059	57154	61245	65333	69416
107	93838	97895	01948	05997	10043
108	0334238	38257	42273	46285	50293
109	74265	78248	82226	86202	90173
110	0413927	17873	21816	25755	29691
111	53230	57141	61048	64952	68852
112	92180	96056	99929	03798	07663
113	0530784	34626	38464	42299	46131
114	69049	72856	76661	80462	84260
115	0606978	10753	14525	18293	22058
116	44580	48322	52061	55797	59530
117	81859	85569	89276	92980	96681
118	0718820	22499	26175	29847	33517
119	55470	59118	62763	66404	70043
120	91812	95430	99045	02656	06265
121	0827854	41441	35026	38608	42187
122	63598	67157	70712	74265	77814
123	99051	02581	06107	09631	13152
124	0934217	37718	41216	44711	48204
125	69100	72573	76043	79511	82975
126	1003705	07151	10594	14034	17471
127	38037	41456	44871	48284	51694
128	72100	75491	78880	82267	85650
129	1105897	09262	12625	15985	19343
130	39434	42773	46110	49444	52776
131	72713	76027	79338	82647	85954
132	1205739	09028	12315	15598	18880
133	38516	41781	45042	48301	51558
134	71048	74288	77525	80760	83993
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136	35389	38581	41771	44959	48144
137	67206	70375	73541	76705	79867

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101	64660	68937	73210	77478	81742
102	0107239	11474	15704	19931	24154
103	49403	53598	57788	61974	66155
104	91163	95317	99467	03613	07755
105	0232525	36639	40750	44857	48960
106	73496	77572	81644	85713	89777
107	0314085	18123	22157	26188	30214
108	54297	58298	62295	66289	70279
109	94141	98106	02066	06023	09977
110	0433623	37551	41476	45398	49315
111	72749	76642	80532	84418	88301
112	0511525	15384	19239	23091	26939
113	49959	53783	57605	61423	65237
114	88055	91846	95634	99419	03200
115	0625820	29578	33334	37086	40834
116	63259	66986	70709	74428	78145
117	0700379	04073	07765	11453	15138
118	37184	40847	44507	48164	51819
119	73679	77312	80942	84568	88192
120	0809870	13473	17073	20669	24263
121	45763	49336	52906	56473	60037
122	81361	84905	88446	91984	95519
123	0916670	20185	23697	27206	30713
124	51694	55180	58665	62146	65624
125	86437	89896	93353	96806	00257
126	1020905	24337	27766	31193	34616
127	55102	58507	61909	65309	68705
128	89031	92410	95785	99159	02529
129	1122698	26050	29400	32747	36092
130	56105	59432	62756	66077	69396
131	89258	92559	95858	99154	02448
132	1222159	25435	28709	31981	35250
133	54813	58065	61314	64561	67806
134	87223	90451	93676	96899	00119
135	1319393	22597	25798	28998	32195
136	51327	54507	57685	60861	64034
137	83027	86184	89339	92492	95643

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138	1398791	01937	05080	08222	11361
139	1430148	33271	36392	39511	42628
140	61280	64381	67480	70577	73671
141	92191	95270	98347	01422	04494
142	1522883	25941	28996	32049	35100
143	53360	56396	59430	62462	65492
144	83625	86640	89653	92663	95672
145	1613680	16674	19666	22656	25644
146	43529	46502	49474	52443	55411
147	73173	76127	79078	82027	84975
148	1702617	05551	08482	11412	14339
149	31863	34776	37688	40598	43506
150	60913	63807	66699	69590	72478
151	89769	92645	95518	98389	01259
152	1818436	21292	24147	26999	29850
153	46914	49752	52588	55422	58254
154	75207	78026	80844	83659	86473
155	1903317	06118	08917	11715	14510
156	31246	34029	36810	39590	42367
157	58997	61762	64525	67287	70047
158	86571	89319	92065	94804	97552
159	2013971	16702	19431	22158	24883
160	41200	43913	46625	49335	52044
161	68259	70955	73650	76344	79035
162	95150	97830	00508	03185	05860
163	2121876	24540	27202	29862	32521
164	48438	51086	53732	56376	59018
165	74839	77471	80100	82729	85355
166	2201081	03696	06310	08922	11533
167	27165	29764	32363	34959	37555
168	53093	55677	58260	60841	63421
169	78867	81436	84004	86570	89134
170	2304489	07043	09596	12146	14696
171	29961	32500	35038	37574	40108
172	55284	57809	60331	62853	65373
173	80461	82971	85479	87986	90491
174	2405492	07988	10482	12974	15465
175	30380	32861	35341	37819	40296

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138	1414498	17632	20765	23895	27022
139	45742	48854	51964	55072	58177
140	76763	79853	82941	86027	89110
141	1507564	10633	13699	16762	19824
142	38149	41195	44240	47282	50322
143	68519	71544	74568	77589	80608
144	98678	01683	04685	07686	10684
145	1628630	31614	34596	37575	40553
146	58376	61340	64301	67261	70218
147	87920	90864	93805	96744	99682
148	1717265	20188	23110	26029	28947
149	46412	49316	52218	55118	58016
150	75365	78250	81132	84013	86892
151	1804126	06992	09856	12718	15578
152	32698	35545	38390	41234	44075
153	61084	63912	66739	69563	72386
154	89285	92095	94993	97710	00514
155	1917304	20096	22886	25675	28461
156	45143	47918	50690	53461	56229
157	72806	75562	78317	81070	83821
158	2000293	03032	05769	08505	11239
159	27607	30329	33049	35768	38485
160	54750	57455	60159	62860	65560
161	81725	84414	87100	89785	92468
162	2108534	11205	13876	16544	19211
163	35178	37833	40487	43139	45790
164	61659	64298	66936	69572	72207
165	87980	90603	93225	95845	98464
166	2214142	16750	19356	21960	24563
167	40148	42740	45331	47920	50507
168	65999	68576	71151	73724	76296
169	91697	94258	96818	99377	01934
170	2317244	19790	22335	24879	27421
171	42641	45173	47703	50232	52759
172	67891	70408	72923	75437	77950
173	92995	95497	97998	00498	02996
174	2417954	20442	22929	25414	27898
175	42771	45245	47718	50189	52658

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176	2455127	57594	60059	62523	64986
177	79733	82186	84637	87087	89536
178	2504200	06639	09077	11513	13949
179	28530	30956	33380	35803	38224
180	52725	55137	57548	59957	62365
181	76786	79185	81582	83978	86373
182	2600714	03099	05484	07867	10248
183	24511	26883	29255	31625	33993
184	48178	50538	52896	55253	57609
185	71717	74064	76410	78754	81097
186	95129	97464	99797	02129	04459
187	2718416	20738	23058	25378	27696
188	41578	43888	46196	48503	50809
189	64618	66915	69211	71506	73800
190	87536	89821	92105	94388	96669
191	2810334	12607	14879	17150	19419
192	33012	35274	37534	39793	42051
193	55573	57823	60071	62319	64565
194	78017	80255	82492	84728	86963
195	2900346	02573	04798	07022	09246
196	22561	24776	26990	29203	31415
197	44662	46866	49069	51271	53471
198	66652	68845	71037	73227	75417
199	88531	90713	92893	95073	97252
200	3010300	12471	14641	16809	18977
201	31961	34121	36280	38438	40595
202	53514	55663	57812	59959	62105
203	74960	77099	79237	81374	83509
204	96302	98430	00557	02684	04809
205	3117539	19657	21774	23889	26004
206	38672	40780	42887	44992	47097
207	59703	61801	63898	65993	68088
208	80633	82721	84807	86893	88977
209	3201463	03540	05617	07692	09767
210	22193	24261	26327	28393	30457
211	42825	44882	46939	48995	51050
212	63359	65407	67454	69500	71545
213	83796	85834	87872	89909	91944

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177	91984	94430	96874	99318	01759
178	2516382	18815	21246	23675	26103
179	40645	43063	45481	47897	50312
180	64772	67177	69582	71984	74386
181	88766	91158	93549	95939	98327
182	2612629	15008	17385	19762	22137
183	36361	38727	41092	43455	45817
184	59964	62317	64669	67020	69369
185	83439	85780	88119	90457	92794
186	2706788	09116	11443	13769	16093
187	30013	32328	34643	36956	39268
188	53114	55417	57719	60020	62320
189	76092	78383	80673	82962	85250
190	98950	01229	03507	05784	08059
191	2821688	23955	26221	28486	30750
192	44307	46563	48817	51070	53322
193	66810	69054	71296	73538	75778
194	89196	91428	93660	95890	98118
195	2911468	13689	15908	18127	20344
196	33626	35835	38044	40251	42457
197	55671	57869	60067	62263	64458
198	77605	79792	81979	84164	86348
199	99429	01605	03781	05955	08128
200	3021144	23309	25474	27637	29799
201	42751	44905	47059	49212	51363
202	64250	66394	68537	70680	72820
203	85644	87778	89910	92042	94172
204	3106933	09056	11178	13300	15420
205	3128118	30231	32343	34454	36563
206	49201	51303	53405	55505	57605
207	70181	72273	74365	76455	78545
208	91061	93143	95224	97305	99384
209	3211840	13913	15984	18055	20124
210	32521	34584	36645	38706	40766
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217	64597	66598	68598	70597	72595
218	84565	86557	88547	90537	92526
219	3404441	06424	08405	10386	12366
220	24227	26200	28173	30145	32116
221	43923	45887	47851	49814	51776
222	63530	65486	67441	69395	71348
223	83049	84996	86942	88887	90832
224	3502480	04419	06356	08293	10229
225	21825	23755	25684	27612	29539
226	41084	43006	44926	46846	48764
227	60259	62171	64083	65994	67905
228	79348	81253	83156	85059	86961
229	98355	00251	02146	04041	05934
230	3617278	19166	21053	22939	24825
231	36120	37999	39878	41756	43634
232	54880	56751	58622	60492	62361
233	73559	75423	77285	79147	81009
234	92159	94014	95869	97723	99576
235	3710679	12526	14373	16219	18065
236	29120	30960	32799	34637	36475
237	47483	49316	51147	52977	54807
238	65770	67594	69418	71240	73063
239	83979	85796	87612	89427	91241
240	3802112	03922	05730	07538	09345
241	20170	21972	23773	25573	27373
242	38154	39948	41741	43534	45326
243	56063	57850	59636	61421	63206
244	73898	75678	77457	79235	81012
245	91661	93433	95205	96975	98746
246	3900935	11116	12880	14644	16407
247	26970	28727	30485	32241	33997
248	44517	46268	48018	49767	51516
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250	79400	81137	82873	84608	86343
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217	74593	76589	78584	80579	82572
218	94514	96502	98488	00473	02458
219	3414345	16323	18301	20277	22252
220	34086	36055	38023	39991	41957
221	53737	55698	57657	59615	61573
222	73300	75252	77202	79152	81101
223	92775	94718	96660	98601	00541
224	3512163	14098	16031	17963	19895
225	31465	33391	35316	37239	39162
226	50682	52599	54515	56431	58345
227	69814	71723	73630	75537	77443
228	88862	90762	92662	94560	96458
229	3607827	09719	11610	13500	15390
230	26709	28593	30476	32358	34239
231	45510	47386	49260	51134	53007
232	64230	66097	67964	69830	71695
233	82869	84728	86587	88445	90302
234	3701428	03280	05131	06981	08830
235	19909	21753	23596	25438	27279
236	38311	40147	41983	43817	45651
237	56636	58464	60292	62119	63944
238	74884	76704	78524	80343	82161
239	93055	94868	96680	98492	00302
240	3811151	12956	14761	16565	18368
241	29171	30969	32767	34563	36359
242	47117	48908	50698	52487	54275
243	64990	66773	68555	70337	72118
244	82789	84565	86340	88114	89888
245	3900515	02284	04052	05819	07585
246	18169	19931	21691	23452	25211
247	35752	37506	39260	41013	42765
248	53264	55011	56758	58504	60249
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255	65402	67105	68807	70508	72209
256	82400	84096	85791	87486	89180
257	99351	01021	02710	04398	06085
258	4116197	17880	19562	21244	22925
259	32998	34674	36350	38025	39700
260	49733	51404	53073	54742	56410
261	66405	68069	69732	71394	73056
262	83013	84670	86327	87983	89638
263	99557	01208	02859	04509	06158
264	4216039	17684	19328	20972	22615
265	32459	34097	35735	37372	39009
266	48816	50449	52081	53712	55342
267	65113	66739	68365	69990	71614
268	81348	82968	84588	86207	87825
269	97523	99137	00751	02364	03976
270	4313638	15246	16853	18460	20067
271	29693	31295	32897	34498	36098
272	45689	47285	48881	50476	52071
273	61626	63217	64807	66396	67985
274	77506	79090	80675	82258	83841
275	93327	94906	96484	98062	99639
276	4409091	10664	12237	13809	15380
277	24798	26365	27932	29499	31065
278	40448	42010	43571	45132	46692
279	56042	57598	59154	60709	62264
280	71580	73131	74681	76231	77780
281	87063	88608	90153	91697	93241
282	4502491	04031	05570	07109	08647
283	17864	19399	20932	22466	23998
284	33183	34712	36241	37769	39296
285	48449	49972	51495	53018	54540
286	63660	65179	66696	68213	69730
287	78819	80332	81844	83356	84868
288	93925	95433	96940	98446	99953
289	4608978	10481	11983	13484	14985

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253	39780	41492	43205	44916	46627
254	56878	58584	60289	61994	63698
255	73909	75608	77307	79005	80703
256	90874	92567	94259	95950	97641
257	4107772	09459	11144	12829	14513
258	24605	26285	27964	29643	31321
259	41374	43047	44719	46391	48063
260	58077	59744	61410	63076	64741
261	74717	76377	78037	79696	81355
262	91293	92947	94601	96254	97906
263	4207806	09454	11101	12748	14394
264	24257	25898	27539	29180	30820
265	40645	42281	43916	45550	47183
266	56972	58601	60230	61858	63486
267	73238	74861	76484	78106	79727
268	89443	91060	92677	94293	95908
269	4305588	07199	08809	10419	12029
270	21672	23278	24883	26487	28090
271	37698	39298	40896	42495	44092
272	53665	55259	56851	58444	60035
273	69573	71161	72748	74334	75920
274	85423	87005	88587	90167	91747
275	4401216	02792	84368	05943	07517
276	16951	18522	20092	21661	23230
277	32630	34195	35759	37322	38885
278	48251	49811	51370	52928	54485
279	63818	65372	66925	68477	70029
280	79329	80877	82424	83971	85517
281	94784	96327	97868	99410	00951
282	4510185	11722	13258	14794	16329
283	25531	27062	28593	30124	31654
284	40823	42349	43875	45400	46924
285	56061	57582	59102	60622	62142
286	71246	72762	74277	75791	77305
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289	16486	17986	19485	20984	22482

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293	68676	70158	71640	73121	74601
294	83473	84950	86427	87903	89378
295	98220	99692	01164	02634	04105
296	4712917	14384	15851	17317	18782
297	27564	29027	30488	31949	33410
298	42163	43620	45076	46533	47988
299	56712	58164	59616	61067	62518
300	71213	72660	74107	75553	76999
301	85665	87108	88550	89991	91432
302	4800069	01507	02945	04381	05818
303	14426	15859	17292	18724	20156
304	28736	30164	31592	33020	34446
305	42998	44422	45845	47268	48690
306	57214	58633	60052	61470	62888
307	71384	72798	74212	75626	77839
308	85507	86917	88326	89735	91144
309	99585	00990	02395	03799	05203
310	4913617	15018	16418	17818	19217
311	27604	29000	30396	31791	33186
312	41546	42938	44329	45720	47110
313	55443	56831	58218	59604	60990
314	69296	70679	72062	73444	74825
315	83106	84484	85862	87240	88617
316	96871	98245	99619	00992	02365
317	5010593	11962	13332	14701	16069
318	24271	25637	27002	28366	29731
319	37907	39268	40629	41989	43349
320	51500	52857	54213	55569	56925
321	65050	66403	67755	69107	70459
322	78559	79907	81255	82603	83950
323	92025	93370	94714	96057	97400
324	5105450	06790	08130	09469	10808
325	18834	20170	21505	22841	24175
326	32176	33508	34840	36171	37502
327	45478	46805	48133	49460	50787

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293	76081	77561	79039	80518	81996
294	90853	92327	93801	95275	96748
295	4705575	07044	08513	09982	11450
296	20247	21711	23175	24639	26102
297	34870	36329	37788	39247	40705
298	49443	50898	52352	53806	55259
299	63968	65418	66867	68316	69765
300	78445	79890	81334	82778	84222
301	92873	94313	95753	97192	98631
302	4807254	08689	10124	11559	12993
303	21587	23018	24448	25878	27307
304	35873	37299	38725	40150	41574
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306	64305	65722	67138	68554	69969
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309	4906607	08010	09412	10814	12216
310	20616	22015	23413	24810	26207
311	34581	35974	37368	38761	40154
312	48500	49890	51279	52667	54056
313	62375	63761	65145	66529	67913
314	76206	77587	78967	80347	81727
315	89994	91370	92746	94121	95496
216	5003737	05109	06481	07852	09222
317	17437	18805	20172	21539	22905
318	31094	32458	33821	35183	36545
319	44709	46068	47426	48785	50142
320	58280	59635	60990	62344	63697
321	71810	73160	74511	75860	77210
322	85297	86644	87990	89335	90680
323	98743	00085	01427	02768	04109
324	5112147	13485	14823	16160	17497
325	25510	26844	28178	29511	30844
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327	52113	53439	54764	56089	57414

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331	98280	99592	00903	02214	03525
332	5211381	12689	13996	15303	16610
333	24442	25746	27050	28353	29656
334	37465	38765	40064	41364	42663
335	50448	51744	53040	54336	55631
336	63393	64685	65977	67269	68560
337	76299	77588	78876	80163	81451
338	89167	90452	91736	93020	94304
339	5301997	03278	04558	05839	07118
340	14789	16066	17343	18619	19896
341	27544	28817	30090	31363	32635
342	40261	41531	42800	44069	45338
343	52941	54207	55473	56738	58003
344	65584	66847	68109	69370	70631
345	78191	79450	80708	81966	83223
346	90761	92016	93271	94525	95779
347	5403295	04546	05797	07048	08298
348	15792	17040	18288	19535	20781
349	28254	29498	30742	31986	33229
350	40680	41921	43161	44401	45641
351	53071	54308	55545	56781	58018
352	65427	66660	67894	69126	70359
353	77747	78977	80207	81436	82665
354	90033	91259	92486	93712	94937
355	5502284	03507	04730	05952	07174
356	14500	15720	16939	18158	19377
357	26682	27899	29115	30330	31545
358	38830	40043	41256	42468	43680
359	50944	52154	53363	54572	55781
360	63025	64231	65437	66643	67848
361	75072	76275	77477	78680	79881
362	87086	88285	89484	90683	91882
363	99066	00262	01458	02654	03849
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331	5204835	06145	07455	08764	10073
332	17916	19222	20528	21833	23138
333	30958	32260	33562	34863	36164
334	43961	45259	46557	47854	49151
335	56925	58220	59513	60807	62100
336	69851	71141	72431	73721	75010
337	82738	84024	85311	86596	87882
338	95587	96870	98152	99434	00716
339	5308398	09677	10955	12234	13512
340	21171	22446	23721	24996	26270
341	33907	35179	36450	37721	38991
342	46606	47874	49141	50408	51675
343	59267	60532	61795	63059	64322
344	71892	73153	74413	75673	76932
345	84481	85737	86994	88250	89506
346	97032	98286	99538	00791	02043
347	5409548	10798	12047	13296	14544
348	22028	23274	24519	25765	27010
349	34472	35714	36956	38198	39439
350	46880	48119	49358	50596	51834
351	59253	60489	61724	62958	64193
352	71591	72823	74055	75286	76517
353	83894	85123	86351	87578	88806
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355	5508396	09618	10839	12059	13280
356	20595	21813	23031	24248	25465
357	32760	33975	35189	36403	37617
358	44892	46103	47314	48524	49735
359	56989	58197	59404	60612	61818
360	69053	70257	71461	72665	73869
361	81083	82284	83485	84686	85886
362	93080	94278	95476	96673	97870
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369	70264	71440	72617	73793	74969
370	82017	83191	84364	85537	86710
371	93739	94910	96080	97249	98419
372	5705429	06597	07764	08930	10097
373	17088	18252	19416	20580	21743
374	28716	29877	31038	32198	33358
375	40313	41471	42628	43786	44943
376	51878	53033	54188	55342	56496
377	63414	64565	65717	66868	68019
378	74918	76067	77215	78363	79511
379	86392	87538	88683	89828	90973
380	97836	98979	00121	01263	02405
381	5809250	10389	11529	12668	13807
382	20634	21770	22907	24043	25179
383	31988	33122	34255	35388	36521
384	43312	44443	45574	46704	47834
385	54607	55735	56863	57990	59117
386	65873	66998	68123	69247	70371
387	77110	78232	79353	80475	81596
388	88317	89436	90555	91674	92792
389	99496	00612	01728	02844	03959
390	5910646	11760	12873	13986	15098
391	21768	22878	23988	25098	26208
392	32861	33968	35076	36183	37290
393	43926	45030	46135	47239	48344
394	54962	56064	57166	58268	59369
395	65971	67070	68169	69268	70367
396	76952	78048	79145	80241	81336
397	87905	88999	90092	91186	92279
398	98831	99922	01013	02103	03193
399	6009729	10817	11905	12993	14081
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369	76144	77320	78495	79669	80843
370	87882	89054	90226	91397	92568
371	99588	00757	01926	03094	04262
372	5711263	12429	13594	14759	15924
373	22906	24069	25231	26393	27555
374	34518	35678	36837	37996	39154
375	46099	47256	48412	49568	50723
376	57650	58803	59956	61109	62261
377	69170	70320	71470	72620	73769
378	80659	81806	82953	84100	85246
379	92118	93262	94406	95550	96693
380	5803547	04688	05829	06969	08110
381	14945	16084	17222	18359	19497
382	26314	27450	28585	29719	30854
383	37654	38786	39918	41050	42181
384	48963	50093	51222	52351	53479
385	60244	61370	62496	63622	64748
386	71495	72618	73742	74865	75987
387	82717	83838	84958	86078	87198
388	93910	95028	96145	97263	98379
389	5905075	06189	07304	08418	09532
390	16210	17322	18434	19546	20657
391	27318	28427	29536	30644	31753
392	38397	39503	40609	41715	42820
393	49447	50551	51654	52757	53860
394	60470	61571	62671	63771	64871
395	71465	72563	73661	74758	75855
396	82432	83527	84622	85717	86811
397	93371	94464	95556	96648	97739
398	6004283	05373	06462	07551	08640
399	15168	16255	17341	18428	19514
400	26025	27109	28193	29277	30361
401	36855	37937	39018	40099	41180
402	47659	48738	49816	50895	51973
403	58435	59512	60587	61663	62739

N°	0	1	2	3	4
404	6063814	64889	65963	67037	68111
405	74550	75622	76694	77766	78837
406	85260	86330	87399	88468	89537
407	95944	97011	98078	99144	00210
408	6106602	07666	08730	09794	10857
409	17233	18295	19356	20417	21478
410	27839	28898	29957	31015	32074
411	38418	39475	40531	41587	42643
412	48972	50026	51080	52133	53187
413	59501	60552	61603	62654	63705
414	70003	71052	72101	73149	74197
415	80481	81527	82573	83619	84665
416	90933	91977	93021	94064	95107
417	6201361	02402	03443	04484	05524
418	11763	12802	13840	14879	15917
419	22140	23177	24213	25249	26284
420	32493	33527	34560	35594	36627
421	42821	43852	44884	45915	46945
422	53125	54154	55182	56211	57239
423	63404	64430	65457	66483	67509
424	73659	74683	75707	76730	77754
425	83889	84911	85933	86954	87975
426	94096	95115	96134	97153	98172
427	6304279	05296	06312	07329	08345
428	14438	15452	16467	17481	18495
429	24573	25585	26597	27609	28620
430	34685	35694	36704	37713	38723
431	44773	45780	46788	47795	48801
432	54837	55843	56848	57852	58857
433	64879	65882	66884	67887	68889
434	74897	75898	76898	77898	78898

N°	5	6	7	8	9
404	6069185	70259	71332	72405	73478
405	79909	80979	82050	83120	84191
406	90605	91674	92742	93809	94877
407	6101276	02342	03407	04472	05537
408	11921	12984	14046	15109	16171
409	22539	23599	24660	25720	26779
410	33132	34189	35247	36304	37361
411	43698	44754	45809	46863	47918
412	54240	55292	56345	57397	58449
413	64755	65805	66855	67905	68954
414	75245	76293	77340	78387	79434
415	6185710	86755	87800	88845	89889
416	96150	97193	98235	99277	00319
417	6206565	07605	08645	09684	10724
418	16955	17992	19030	20067	21104
419	27320	28355	29390	30424	31459
420	37660	38693	39725	40757	41789
421	47976	49006	50036	51066	52095
422	58267	59295	60322	61350	62377
423	68534	69560	70585	71610	72634
424	78777	79800	80823	81845	82867
425	88996	90016	91037	92057	93076
426	99190	00209	01226	02244	03262
427	6309361	10377	11393	12408	13423
428	19508	20522	21535	22548	23560
429	29632	30643	31654	32664	33674
430	39732	40740	41749	42757	43765
431	49808	50814	51820	52826	53832
432	59861	60865	61869	62873	63876
433	69891	70893	71894	72895	73897
434	79898	80897	81896	82895	83894

N°	0	1	2	3	4	Dif
435	638.4893	5891	6889	7887	8884	998
436	639.4865	5861	6857	7852	8847	995
437	640.4814	5808	6802	7795	8788	993
438	641.4741	5733	6724	7715	8705	991
439	642.4645	5634	6623	7612	8601	989
440	643.4527	5514	6500	7487	8473	986
441	644.4386	5371	6355	7339	8323	984
442	645.4223	5205	6187	7169	8151	982
443	646.4037	5018	5998	6977	7957	980
444	647.3830	4808	5786	6763	7741	978
445	648.3600	4576	5552	6527	7502	975
446	649.3349	4322	5296	6269	7242	973
447	650.3075	4047	5018	5989	6960	971
448	651.2780	3749	4719	5687	6656	969
449	652.2463	3431	4397	5364	6331	966
450	653.2125	3090	4055	5019	5984	964
451	654.1765	2728	3691	4653	5616	962
452	655.1384	2345	3306	4266	5226	960
453	656.0982	1941	2899	3857	4815	958
454	657.0559	1515	2471	3427	4383	956
455	658.0114	1068	2023	2977	3930	954
456	9648	0601	1553	2505	3456	952
457	659.9162	0112	1062	2012	2962	950
458	660.8655	9603	0551	1499	2446	947
459	661.8127	9073	0019	0964	1910	945
460	662.7578	8522	9466	0410	1353	943
461	663.7009	7951	8893	9835	0776	941
462	664.6420	7360	8299	9239	0178	939
463	665.5810	6748	7686	8623	9560	937
464	666.5180	6116	7051	7987	8922	935
465	667.4530	5463	6397	7331	8264	933
466	668.3859	4791	5723	6654	7585	931
467	669.3169	4099	5028	5958	6887	929
468	670.2459	3386	4314	5242	6169	927
469	671.1728	2654	3580	4506	5431	925
470	672.0979	1903	2826	3750	4673	923
471	673.0209	1131	2053	2974	3896	921
472	9420	0340	1260	2179	3099	919

N°	5	6	7	8	9	Dif
435	638.9882	0879	1876	2872	3869	998
436	639.9842	0837	1832	2826	3820	995
437	640.9781	0773	1765	2758	3749	993
438	641.9696	0686	1676	2666	3656	991
439	642.9589	0577	1565	2552	3540	989
440	643.9459	0445	1431	2416	3401	986
441	644.9307	0291	1274	2257	3240	984
442	645.9133	0114	1095	2076	3057	982
443	646.8936	9915	0894	1873	2851	980
444	647.8718	9695	0671	1648	2624	978
445	648.8477	9452	0426	1401	2375	975
446	649.8215	9187	0160	1132	2104	973
447	650.7930	8901	9871	0841	1811	971
448	651.7624	8593	9561	0528	1496	969
449	652.7297	8263	9229	0195	1160	966
450	653.6948	7912	8876	9839	0802	964
451	654.6578	7539	8501	9462	0423	962
452	655.6186	7145	8105	9064	0023	960
453	656.5773	6730	7688	8645	9602	958
454	657.5339	6294	7250	8205	9159	956
455	658.4884	5837	6790	7743	8696	954
456	659.4408	5359	6310	7261	8212	952
457	660 3911	4860	5809	6758	7706	950
458	661.3393	4341	5287	6234	7181	947
459	662.2855	3800	4745	5690	6634	945
460	663.2296	3239	4182	5125	6067	943
461	664.1717	2658	3599	4539	5480	941
462	665.1117	2056	2995	3934	4872	939
463	666.0497	1434	2371	3307	4244	937
464	.9857	0792	1727	2661	3595	935
465	667.9197	0130	1062	1995	2927	933
466	668.8516	9447	0378	1308	2239	931
467	669.7816	8745	9674	0602	1530	929
468	670.7096	8023	8950	9876	0802	927
469	671.6356	7281	8206	9130	0054	925
470	672.5596	6519	7442	8365	9287	923
471	673.4817	5738	6659	7579	8500	921
472	674.4018	4937	5856	6775	7693	919

N°	0	1	2	3	4	Dif
473	674.8611	9529	0447	1365	2283	917
474	675.7783	8700	9615	0531	1447	915
475	676.6936	7850	8764	9678	0592	913
476	677.6070	6982	7894	8806	9718	911
477	678.5184	6094	7004	7914	8824	910
478	679.4279	5187	6096	7004	7912	908
479	680.3355	4262	5168	6074	6980	906
480	681.2412	3317	4222	5126	6030	904
481	682.1451	2354	3256	4159	5061	902
482	683.0470	1371	2272	3173	4073	900
483	9471	0370	1269	2168	3066	898
484	684.8454	9351	0248	1145	2041	896
485	685.7417	8313	9208	0103	0998	895
486	686.6363	7256	8150	9043	9936	893
487	687.5290	6181	7073	7964	8855	891
488	688.4198	5088	5978	6867	7757	889
489	689.3089	3977	4864	5752	6640	887
490	690.1961	2847	3733	4619	5505	885
491	691.0815	1699	2584	3468	4352	884
492	9651	0534	1416	2298	3180	882
493	692.8469	9350	0231	1111	1991	880
494	693.7269	8149	9027	9906	0785	878
495	694.6052	6929	7806	8683	9560	877
496	695.4817	5692	6568	7443	8318	875
497	696.3564	4438	5311	6185	7058	873
498	697.2293	3165	4037	4909	5780	872
499	698.1005	1876	2746	3616	4485	870
500	9700	0569	1437	2305	3173	868
501	699.8377	9244	0111	0977	1843	866
502	700.7037	7902	8767	9632	0496	864
503	701.5680	6543	7406	8269	9132	863
504	702.4305	5167	6028	6890	7751	861
505	703.2914	3774	4633	5493	6352	860
506	704.1505	2363	3221	4079	4937	858
507	705.0080	0936	1792	2649	3505	856
508	8637	9492	0347	1201	2055	854
509	706.7178	8031	8884	9737	0589	852
510	707.5702	6553	7405	8256	9107	851

N°	5	6	7	8	9	Dif
473	675.3200	4117	5034	5951	6867	917
474	676.2362	3277	4192	5107	6022	915
475	677.1505	2418	3332	4244	5157	913
476	678.0629	1540	2452	3362	4273	911
477	9734	0643	1552	2461	3370	910
478	679.8819	9727	0634	1541	2448	908
479	680.7886	8792	9697	0602	1507	906
480	681.6934	7838	8741	9645	0548	904
481	682.5963	6865	7766	8668	9569	902
482	683.4973	5873	6773	7673	8572	900
483	684.3965	4863	5761	6659	7556	898
484	685.2938	3834	4730	5626	6522	896
485	686.1892	2787	3681	4575	5469	895
486	687.0828	1721	2613	3506	4398	893
487	9746	0637	1528	2418	3308	891
488	688.8646	9535	0423	1312	2200	889
489	689.7527	8414	9301	0188	1074	887
490	690.6390	7275	8161	9046	9930	885
491	691.5235	6119	7002	7885	8768	884
492	692.4062	4944	5826	6707	7588	882
493	693.2872	3752	4631	5511	6390	880
494	694.1663	2541	3419	4297	5175	878
495	695.0437	1312	2189	3065	3941	877
496	9193	0067	0942	1816	2690	875
497	696.7931	8804	9676	0549	1421	873
498	697.6652	7523	8394	9264	0135	872
499	698.5355	6224	7093	7963	8831	870
500	699.4041	4908	5776	6643	7510	868
501	700.2709	3575	4441	5307	6172	866
502	701.1361	2225	3089	3953	4816	864
503	9995	0857	1720	2582	3444	863
504	702.8612	9472	0333	1193	2054	861
505	703.7212	8071	8930	9788	0647	860
506	704.5794	6652	7509	8366	9223	858
507	705.4360	5216	6072	6927	7782	856
508	706.2910	3764	4617	5471	6325	854
509	707.1442	2294	3146	3998	4850	852
510	9957	0808	1659	2509	3359	851

N°	0	1	2	3	4	Dif
511	708.4209	5059	5908	6758	7607	849
512	709.2700	3548	4396	5244	6091	847
513	710.1174	2020	2866	3714	4559	846
514	9631	0476	1321	2165	3010	844
515	711.8072	8915	9759	0601	1444	843
516	712.6497	7339	8180	9021	9862	841
517	713.4905	5745	6585	7425	8264	839
518	714.3298	4136	4974	5812	6650	838
519	715.1674	2510	3347	4183	5019	836
520	716.0033	0869	1703	2538	3373	835
521	8377	9211	0044	0877	1710	833
522	717.6705	7537	8369	9200	0032	831
523	718.5017	5847	6677	7507	8337	830
524	719.3313	4142	4970	5799	6627	828
525	720.1593	2420	3247	4074	4901	826
526	9857	0683	1508	2334	3159	825
527	721.8106	8930	9754	0578	1401	823
528	722.6339	7162	7984	8806	9628	822
529	723.4557	5378	6198	7019	7839	820
530	724.2759	3578	4397	5216	6035	818
531	725.0945	1763	2581	3398	4216	817
532	9116	9933	0749	1565	2380	815
533	726.7272	8087	8901	9716	0530	814
534	727.5413	6226	7039	7852	8664	812
535	728.3538	4350	5161	5972	6784	811
536	729.1648	2458	3268	4078	4888	809
537	9743	0552	1360	2168	2977	808
538	730.7823	8630	9437	0244	1051	806
539	731.5888	6693	7499	8304	9109	805
540	732.3938	4742	5546	6350	7153	804
541	733.1973	2775	3578	4380	5183	802
542	9993	0794	1595	2396	3197	801
543	734.7998	8798	9598	0397	1196	799
544	735.5989	6787	7585	8383	9181	798
545	736.3965	4762	5558	6355	7151	796
546	737.1926	2722	3517	4312	5107	795
547	9873	0667	1461	2254	3048	793
548	738.7806	8598	9390	0182	0974	792

N°	5	6	7	8	9	Dif
511	708.8456	9305	0154	1003	1851	849
512	709.6939	7786	8633	9480	0327	847
513	710.5404	6250	7096	7941	8788	846
514	711.3854	4698	5542	6385	7229	844
515	712.2287	3129	3971	4813	5655	843
516	713.0703	1544	2385	3225	4065	841
517	9104	9943	0782	1620	2459	839
518	714.7488	8325	9162	0000	0837	838
519	715.5856	6691	7527	8363	9198	836
520	716.4207	5042	5876	6710	7544	835
521	717.2543	3376	4208	5041	5873	833
522	718.0863	1694	2525	3356	4186	831
523	9167	9996	0826	1655	2484	830
524	719.7455	8283	9111	9938	0766	828
525	720.5727	6554	7380	8206	9032	826
526	721.3984	4809	5633	6458	7282	825
527	722.2225	3048	3871	4694	5517	823
528	723.0450	1272	2093	2914	3736	822
529	8660	9480	0300	1120	1939	820
530	724.6854	7672	8491	9309	0127	818
531	725.5033	5850	6667	7483	8300	817
532	726.3196	4012	4827	5642	6457	815
533	727.1344	2158	2972	3786	4599	814
534	9477	0290	1102	1914	2726	812
535	728.7595	8406	9216	0027	0838	811
536	729.5697	6507	7316	8125	8934	809
537	730.3785	4593	5400	6208	7015	808
538	731.1857	2663	3470	4276	5082	806
539	9914	0719	1524	2329	3133	805
540	732.7957	8760	9564	0367	1170	804
541	733.5985	6787	7588	8390	9192	802
542	734.3997	4798	5598	6398	7198	801
543	735.1995	2794	3593	4392	5191	799
544	9979	0776	1574	2371	3168	798
545	736.7948	8744	9540	0335	1131	796
546	737.5902	6696	7491	8285	9079	795
547	738.3841	4634	5427	6220	7013	793
548	739.1766	2558	3350	4141	4932	792

N°	0	1	2	3	4	Dif
587	768.6381	7121	7860	8600	9339	739
588	769.3773	4512	5250	5988	6727	738
589	770.1153	1890	2627	3364	4101	737
590	8520	9256	9992	0728	1463	736
591	771.5875	6610	7344	8079	8813	735
592	772.3217	3951	4684	5417	6150	733
593	773.0547	1279	2011	2743	3475	732
594	7864	8595	9326	0057	0788	731
595	774.5170	5899	6629	7359	8088	729
596	775.2463	3191	3920	4648	5376	728
597	9743	0471	1198	1925	2652	727
598	776.7012	7738	8464	9190	9916	726
599	777.4268	4993	5718	6443	7167	724
600	778.1512	2236	2960	3683	4407	723
601	8745	9467	0190	0912	1634	722
602	779.5965	6686	7407	8129	8850	721
603	780.3173	3893	4613	5333	6053	720
604	781.0369	1088	1807	2526	3245	719
605	7554	8271	8989	9707	0424	717
606	782.4726	5443	6159	6876	7592	716
607	783.1887	2602	3318	4033	4748	715
608	9036	9750	0464	1178	1892	714
609	784.6173	6886	7599	8312	9024	713
610	785.3298	4010	4722	5434	6145	712
611	786.0412	1123	1833	2544	3254	710
612	7514	8224	8933	9643	0352	709
613	787.4605	5313	6021	6730	7438	708
614	788.1684	2391	3098	3805	4512	707
615	8751	9457	0163	0869	1575	706
616	789.5807	6512	7217	7922	8626	705
617	790.2853	3555	4259	4963	5666	703
618	9885	0587	1290	1992	2695	702
619	791.6906	7608	8309	9011	9712	701
620	792.3917	4617	5318	6018	6718	700
621	793.0916	1615	2313	3013	3712	699
622	7904	8602	9300	9998	0696	698
623	794.4880	5577	6274	6971	7668	696
624	795.1846	2542	3238	3933	4629	695

N°	5	6	7	8	9	*Dif*
587	769.0079	0818	1557	2296	3035	739
588	7465	8203	8940	9678	0415	738
589	770.4838	5575	6311	7048	7784	737
590	771.2199	2934	3670	4405	5140	736
591	9547	0282	1016	1750	2483	735
592	772.6883	7616	8349	9082	9814	733
593	773.4207	4939	5670	6402	7133	732
594	774.1519	2249	2979	3710	4440	731
595	8818	9547	0276	1005	1734	729
596	775.6104	6832	7560	8288	9016	728
597	776.3379	4106	4833	5559	6285	727
598	777.0641	1367	2093	2818	3543	726
599	7892	8616	9340	0065	0789	724
600	778.5130	5853	6576	7299	8022	723
601	779.2356	3078	3800	4522	5243	722
602	9570	0291	1012	1732	2453	721
603	780.6773	7492	8212	8931	9650	720
604	781.3963	4681	5400	6118	6836	719
605	782.1141	1859	2576	3293	4009	717
606	8308	9024	9740	0456	1171	716
607	783.5463	6178	6892	7607	8321	715
608	784.2606	3319	4033	4746	5460	714
609	9737	0450	1162	1874	2586	713
610	785.6857	7568	8279	8990	9701	712
611	786.3965	4675	5385	6095	6804	710
612	787.1061	1770	2479	3188	3896	709
613	8146	8853	9561	0269	0976	708
614	788.5219	5926	6632	7339	8045	707
615	789.2281	2986	3691	4397	5102	706
616	9331	0035	0739	1444	2148	705
617	790.6370	7073	7776	8479	9182	703
618	791.3397	4099	4801	5503	6205	702
619	792.0413	1114	1815	2516	3216	701
620	7418	8118	8817	9517	0217	700
621	793.4411	5110	5809	6507	7206	699
622	794.1394	2091	2789	3486	4183	698
623	8365	9061	9757	0454	1150	696
624	795.5324	6020	6715	7410	8105	695

N°	0	1	2	3	4	Dif
625	795.8800	9495	0190	0884	1579	694
626	796.5743	6437	7131	7824	8517	693
627	797.2675	3368	4060	1753	5445	692
628	9596	0288	0970	1671	2362	691
629	798.6566	7197	7887	8577	9267	690
630	799.3405	4095	4784	5473	6162	689
631.	800.0294	0982	1670	2358	3046	688
632	7171	7858	8545	9232	9919	687
633	801.4037	4723	5409	6095	6781	686
634	802.0893	1577	2262	2947	3632	684
635	7737	8421	9105	9789	0472	683
636	803.4571	5254	5937	6619	7302	682
637	804.1394	2076	2758	3439	4121	681
638	8207	8887	9568	0248	0929	680
639	805.5009	5688	6368	7047	7726	679
640	806.1800	2478	3157	3835	4513	678
641	8580	9258	9935	0612	1290	677
642	807.5350	6027	6703	7379	8055	676
643	808.2110	2785	3460	4135	4811	675
644	8859	9533	0207	0881	1555	674
645	809.5597	6270	6944	7617	8290	673
646	810.2325	2997	3669	4342	5013	672
647	9043	9714	0385	1056	1727	671
648	811.5750	6420	7090	7760	8430	670
649	812.2447	3116	3785	4454	5123	669
650	9134	9802	0470	1137	1805	668
651	813.5810	6477	7144	7811	8477	667
652	814.2476	3142	3808	4474	5139	666
653	9132	9797	0462	1127	1791	665
654	815.5777	6441	7105	7769	8433	664
655	816 2413	3076	3739	4402	5064	663
656	9038	9700	0362	1024	1686	662
657	817.5654	6315	6975	7636	8297	660
658	818.2259	29 9	3579	4239	4898	659
659	8854	9513	0172	0831	1489	658
660	819.5439	6097	6755	7413	8071	658
661	820.2015	2672	3328	3985	4642	657
662	8580	9236	9892	0548	1203	655

N°	5	6	7	8	9	Dif
625	796.2273	2967	3662	4356	5049	694
626	9211	9904	0597	1290	1983	693
627	797.6137	6829	7521	8213	8905	692
628	798.3053	3744	4435	5125	5816	691
629	9957	0647	1337	2027	2716	690
630	799.6851	7540	8228	8917	9605	689
631	800.3731	4421	5109	5796	6484	688
632	801.0605	1292	1978	2665	3351	687
633	7466	8152	8837	9522	0207	686
634	802.4316	5001	5685	6369	7053	684
635	803.1155	1839	2522	3205	3888	683
636	7984	8666	9348	0030	0712	682
637	804.4802	5483	6164	6845	7526	681
638	805.1609	2289	2969	3649	4329	680
639	8405	9085	9763	0442	1121	679
640	806.5191	5869	6547	7225	7903	678
641	807.1967	2644	3320	3997	4674	677
642	8731	9407	0083	0759	1434	676
643	808.5485	6160	6835	7510	8184	675
644	809.2229	2903	3577	4250	4924	674
645	8962	9635	0308	0980	1653	673
646	810.5685	6357	7029	7700	8371	672
647	811.2398	3068	3739	4409	5080	671
648	9100	9769	0439	1108	1778	670
649	812.5792	6460	7129	7797	8465	669
650	813.2473	3141	3808	4475	5143	668
651	9144	9811	0477	1144	1810	667
652	814.5805	6471	7136	7801	8467	666
653	815.2456	3120	3785	4449	5113	665
654	9096	9760	0423	1087	1750	664
655	816.5727	6389	7052	7714	8376	663
656	817.2347	3009	3670	4331	4993	662
657	8958	9618	0278	0939	1599	660
658	818.5558	6217	6877	7536	8195	659
659	819.2148	2806	3465	4123	4781	658
660	8728	9386	0043	0700	1358	658
661	820.5298	5955	6611	7268	7924	657
662	821.1859	2514	3170	3825	4480	656

No	0	1	2	3	4	Diff
663	821.5135	5790	6445	7100	7755	655
664	822.1681	2335	2989	3642	4296	654
665	8216	8869	9522	0175	0828	653
666	823.4742	5394	6046	6698	7350	652
667	824.1258	1909	2560	3211	3862	651
668	7765	8415	9065	9715	0364	650
669	825.4261	4910	5559	6208	6857	649
670	826.0748	1396	2044	2692	3340	648
671	7225	7872	8519	9166	9813	647
672	827.3693	4339	4985	5631	6277	646
673	828.0151	0796	1441	2086	2731	645
674	6599	7243	7887	8532	9176	644
675	829.3038	3681	4324	4967	5611	643
676	9467	0109	0752	1394	2036	642
677	830.5887	6528	7169	7811	8452	641
678	831.2297	2937	3578	4218	4858	640
679	8698	9337	9977	0616	1255	639
680	832.5089	5728	6366	7005	7643	638
681	833.1471	2109	2746	3384	4021	637
682	7844	8480	9117	9754	0390	636
683	834.4207	4843	5479	6114	6750	635
684	835.0561	1196	1831	2465	3100	635
685	6906	7540	8174	8807	9441	634
686	836.3241	3874	4507	5140	5773	633
687	9567	0199	0832	1463	2095	632
688	837.5884	6516	7147	7778	8409	631
689	838.2192	2822	3453	4083	4713	630
690	8491	9120	9750	0379	1008	629
691	839.4780	5409	6037	6666	7294	628
692	840.1061	1688	2316	2943	3571	627
693	7332	7959	8586	9212	9838	626
694	841.3595	4220	4846	5472	6097	625
695	9848	0473	1098	1722	2347	624
696	842.6092	6716	7340	7964	8588	624
697	843.2328	2951	3574	4197	4819	623
698	8554	9176	9798	0420	1042	622
699	844.4772	5393	6014	6635	7256	621
700	845.0980	1601	2221	2841	3461	620

N°	5	6	7	8	9	Dif
663	821.8409	9064	9718	0372	1027	655
664	822.4950	5603	6257	6910	7563	654
665	823.1481	2133	2786	3438	4090	653
666	8002	8653	9305	9956	0607	652
667	824.4513	5163	5814	6464	7114	651
668	825.1014	1664	2313	2963	3612	650
669	7506	8154	8803	9451	0100	649
670	826.3988	4635	5283	5931	6578	648
671	827.0460	1107	1753	2400	3046	647
672	6923	7569	8214	8860	9505	646
673	828.3376	4021	4665	5310	5955	645
674	9820	0463	1107	1751	2394	644
675	829.6254	6896	7539	8182	8824	643
676	830.2678	3320	3962	4604	5245	642
677	9093	9734	0375	1016	1656	641
678	831.5499	6139	6778	7418	8058	640
679	832.1895	2534	3173	3812	4450	639
680	8281	8919	9558	0195	0833	638
681	833.4659	5296	5933	6570	7207	637
682	834.1027	1663	2299	2935	3571	636
683	7385	8021	8656	9291	9926	635
684	835.3735	4369	5003	5638	6272	635
685	836.0075	0708	1341	1975	2608	634
686	6405	7038	7670	8303	8935	633
687	837.2727	3359	3990	4622	5253	632
688	9039	9670	0301	0931	1562	631
689	838.5343	5973	6602	7232	7861	630
690	039.1637	2266	2895	3523	4152	629
691	7922	8550	9178	9806	0433	628
692	840.4198	4825	5452	6079	6706	627
693	841.0465	1091	1717	2343	2969	626
694	6723	7348	7973	8598	9223	625
695	842.2971	3596	4220	4844	5468	624
696	9211	9835	0458	1081	1705	624
697	843.5442	6065	6687	7310	7932	623
698	844.1664	2286	2907	3529	4150	622
699	7877	8498	9119	9739	0360	621
700	845.4081	4701	5521	5941	6561	620

N°	0	1	2	3	4	Diff
701	845.7180	7800	8419	9038	9658	619
702	846.3371	3990	4608	5227	5845	618
703	9553	0171	0789	1406	2024	618
704	847.5727	6343	6960	7577	8193	617
705	848.1891	2507	3123	3739	4355	616
706	8047	8662	9277	9892	0507	615
707	849.4194	4808	5423	6037	6651	614
708	850.0333	0946	1559	2172	2786	613
709	6462	7075	7687	8300	8912	612
710	851.2583	3195	3807	4418	5030	611
711	8696	9307	9917	0528	1139	611
712	852.4800	5410	6020	6629	7239	610
713	853.0895	1504	2113	2722	3331	609
714	6982	7590	8198	8807	9414	608
715	854 3060	3668	4275	4882	5489	607
716	9130	9737	0343	0950	1556	606
717	855.5192	5797	6403	7008	7614	605
718	856.1244	1849	2454	3059	3663	604
719	7289	7893	8497	9101	9704	604
720	857.3325	3928	4531	5134	5737	603
721	9353	9955	0557	1159	1761	602
722	858.5372	5973	6575	7176	7777	601
723	859.1383	1984	2584	3185	3785	600
724	7386	7985	8585	9185	9784	600
725	860.3380	3979	4578	5177	5776	599
726	9366	9964	0562	1160	1758	598
727	861.5344	5941	6539	7136	7733	597
728	862.1314	1910	2507	3103	3699	596
729	7275	7871	8467	9062	9658	595
730	863.3229	3823	4418	5013	5608	595
731	9174	9768	0362	0956	1550	594
732	864.5111	5704	6297	6890	7483	593
733	865.1040	1632	2225	2817	3409	592
734	6961	7552	8144	8735	9327	591
735	866.2873	3464	4055	4646	5236	590
736	8778	9368	9958	0548	1138	590
737	867.4675	5264	5853	6442	7031	589
738	868.0564	1152	1740	2329	2917	588

N°	5	6	7	8	9	Dif
701	846.0277	0896	1515	2134	2752	619
702	6463	7081	7700	8318	8935	618
703	847.2641	3258	3876	4493	5110	618
704	8810	9426	0043	0659	1275	617
705	848.4979	5586	6201	6817	7432	616
706	849.1122	1736	2351	2965	3580	615
707	7264	7878	8492	9106	9719	614
708	850.3399	4011	4624	5237	5850	613
709	9524	0136	0748	1360	1972	612
710	851.5641	6252	6863	7474	8085	611
711	852.1749	2359	2970	3580	4190	611
712	7849	8458	9068	9677	0286	610
713	853.3940	4548	5157	5765	6374	609
714	854.0022	0630	1238	1845	2453	608
715	6096	6703	7310	7917	8524	607
716	855.2162	2768	3374	3980	4586	606
717	8219	8824	9429	0035	0640	605
718	856.4268	4872	5476	6081	6685	604
719	857.0308	0912	1515	2118	2722	604
720	6340	6943	7545	8148	8750	603
721	858.2363	2965	3567	4169	4770	602
722	8379	8980	9581	0181	0782	601
723	859.4385	4986	5586	6186	6786	600
724	860.0384	0983	1583	2182	2781	600
725	6374	6973	7571	8170	8768	599
726	861.2356	2954	3552	4149	4747	598
727	8330	8927	9524	0121	0717	597
728	862.4296	4892	5488	6084	6680	596
729	863.0253	0848	1443	2039	2634	595
730	6202	6797	7391	7985	8580	595
731	864.2143	2737	3331	3924	4517	594
732	8076	8669	9262	9855	0447	593
733	865.4001	4593	5185	5777	6369	592
734	9918	0509	1100	1691	2282	592
735	866.5827	6417	7008	7598	8188	590
736	867.1728	2317	2907	3496	4086	590
737	7620	8209	8798	9387	9975	589
738	868.3505	4093	4681	5269	5857	588

N°	0	1	2	3	4	Dif
739	868.6444	7032	7620	8207	8794	587
740	869.2317	2904	3491	4077	4664	586
741	8182	8768	9354	9940	0526	586
742	870.4039	4624	5210	5795	6380	585
743	9888	0473	1057	1641	2226	584
744	871.5729	6313	6897	7480	8064	583
745	872.1563	2146	2728	3311	3894	582
746	7388	7970	8552	9134	9716	582
747	873.3206	3787	4369	4950	5531	581
748	9016	9597	0177	0757	1338	580
749	874.4818	5398	5978	6557	7137	579
750	875.0613	1192	1771	2349	2928	579
751	6399	6978	7556	8134	8712	578
752	876.2178	2756	3333	3911	4488	577
753	7950	8526	9103	9680	0256	576
754	877.3713	4289	4865	5441	6017	576
755	9470	0045	0620	1195	1770	575
756	878.5218	5792	6367	6941	7515	574
757	879.0959	1532	2106	2680	3253	573
758	6692	7265	7838	8411	8983	572
759	880.2418	2990	3562	4134	4706	572
760	8136	8707	9279	9850	0421	571
761	881.3847	4417	4988	5558	6129	570
762	9550	0120	0689	1259	1829	569
763	882.5245	5815	6384	6953	7522	569
764	883.0934	1502	2070	2639	3207	568
765	6614	7182	7750	8317	8885	567
766	884.2288	2855	3421	3988	4555	567
767	7954	8520	9086	9652	0218	566
768	885.3612	4178	4743	5308	5874	565
769	9263	9828	0393	0957	1522	564
770	886.4907	5471	6035	6599	7163	564
771	887.0544	1107	1670	2233	2796	563
772	6173	6736	7298	7860	8423	562
773	888.1795	2357	2918	3480	4042	561
774	7410	7971	8532	9093	9653	561
775	889.3017	3577	4138	4694	5258	560
776	8617	9177	9736	0296	0855	559

N°	5	6	7	8	9	Dif
739	868.9382	9969	0556	1143	1730	587
740	869.5251	5837	6423	7010	7596	586
741	870.1112	1697	2283	2868	3454	586
742	6965	7549	8134	8719	9304	585
743	871.2810	3394	3978	4562	5146	584
744	8647	9230	9814	0397	0980	583
745	872.4476	5059	5641	6224	6806	582
746	873.0298	0880	1462	2043	2625	582
747	6112	6693	7274	7855	8435	581
748	874.1918	2498	3078	3658	4238	580
749	7716	8296	8875	9454	0034	579
750	875.3507	4086	4664	5243	5821	579
751	9290	9868	0446	1023	1601	578
752	876.5065	5642	6219	6796	7373	577
753	877.0833	1409	1985	2561	3137	576
754	6592	7168	7743	8319	8894	576
755	878.2345	2919	3494	4069	4643	575
756	8089	8663	9237	9811	0385	574
757	879.3826	4400	4973	5546	6119	573
758	9556	0128	0701	1273	1846	572
759	880.5278	5850	6421	6993	7564	572
760	881.0992	1563	2134	2705	3276	571
761	6699	7269	7840	841[illegible]	8980	570
762	882.2398	2968	3537	4107	4676	569
763	8090	8659	9228	9797	0365	569
764	883.3775	4343	4911	5479	6047	568
765	9452	0019	0586	1154	1721	567
766	884.5122	5688	6255	6821	7387	567
767	885.0784	1350	1915	2481	3047	566
768	6439	7004	7569	8134	8699	565
769	886.2086	2651	3215	3779	4343	564
770	7726	8290	8854	9417	9980	564
771	887.3359	3922	4485	5048	5610	563
772	8985	9547	0109	0671	1233	562
773	888.4603	5165	5726	6287	6848	561
774	889.0214	0775	1336	1896	2457	561
775	5818	6378	6938	7498	8058	560
776	890.1415	1974	2533	3092	3651	559

N°	0	1	2	3	4	Dif
739	868.6444	7032	7620	8207	8794	587
740	869.2317	2904	3491	4077	4664	586
741	8182	8768	9354	9940	0526	586
742	870.4039	4624	5210	5795	6380	585
743	9888	0473	1057	1641	2226	584
744	871.5729	6313	6897	7480	8064	583
745	872.1563	2146	2728	3311	3894	582
746	7388	7970	8552	9134	9716	582
747	873.3206	3787	4369	4950	5531	581
748	9016	9597	0177	0757	1338	580
749	874.4818	5398	5978	6557	7137	579
750	875.0613	1192	1771	2349	2928	579
751	6399	6978	7556	8134	8712	578
752	876.2178	2756	3333	3911	4488	577
753	7950	8526	9103	9680	0256	576
754	877.3713	4289	4865	5441	6017	576
755	9470	0045	0620	1195	1770	575
756	878.5218	5792	6367	6941	7515	574
757	879.0959	1532	2106	2680	3253	573
758	6692	7265	7838	8411	8983	572
759	880.2418	2990	3562	4134	4706	572
760	8136	8707	9279	9850	0421	571
761	881.3847	4417	4988	5558	6129	570
762	9550	0120	0689	1259	1829	569
763	882.5245	5815	6384	6953	7522	569
764	883.0934	1502	2070	2639	3207	568
765	6614	7182	7750	8317	8885	567
766	884.2288	2855	3421	3988	4555	567
767	7954	8520	9086	9652	0218	566
768	885.3612	4178	4743	5308	5874	565
769	9263	9828	0393	0957	1522	564
770	886.4907	5471	6035	6599	7163	564
771	887.0544	1107	1670	2233	2796	563
772	6173	6736	7298	7860	8423	562
773	888.1795	2357	2918	3480	4042	561
774	7410	7971	8532	9093	9653	561
775	889.3017	3577	4138	4694	5258	560
776	8617	9177	9736	0296	0855	559

N°	5	6	7	8	9	Dif
777	890.7004	7563	8121	8679	9238	559
778	891 2586	3144	3702	4259	4817	558
779	8161	8718	9275	9832	0389	557
780	892.3729	4285	4842	5398	5954	556
781	9290	9846	0401	0957	1512	556
782	893.4843	5398	5953	6508	706	555
783	894.0390	0944	1498	2053	2607	554
784	5929	6483	7037	7590	8143	554
785	895.1462	2015	2568	3120	3673	553
786	6987	7539	8092	8644	9195	552
787	896.2506	3057	3608	4160	4711	552
788	8017	8568	9118	9669	0220	551
789	897.3521	4071	4621	5171	5721	550
790	9019	9568	0117	0667	1216	549
791	898.4509	5058	5606	6155	6703	549
792	9993	0541	1089	1636	2184	548
793	899.5469	6017	6564	7111	7658	547
794	900.0939	1486	2032	2579	3125	547
795	6402	6948	7494	8039	8585	546
796	901.1858	2403	2948	3493	4038	545
797	7307	7851	8396	8940	9485	544
798	902.2749	3293	3837	4381	4924	544
799	8185	8728	9271	9814	0357	543
800	903.3613	4156	4698	5241	5783	543
801	9035	9577	0119	0661	1202	542
802	904.4450	4992	5533	6074	6615	541
803	9859	0399	0940	1480	2020	540
804	905.5260	5800	6340	6880	7419	540
805	906.0655	1195	1734	2273	2812	539
806	6044	6582	7121	7659	8197	539
807	907.1425	1963	2501	3038	3576	538
808	6800	7337	7874	8411	8948	537
809	908.2169	2705	3241	3778	4314	536
810	7530	8066	8602	9137	9673	536
811	909.2885	3420	3955	4490	5025	535
812	8234	8768	9303	9837	0371	534
813	910.3576	4109	4643	5177	5710	534
814	8911	9444	9977	0510	1043	533

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815	911.1576	2109	2642	3174	3707	533
816	6902	7434	7966	8498	9030	532
817	912.2221	2752	3284	3815	4346	531
818	7533	8064	8595	9126	9656	530
819	913.2839	3369	3899	4430	4960	530
820	8139	8668	9198	9727	0257	529
821	914.3432	3961	4489	5018	5547	529
822	8718	9246	9775	0303	0831	528
823	915.3998	4526	5054	5581	6109	527
824	9272	9799	0326	0853	1380	527
825	916.4539	5066	5592	6118	6645	526
826	9800	0326	0852	1378	1903	526
827	917.5055	5580	6105	6630	7155	525
828	918.0303	0828	1352	1877	2401	524
829	5545	6069	6593	7117	7640	524
830	919.0781	1304	1827	2350	2873	523
831	6010	6533	7055	7578	8100	522
832	920.1233	1755	2277	2799	3321	522
833	6450	6971	7493	8014	8535	521
834	921.1661	2181	2702	3222	3743	521
835	6865	7385	7905	8425	8945	520
836	922.2063	2582	3102	3621	4140	519
837	7255	7773	8292	8811	9330	518
838	923.2440	2958	3477	3995	4513	518
839	7620	8137	8655	9172	9690	517
840	924.2793	3310	3827	4344	4860	517
841	7960	8476	8993	9509	0025	516
842	925.3121	3637	4152	4668	5184	516
843	8276	8791	9306	9821	0336	515
844	926.3424	3939	4453	4968	5482	514
845	8567	9081	9595	0109	0622	514
846	927.3704	4217	4730	5243	5757	513
847	8834	9347	9859	0372	0885	512
848	928.3959	4471	4983	5495	6007	512
849	9077	9588	0100	0611	1123	511
850	929.4189	4700	5211	5722	6233	511
851	9296	9806	0316	0826	1336	510
852	930.4396	4906	5415	5925	6434	509

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818	913.0189	0717	1248	1778	2309	530
819	5490	6019	6549	7079	7609	530
820	914.0786	1315	1844	2373	2903	529
821	6076	6604	7133	7661	8190	529
822	915.1359	1887	2415	2943	3471	528
823	6636	7163	7691	8218	8745	527
824	916.1907	2433	2960	3487	4013	527
825	7171	7697	8223	8749	9275	526
826	917.2429	2954	3479	4005	4530	526
827	7680	8205	8730	9254	9779	525
828	918.2925	3449	3973	4497	5021	524
829	8164	8687	9211	9734	0258	524
830	919.3396	3919	4442	4965	5488	523
831	8623	9145	9667	0189	0711	522
832	920.3842	4364	4886	5407	5929	522
833	9056	9577	0098	0619	1140	521
834	921.4263	4784	5304	5824	6345	521
835	9465	9984	0504	1024	1543	520
836	922.4659	5179	5698	6217	6736	519
837	9848	0367	0885	1404	1922	518
838	923.5031	5549	6066	6584	7102	518
839	924.0207	0724	1242	1759	2276	517
840	5377	5894	6410	6927	7444	517
841	925.0541	1057	1573	2089	2605	516
842	5699	6215	6730	7245	7761	516
843	926.0851	1366	1880	2395	2910	515
844	5997	6511	7025	7539	8035	514
845	927.1136	1650	2163	2677	3190	514
846	6270	6783	7296	7808	8321	513
847	928.1397	1909	2422	2934	3446	512
848	6518	7030	7542	8054	8565	512
849	929.1634	2145	2656	3167	3678	511
850	6743	7254	7764	8275	8785	511
851	930.1847	2357	2866	3376	3886	510
852	6944	7453	7963	8472	8981	509

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855	9661	0169	0677	1185	1692	508
856	932.4738	5245	5752	6259	6767	507
857	9808	0315	0822	1328	1835	506
858	933.4873	5379	5885	6391	6897	106
859	9932	0437	0943	1448	1953	505
860	934.4985	5489	5994	6499	7004	505
861	935.0032	0536	1040	1544	2049	504
862	5073	5576	6080	6584	7087	503
863	936,0108	0611	1114	1617	2120	503
864	5137	5640	6143	6645	7148	502
865	937.0161	0663	1165	1667	2169	502
866	5179	5680	6182	6683	7184	501
867	938.0191	0692	1193	1693	2194	501
868	5197	5698	6198	6698	7198	500
869	939.0198	0697	1197	1697	2197	499
870	5193	5694	6191	6690	7189	499
871	940.0182	0680	1179	1677	2176	498
872	5165	5663	6161	6659	7157	498
873	941.0142	0640	1137	1635	2132	497
874	5114	5611	6108	6605	7101	497
875	942.0081	0577	1073	1569	2065	496
876	5041	5537	6032	6528	7024	496
877	9996	0491	0986	1481	1976	495
878	943.4945	5440	5934	6429	6923	494
879	9889	0383	0877	1371	1865	494
880	944.4827	5320	5814	6307	6800	493
881	9759	0252	0745	1238	1730	493
882	945.4686	5178	5671	6163	6655	492
883	9607	0099	0591	1082	1574	492
884	946.4523	5014	5505	5996	6487	491
885	9433	9923	0414	0905	1395	490
886	947.4337	4827	5317	5807	6297	490
887	9236	9726	0215	0705	1194	489
888	948.4130	4619	5108	5597	6085	489
889	9018	9506	9995	0483	0971	488
890	949.3900	4388	4876	5364	5852	488

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854	7121	7629	8137	8645	9153	508
855	932.2200	2708	3215	3723	4230	508
856	7274	7781	8288	8795	9301	507
857	933.2341	2848	3354	3860	4367	506
858	7403	7909	8415	8920	9426	506
859	934.2404	2964	3469	3974	4479	505
860	7509	8013	8518	9023	9527	505
861	935.2553	3057	3561	4065	4569	504
862	7591	8095	8598	9101	9605	503
863	936.2623	3126	3629	4132	4635	503
864	7650	8152	8655	9157	9659	502
865	937.2671	3172	3674	4176	4677	502
866	7686	8187	8688	9189	9690	501
867	938.2695	3195	3696	4196	4697	501
868	7698	8198	8698	9198	9698	500
869	939.2696	3195	3695	4194	4693	499
870	7688	8187	8685	9184	9683	499
871	940.2674	3172	3670	4169	4667	498
872	7654	8152	8650	9147	9645	498
873	941.2629	3126	3623	4120	4617	497
874	7598	8095	8591	9088	9584	497
875	942.2562	3058	3553	4049	4545	496
876	7519	8015	8510	9005	9501	496
877	943.2471	2966	3461	3956	4450	495
878	7418	7912	8406	8900	9395	494
879	944.2358	2852	3346	3840	4333	494
880	7294	7787	8280	8773	9266	493
881	945.2223	2716	3208	3701	4193	493
882	7147	7639	8131	8623	9115	492
883	946.2066	2557	3049	3540	4031	492
884	6978	7469	7960	8451	8942	491
885	947.1886	2376	2866	3357	3847	490
886	6787	7277	7767	8257	8747	490
887	948.1684	2173	2662	3151	3641	489
888	6574	7063	7552	8040	8529	489
889	949.1460	1948	2436	2924	3412	488
890	6339	6827	7315	7802	8290	488

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892	950.3649	4135	4622	5109	5596	487
893	8515	9001	9487	9973	0459	486
894	951.3375	3861	4347	4832	5318	486
895	8230	8716	9201	9686	0171	485
896	952.3080	3565	4049	4534	5018	485
897	7924	8409	8893	9377	9861	484
898	953.2763	3247	3731	4214	4697	484
899	7597	8080	8563	9046	9529	483
900	954.2425	2908	3390	3873	4355	482
901	7248	7730	8212	8694	9176	482
902	955.2065	2547	3028	3510	3991	481
903	6878	7358	7839	8320	8801	481
904	956.1684	2165	2645	3125	3606	480
905	6486	6966	7445	7925	8405	480
906	957.1282	1761	2241	2720	3199	479
907	6073	6552	7030	7509	7988	479
908	958.0858	1337	1815	2293	2771	478
909	5639	6117	6594	7072	7549	478
910	959.0414	0891	1368	1845	2322	477
911	5184	5660	6137	6614	7090	477
912	9948	0425	0901	1377	1853	476
913	960.4708	5183	5659	6135	6610	476
914	9462	9937	0412	0887	1362	475
915	961.4211	4686	5160	5635	6109	474
916	8955	9429	9903	0377	0851	474
917	962.3693	4167	4640	5114	5587	473
918	8427	8900	9373	9846	0319	473
919	963.3155	3628	4100	4573	5045	472
920	7878	8350	8822	9294	9766	472
921	964.2596	3068	3539	4011	4482	471
922	7309	7780	8251	8722	9193	471
923	965.2017	2488	2958	3428	3899	470
924	6720	7190	7660	8130	8599	470
925	966.1417	1887	2356	2826	3295	469
926	6110	6579	7048	7517	7985	469
927	967.0797	1266	1734	2203	2671	468
928	5480	5948	6416	6884	7351	468

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892	6082	6569	7055	7542	8028	487
893	951.0946	1432	1918	2404	2889	486
894	5803	6289	6774	7260	7745	486
895	952.0656	1141	1626	2111	2595	485
896	5503	5987	6472	6956	7440	485
897	953.0345	0828	1312	1796	2280	484
898	5181	5664	6147	6631	7114	484
899	954 0012	0494	0977	1460	1943	483
900	4837	5319	5802	6284	6766	482
901	9657	0139	0621	1102	1584	482
902	655.4472	4953	5434	5916	6397	481
903	9282	9762	0243	0723	1204	481
904	956 4086	4566	5046	5526	6006	480
905	8885	9364	9844	0323	0803	480
906	957.3678	4157	4636	5115	5594	479
907	8466	8945	9423	9902	0380	479
908	958.3249	3727	4205	4683	5161	478
909	8027	8505	8982	9459	9937	478
910	859.2800	3276	3753	4230	4707	477
911	7567	8043	8520	8996	9472	477
912	960.2329	2805	3281	3756	4232	476
913	7086	7561	8036	8512	8987	476
914	961.1837	2312	2787	3262	3736	475
915	6583	7058	7532	8006	8481	474
916	962.1325	1799	2272	2746	3220	474
917	6061	6534	7007	7481	7954	473
918	963.0792	1264	1737	2210	2683	473
919	5517	5990	6462	6934	7406	472
920	964.0238	0710	1181	1653	2125	472
921	4953	5425	5896	6367	6838	471
922	9664	0135	0605	1076	1546	471
923	965.4369	4839	5309	5780	6250	470
924	9069	9539	0009	0478	0948	470
925	966.3764	4233	4703	5172	5641	469
926	8454	8923	9392	9860	0329	469
927	967.3139	3607	4076	4544	5012	468
928	7819	8287	8754	9222	9690	468

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931	9497	9963	0430	0896	1362	466
932	969.4159	4625	5091	5557	6023	466
933	8816	9282	9747	0213	0678	465
934	970.3469	3934	4399	4863	5328	465
935	8116	8581	9045	9509	9974	464
936	971.2758	3222	3686	4150	4614	464
937	7396	7859	8323	8786	9249	463
938	972.2028	2491	2954	3417	3880	463
939	6656	7118	7581	8043	8506	462
940	973.1279	1741	2202	2664	3120	462
941	5896	6358	6819	7281	7742	461
942	974.0509	0970	1431	1892	2353	461
943	5117	5577	6038	6498	6959	460
944	9720	0180	0640	1100	1560	460
945	975.4318	4778	5237	5697	6156	459
946	8911	9370	9829	0288	0747	459
947	976.3500	3958	4417	4875	5334	458
948	8083	8541	9000	9458	9915	458
949	977.2662	3120	3577	4035	4492	457
950	7236	7693	8150	8607	9064	457
951	978.1805	2262	2718	3175	3631	457
952	6369	6826	7282	7738	8194	456
953	979.0929	1385	1840	2296	2751	456
954	5484	5939	6394	6849	7304	455
955	980.0034	0488	0943	1398	1852	455
956	4579	5033	5487	5942	6396	454
957	9119	9573	0027	0481	0934	454
958	981.3655	4108	4562	5015	5468	453
959	8186	8639	9092	9544	9997	453
960	982.2712	3165	3617	4069	4522	452
961	7234	7686	8138	8589	9041	452
962	983.1751	2202	2654	3105	3556	451
963	6263	6714	7165	7616	8066	451
964	984.0770	1221	1671	2122	2572	450
965	5273	5723	6173	6623	7073	450
966	9771	0221	0670	1120	1569	449

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930	7164	7630	8097	8564	9030	467
931	969.1829	2295	2761	3227	3693	466
932	6488	6954	7420	7885	8351	466
933	970.1143	1608	2074	2539	3004	465
934	5793	6258	6722	7187	7652	465
935	971.0438	0902	1366	1830	2294	464
936	5078	5542	6005	6469	6932	464
937	9713	0176	0639	1102	1565	463
938	972.4343	4805	5268	5731	6193	463
939	8968	9430	9892	0354	0816	462
940	973.3588	4050	4511	4973	5435	462
941	8203	8664	9126	9587	0048	461
942	974.2814	3274	3735	4196	4656	461
943	7419	7879	8340	8800	9260	460
944	975.2020	2479	2939	3399	3858	460
945	6615	7075	7534	7993	8452	459
946	976.1206	1665	2124	2582	3041	459
947	5792	6251	6709	7167	7625	458
948	977.0373	0831	1289	1747	2204	458
949	4950	5407	5864	6322	6779	457
950	9521	9978	0435	0892	1348	457
951	978.4088	4544	5001	5457	5913	457
952	8650	9106	9562	0017	0473	456
953	979.3205	3662	4118	4573	5028	456
954	7759	8214	8669	9124	9579	455
955	980.2307	2761	3216	3670	4125	455
956	6850	7304	7758	8212	8666	454
957	981.1388	1841	2295	2748	3202	454
958	5921	6374	6827	7280	7733	453
959	982.0450	0902	1355	1807	2260	453
960	4974	5426	5878	6330	6782	452
961	983.9493	9945	0396	0848	1299	452
962	4007	4459	4910	5361	5812	451
963	8517	8968	9419	9869	0320	451
964	984.3022	3473	3923	4373	4823	450
965	7523	7973	8422	8872	9322	450
966	985.2019	2468	2917	3366	3816	449

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968	8754	9202	9651	0099	0548	449
969	986.3238	3686	4134	4582	5030	448
970	7717	8165	8613	9060	9508	448
971	987.2192	2640	3087	3534	3981	447
972	6663	7109	7556	8003	8450	447
973	988.1128	1575	2021	2467	2913	446
974	5590	6035	6481	6927	7373	446
975	989.0046	0492	0937	1382	1828	445
976	4498	4943	5388	5833	6278	445
977	8946	9390	9835	0279	0723	444
978	990.3389	3833	4277	4721	5164	444
979	7827	8271	8714	9158	9601	443
980	991.2261	2704	3147	3590	4033	443
981	6690	7133	7575	8018	8461	442
982	992.1115	1557	1999	2441	2884	442
983	5535	5977	6419	6860	7302	442
984	9951	0392	0834	1275	1716	441
985	993.4362	4803	5244	5685	6126	441
986	8769	9209	9650	0090	0531	440
987	994.3171	3611	4051	4491	4931	440
988	7569	8009	8448	8888	9327	440
989	995.1963	2402	2841	3280	3719	439
990	6352	6791	7229	7668	8106	439
991	996.0736	1175	1613	2051	2489	438
992	5117	5554	5992	6430	6867	438
993	9492	9930	0367	0804	1241	437
994	997.3864	4301	4737	5174	5611	437
995	8231	8667	9104	9540	9976	436
996	998.2593	3029	3465	3901	4337	436
997	6951	7387	7823	8258	8694	435
998	999.1305	1740	2176	2611	3046	435
999	5655	6089	6524	6959	7393	434

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968	986.0996	1445	1893	2341	2790	449
969	5478	5926	6374	6822	7270	448
970	9955	0403	0850	1298	1745	448
971	987.4428	4875	5322	5769	6216	447
972	8896	9343	9789	0236	0682	447
973	988.3360	3806	4252	4698	5144	446
974	7818	8264	8710	9155	9601	446
975	989.2273	2718	3163	3608	4053	445
976	6722	7167	7612	8057	8501	445
977	990.1168	1612	2056	2500	2944	444
978	5608	6052	6496	6940	7383	444
979	991.0044	0488	0931	1374	1818	443
980	4476	4919	5362	5805	6247	443
981	8903	9345	9788	0230	0673	442
982	992.3326	3768	4210	4651	5093	442
983	7744	8185	8627	9068	9510	442
984	993.2157	2598	3039	3480	3921	441
985	6566	7007	7447	7888	8329	441
986	994.0971	1411	1851	2291	2731	440
987	5371	5811	6250	6690	7130	440
988	9767	0206	0645	1085	1524	440
989	995.4158	4597	5036	5474	5913	439
990	8545	8983	9422	9860	0298	439
991	996.2927	3365	3803	4241	4679	438
992	7305	7743	8180	8618	9055	438
993	997.1679	2116	2553	2990	3427	437
994	6048	6484	6921	7358	7794	437
995	998.0413	0849	1285	1721	2157	436
996	4773	5209	5645	6080	6516	436
997	9129	9564	0000	0435	0870	435
998	999.3481	3916	4350	4785	5220	435
999	7828	8262	8697	9131	9566	434

N°	0	1	2	3	4	Dif
967	985.4265	4714	5163	5612	6061	449
968	8754	9202	9651	0099	0548	449
969	986.3238	3686	4134	4582	5030	448
970	7717	8165	8613	9060	9508	448
971	987.2192	2640	3087	3534	3981	447
972	6663	7109	7556	8003	8450	447
973	988.1128	1575	2021	2467	2913	446
974	5590	6035	6481	6927	7373	446
975	989.0046	0492	0937	1382	1828	445
976	4498	4943	5388	5833	6278	445
977	8946	9390	9835	0279	0723	444
978	990.3389	3833	4277	4721	5164	444
979	7827	8271	8714	9158	9601	443
980	991.2261	2704	3147	3590	4033	443
981	6690	7133	7575	8018	8461	442
982	992.1115	1557	1999	2441	2884	442
983	5535	5977	6419	6860	7302	442
984	9951	0392	0834	1275	1716	441
985	993.4362	4803	5244	5685	6126	441
986	8769	9209	9650	0090	0531	440
987	994.3171	3611	4051	4491	4931	440
988	7569	8009	8448	8888	9327	440
989	995.1963	2402	2841	3280	3719	439
990	6352	6791	7229	7668	8106	439
991	996.0736	1175	1613	2051	2489	438
992	5117	5554	5992	6430	6867	438
993	9492	9930	0367	0804	1241	437
994	997.3864	4301	4737	5174	5611	437
995	8231	8667	9104	9540	9976	436
996	998.2593	3029	3465	3901	4337	436
997	6951	7387	7823	8258	8694	435
998	999.1305	1740	2176	2611	3046	435
999	5655	6089	6524	6959	7393	434

o Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	7,9408419	9,9999835	7,9408584	12,0591416	30
31	7,9550819	9,9999823	7,9550996	12,0449004	29
32	7,9688698	9,9999812	7,9688886	12,0311114	28
33	7,9822334	9,9999800	7,9822534	12,0177466	27
34	7,9951980	9,9999788	7,9952192	12,0047808	26
35	8,0077867	9,9999775	8,0078092	11,9921908	25
36	8,0200207	9,9999762	8,0208445	11,9799555	24
37	8,0319195	9,9999748	8,0319446	11,9680554	23
38	8,0435009	9,9999735	8,0435274	11,9564726	22
39	8,0547814	9,9999721	8,0548094	11,9451906	21
40	8,0657763	9,9999706	8,0658057	11,9341943	20
41	8,0764997	9,9999691	8,0765306	11,9234694	19
42	8,0869646	9,9999676	8,0869970	11,9130030	18
43	8,0971832	9,9999660	8,0972172	11,9027828	17
44	8,1071669	9,9999644	8,1072025	11,8927975	16
45	8,1169262	9,9999628	8,1169634	11,8830366	15
46	8,1264710	9,9999611	8,1265099	11,8734901	14
47	8,1358104	9,9999594	8,1358510	11,8641490	13
48	8,1449532	9,9999577	8,1449956	11,8550044	12
49	8,1539075	9,9999559	8,1539516	11,8460484	11
50	8,1626808	9,9999541	8,1627267	11,8372733	10
51	8,1712804	9,9999522	8,1713282	11,8286718	9
52	8,1797129	9,9999503	8,1797626	11,8202374	8
53	8,1879848	9,9999484	8,1880364	11,8119636	7
54	8,1961020	9,9999464	8,1961556	11,8038444	6
55	8,2040703	9,9999444	8,2041259	11,7958741	5
56	8,2118949	9,9999424	8,2119526	11,7880474	4
57	8,2195811	9,9999403	8,2196408	11,7803592	3
58	8,2271335	9,9999382	8,2271953	11,7728047	2
59	8,2345568	9,9999360	8,2346208	11,7653792	1
60	8,2418553	9,9999338	8,2419215	11,7580785	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

89 Degrees.

1 Degree.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	8,2418553	9,9999338	8,2419215	11,7580785	60
1	8,2490332	9,9999316	8,2491015	11,7508985	59
2	8,2560943	9,9999294	8,2561649	11,7438351	58
3	8,2630424	9,9999271	8,2631153	11,7368847	57
4	8,2698810	9,9999247	8,2699563	11,7300437	56
5	8,2766136	9,9999224	8,2766912	11,7233088	55
6	8,2832434	9,9999200	8,2833234	11,7166766	54
7	8,2897734	9,9999175	8,2898559	11,7101441	53
8	8,2962067	9,9999150	8,2962917	11,7037083	52
9	8,3025460	9,9999125	8,3026335	11,6973665	51
10	8,3087941	9,9999100	8,3088842	11,6911158	50
11	8,3149536	9,9999074	8,3150462	11,6849538	49
12	8,3210269	9,9999047	8,3211221	11,6788779	48
13	8,3270163	9,9999021	8,3271143	11,6728857	47
14	8,3329243	9,9998994	8,3330249	11,6669751	46
15	8,3387529	9,9998966	8,3388563	11,6611437	45
16	8,3445043	9,9998939	8,3446105	11,6553895	44
17	8,3501805	9,9998911	8,3502895	11,6497105	43
18	8,3557835	9,9998882	8,3558953	11,6441047	42
19	8,3613150	9,9998853	8,3614297	11,6385703	41
20	8,3667769	9,9998824	8,3668945	11,6331055	40
21	8,3721710	9,9998794	8,3722915	11,6277085	39
22	8,3774988	9,9998764	8,3776223	11,6223777	38
23	8,3827620	9,9998734	8,3828886	11,6171114	37
24	8,3879622	9,9998703	8,3880918	11,6119082	36
25	8,3931008	9,9998672	8,3932336	11,6067664	35
26	8,3981793	9,9998641	8,3983152	11,6016848	34
27	8,4031990	9,9998609	8,4033381	11,5966619	33
28	8,4081614	9,9998577	8,4083037	11,5916963	32
29	8,4130676	9,9998544	8,4132132	11,5867868	31
30	8,4179190	9,9998512	8,4180679	11,5819321	30
	Sine Com.	Sine.	Tan. Com.	Tang.	Min.

88 Degrees.

1 Degree.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	8,4179190	9,9998512	8,4180679	11,5819321	30
31	8,4227168	9,9998478	8,4228690	11,5771310	29
32	8,4274621	9,9998445	8,4276176	11,5723824	28
33	8,4321561	9,9998411	8,4323150	11,5676850	27
34	8,4367999	9,9998367	8,4369622	11,5630378	26
35	8,4413944	9,9998342	8,4415603	11,5584397	25
36	8,4459409	9,9998306	8,4461103	11,5538897	24
37	8,4504402	9,9998271	8,4506131	11,5493869	23
38	8,4548934	9,9998235	8,4550699	11,5449301	22
39	8,4593013	9,9998199	8,4594814	11,5405186	21
40	8,4636649	9,9998162	8,4638486	11,5361514	20
41	8,4679850	9,9998125	8,4681725	11,5318275	19
42	8,4722626	9,9998088	8,4724538	11,5275462	18
43	8,4764984	9,9998050	8,4766933	12,5233067	17
44	8,4806932	9,9998012	8,4808920	11,5191080	16
45	8,4848479	9,9997974	8,4850505	11,5149495	15
46	8,4889632	9,9997935	8,4891696	11,5108304	14
47	8,4930398	9,9997896	8,4932502	11,5067498	13
48	8,4970784	9,9997856	8,4972928	11,5027072	12
49	8,5010798	9,9997817	8,5012982	11,4987018	11
50	8,5050447	9,9997776	8,5052671	11,4947329	10
51	8,5089736	9,9997736	8,5092001	11,4907999	9
52	8,5128673	9,9997695	8,5130978	11,4869022	8
53	8,5167264	9,9997653	8,5169610	11,4830390	7
54	8,5205514	9,9997612	8,5207902	11,4792098	6
55	8,5243430	9,9997570	8,5245860	11,4754140	5
56	8,5281017	9,9997527	8,5283490	11,4716510	4
57	8,5318281	9,9997484	8,5320797	11,4679203	3
58	8,5355228	9,9997441	8,5357787	11,4642213	2
59	8,5391863	9,9997398	8,5394466	11,4605534	1
60	8,5428192	9,9997354	8,5430838	11,4569162	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

88 Degrees.

2 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	8,5428192	9,9997354	8,5430838	11,4569162	60
1	8,5464218	9,9997309	8,5466909	11,4533091	59
2	8,5499948	9,9997265	8,5502683	11,4497317	58
3	8,5535386	9,9997220	8,5538166	11,4461834	57
4	8,5570536	9,9997174	8,5573362	11,4426638	56
5	8,5605404	9,9997128	8,5608276	11,4391724	55
6	8,5639994	9,9997082	8,5642912	11,4357088	54
7	8,5674310	9,9997036	8,5677275	11,4322725	53
8	8,5708357	9,9996989	8,5711368	11,4288632	52
9	8,5742139	9,9996942	8,5745197	11,4254803	51
10	8,5775660	9,9996894	8,5778766	11,4221234	50
11	8,5808923	9,9996846	8,5812077	11,4187923	49
12	8,5841933	9,9996798	8,5845136	11,4154864	48
13	8,5874694	9,9996749	8,5877945	11,4122055	47
14	8,5907209	9,9996700	8,5910509	11,4089491	46
15	8,593948[illegible]	9,9996650	8,5942832	11,4057168	45
16	8,5971517	9,9996601	8,5974917	11,4025083	44
17	8,6003317	9,9996550	8,6006767	11,3993233	43
18	8,6034856	9,9996500	8,6038[illegible]86	11,3961614	42
19	8,6066220	9,9996449	8,6069777	11,3930223	41
20	8,609734	9,9996398	8,6100943	11,3899057	40
21	8,612823[illegible]	9,9996346	8,6131889	11,3868111	39
22	8,6158910	9,9996294	8,6162616	11,3837384	38
23	8,6189369	9,9996242	8,6193127	11,3806873	37
24	8,6219616	9,9996189	8,6223427	11,3776573	36
25	8,6249653	9,9996136	8,6253518	11,3746482	35
26	8,6279484	9,9996082	8,6283402	11,3716598	34
27	8,6309111	9,9996028	8,6313083	11,3686917	33
28	8,6338537	9,9995974	8,6342563	11,3657437	32
29	8,6367764	9,9995919	8,6371845	11,3628155	31
30	8,6396796	9,9995865	8,6400931	11,3599069	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

87 Degrees.

2 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	8,6396796	9,9995865	8,6400934	11,3599069	30
31	8,6425634	9,9995809	8,6429825	11,3570175	29
32	8,6454282	9,9995753	8,6458528	11,3541472	28
33	8,6482742	9,9995697	8,6487044	11,3512956	27
34	8,6511016	9,9995641	8,6515375	11,3484625	26
35	8,6539107	9,9995584	8,6543522	11,3456478	25
36	8,6567017	9,9995527	8,6571490	11,3428510	24
37	8,6594748	9,9995469	8,6599279	11,3400721	23
38	8,6622303	9,9995411	8,6626891	11,3373109	22
39	8,6649684	9,9995353	8,6654331	11,3345669	21
40	8,6676893	9,9995295	8,6681598	11,3318402	20
41	8,6703932	9,9995236	8,6708697	11,3291303	19
42	8,6730804	9,9995176	8,6735628	11,3264372	18
43	8,6757510	9,9995116	8,6762393	11,3237607	17
44	8,6784052	9,9995056	8,6788996	11,3211004	16
45	8,6810433	9,9994996	8,6815437	11,3184563	15
46	8,6836654	9,9994935	8,6841719	11,3158281	14
47	8,6862718	9,9994874	8,6867844	11,3132156	13
48	8,6888625	9,9994812	8,6893813	11,3106187	12
49	8,6914379	9,9994750	8,6919629	11,3080371	11
50	8,6939980	9,9994688	8,6945292	11,3054708	10
51	8,6965431	9,9994625	8,6970806	11,3029194	9
52	8,6990734	9,9994562	8,6996172	11,3003828	8
53	8,7015889	9,9994498	8,7021390	11,2978610	7
54	8,7040899	9,9994435	8,7046465	11,2953535	6
55	8,7065766	9,9994370	8,7071395	11,2928605	5
56	8,7090490	9,9994306	8,7096185	11,2903815	4
57	8,7115075	9,9994241	8,7120834	11,2879166	3
58	8,7139520	9,9994176	8,7145345	11,2854655	2
59	8,7163829	9,9994110	8,7169719	11,2830281	1
60	8,7188002	9,9994044	8,7193958	11,2806042	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

87 Degrees.

3 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	8,7188002	9,9994044	8,7193958	11,2806042	60
1	8,7212040	9,9993978	8,7218063	11,2781937	59
2	8,7235946	9,9993911	8,7242035	11,2757965	58
3	8,7259721	9,9993844	8,7265877	11,2734123	57
4	8,7283366	9,9993776	8,7289589	11,2710411	56
5	8,7306882	9,9993708	8,7313174	11,2686826	55
6	8,7330272	9,9993640	8,7336631	11,2663369	54
7	8,7353535	9,9993572	8,7359964	11,2640036	53
8	8,7376675	9,9993503	8,7383172	11,2616828	52
9	8,7399691	9,9993433	8,7406258	11,2593742	51
10	8,7422586	9,9993364	8,7429222	11,2570778	50
11	8,7445360	9,9993293	8,7452067	11,2547933	49
12	8,7468015	9,9993223	8,7474792	11,2525208	48
13	8,7490553	9,9993152	8,7497400	11,2502600	47
14	8,7512973	9,9993081	8,7519892	11,2480108	46
15	8,7535278	9,9993009	8,7542269	11,2457731	45
16	8,7557469	9,9992938	8,7564531	11,2435469	44
17	8,7579546	9,9992865	8,7586681	11,2413319	43
18	8,7601512	9,9992793	8,7608719	11,2391281	42
19	8,7623366	9,9992720	8,7630647	11,2369353	41
20	8,7645111	9,9992646	8,7652465	11,2347535	40
21	8,7666747	9,9992572	8,7674175	11,2325825	39
22	8,7688275	9,9992498	8,7695777	11,2304223	38
23	8,7709697	9,9992424	8,7717274	11,2282726	37
24	8,7731014	9,9992349	8,7738665	11,2261335	36
25	8,7752226	9,9992274	8,7759952	11,2240048	35
26	8,7773334	9,9992198	8,7781136	11,2218864	34
27	8,7794340	9,9992122	8,7802218	11,2197782	33
28	8,7815244	9,9992046	8,7823199	11,2176801	32
29	8,7836048	9,9991969	8,7844079	11,2155921	31
30	8,7856753	9,9991892	8,7864861	11,2135139	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

86 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan.Comp.	
30	8,7856753	9,9991892	8,7864861	11,2135139	30
31	8,7877359	9,9991815	8,7885544	11,2114456	29
32	8,7897867	9,9991737	8,7906130	11,2093870	28
33	8,7918278	9,9991659	8,7926620	11,2073380	27
34	8,7938594	9,9991580	8,7947014	11,2052986	26
35	8,7958814	9,9991501	8,7967313	11,2032687	25
36	8,7978941	9,9991422	8,7987519	11,2012481	24
37	8,7998974	9,9991342	8,8007632	11,1992368	23
38	8,8018915	9,9991262	8,8927653	11,1972347	22
39	8,8038764	9,9991182	8,8047583	11,1952417	21
40	8,8058523	9,9991101	8,8067422	11,1932578	20
41	8,8078192	9,9991020	8,8087172	11,1912828	19
42	8,8097772	9,9990938	8,8106834	11,1893166	18
43	8,8117264	9,9990856	8,8126407	11,1873593	17
44	8,8136668	9,9990774	8,8145894	11,1854106	16
45	8,8155985	9,9990691	8,8165294	11,1834706	15
46	8,8175217	9,9990608	8,8184608	11,1815392	14
47	8,8194363	9,9990525	8,8203838	11,1796162	13
48	8,8213425	9,9990441	8,8222984	11,1777016	12
49	8,8232404	9,9990357	8,8242046	11,1757954	11
50	8,8251299	9,9990273	8,8261026	11,1738974	10
51	8,8270112	9,9990188	8,8279924	11,1720076	9
52	8,8288844	9,9990103	8,8298741	11,1701259	8
53	8,8307495	9,9990017	8,8317478	11,1682522	7
54	8,8326066	9,9989931	8,8336134	11,1663866	6
55	8,8344557	9,9989845	8,8354712	11,1645288	5
56	8,8362969	9,9989758	8,8373211	11,1626789	4
57	8,8381304	9,9989671	8,8391633	11,1608367	3
58	8,8399561	9,9989584	8,8409977	11,1590023	2
59	8,8417741	9,9989496	8,8428245	11,1571755	1
60	8,8435845	9,9989408	8,8446437	11,1553563	0
	Sine Com.	Sine	Tan. Com.	Tangent.	Min.

4 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan.Comp.	
0	8,8435845	9,9989408	8,8446437	11,1553563	60
1	8,8453874	9,9989319	8,8464554	11,1535446	59
2	8,8471827	9,9989230	8,8482597	11,1517403	58
3	8,8489707	9,9989141	8,8500566	11,1499434	57
4	8,8507512	9,9989052	8,8518461	11,1481539	56
5	8,8525245	9,9988962	8,8536283	11,1463717	55
6	8,8542905	9,9988871	8,8554034	11,1445966	54
7	8,8560493	9,9988780	8,8571713	11,1428287	53
8	8,8578010	9,9988689	8,8589321	11,1410679	52
9	8,8595457	9,9988598	8,8606859	11,1393141	51
10	8,8612833	9,9988506	8,8624327	11,1375673	50
11	8,8630139	9,9988414	8,8641725	11,1358275	49
12	8,8647376	9,9988321	8,8659055	11,1340945	48
13	8,8664545	9,9988228	8,8676317	11,1323683	47
14	8,8681646	9,9988135	8,8693511	11,1306489	46
15	8,8698680	9,9988041	8,8710638	11,1289362	45
16	8,8715646	9,9987947	8,8727699	11,1272301	44
17	8,8732546	9,9987853	8,8744694	11,1255306	43
18	8,8749381	9,9987758	8,8761623	11,1238377	42
19	8,8766150	9,9987663	8,8778487	11,1221513	41
20	8,8782854	9,9987567	8,8795286	11,1204714	40
21	8,8799493	9,9987471	8,8812022	11,1187978	39
22	8,8816069	9,9987375	8,8828694	11,1171306	38
23	8,8832581	9,9987278	8,8845303	11,1154697	37
24	8,8849031	9,9987181	8,8861850	11,1138150	36
25	8,8865418	9,9987084	8,8878334	11,1121666	35
26	8,8881743	9,9986986	8,8894757	11,1105243	34
27	8,8898007	9,9986888	8,8911119	11,1088881	33
28	8,8914209	9,9986790	8,8927420	11,1072580	32
29	8,8930351	9,9986691	8,8943660	11,1056340	31
30	8,8946433	9,9986591	8,8959842	11,1040158	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

85 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan.Comp.	
30	8,8946433	9,9986591	8,8959842	11,1040158	30
31	8,8962455	9,9986492	8,8975963	11,1024037	29
32	8,8978418	9,9986392	8,8992026	11,1007974	28
33	8,8994322	9,9986292	8,9008030	11,0991970	27
34	8,9010168	9,9986191	8,9023977	11,0970023	26
35	8,9025955	9,9986090	8,9039866	11,0960134	25
36	8,9041685	9,9985988	8,9055697	11,0944303	24
37	8,9057358	9,9985886	8,9071472	11,0928528	23
38	8,9072975	9,9985784	8,9087190	11,0912810	22
39	8,9088535	9,9985682	8,9102853	11,0897147	21
40	8,9104039	9,9985579	8,9118460	11,0881540	20
41	8,9119487	9,9985475	8,9134012	11,0865988	19
42	8,9134881	9,9985372	8,9149509	11,0850491	18
43	8,9150219	9,9985268	8,9164952	11,0835048	17
44	8,9165504	9,9985163	8,9180340	11,0819660	16
45	8,9180734	9,9985058	8,9195675	11,0804325	15
46	8,9195911	9,9984953	8,9210957	11,0789043	14
47	8,9211034	9,9984848	8,9226186	11,0773814	13
48	8,9226105	9,9984742	8,9241363	11,0758637	12
49	8,9241123	9,9984636	8,9256487	11,0743513	11
50	8,9256069	9,9984529	8,9271560	11,0728440	10
51	8,9271003	9,9984422	8,9286581	11,0713419	9
52	8,9285866	9,9984315	8,9301552	11,0698448	8
53	8,9300678	9,9984207	8,9316471	11,0683529	7
54	8,9315439	9,9984099	8,9331340	11,0668660	6
55	8,9330150	9,9983990	8,9346160	11,0653840	5
56	8,9344811	9,9983881	8,9360929	11,0639071	4
57	8,9359422	9,9983772	8,9375650	11,0624350	3
58	8,9373983	9,9983663	8,9390321	11,0609679	2
59	8,9388496	9,9983553	8,9404944	11,0595056	1
60	8,9402960	9,9983442	8,9419518	11,0580482	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	8,9402960	9,9983442	8,9419518	11,0580482	60
1	8,9417376	9,9983332	8,9434044	11,0565956	59
2	8,9431743	9,9983220	8,9448523	11,0551477	58
3	8,9446063	9,9983109	8,9462954	11,0537046	57
4	8,9460335	9,9982997	8,9477338	11,0522662	56
5	8,9474561	9,9982885	8,9491676	11,0508324	55
6	8,9488739	9,9982772	8,9505967	11,0494033	54
7	8,9502871	9,9982660	8,9520211	11,0479789	53
8	8,9516957	9,9982546	8,9534410	11,0465590	52
9	8,9530996	9,9982433	8,9548564	11,0451436	51
10	8,9544991	9,9982318	8,9562672	11,0437328	50
11	8,9558940	9,9982204	8,9576735	11,0423265	49
12	8,9572843	9,9982089	8,9590754	11,0409246	48
13	8,9586703	9,9981974	8,9604728	11,0395272	47
14	8,9600517	9,9981859	8,9618659	11,0381341	46
15	8,9614288	9,9981743	8,9632545	11,0367455	45
16	8,9628014	9,9981629	8,9646388	11,0353612	44
17	8,9641697	9,9981510	8,9660188	11,0339812	43
18	8,9655337	9,9981393	8,9673944	11,0326056	42
19	8,9668934	9,9981275	8,9687658	11,0312342	41
20	8,9682487	9,9981158	8,9701330	11,0298670	40
21	8,9695999	9,9981040	8,9714959	11,0285041	39
22	8,9709468	9,9980921	8,9728547	11,0271453	38
23	8,9722895	9,9980802	8,9742092	11,0257908	37
24	8,9736280	9,9980683	8,9755597	11,0244403	36
25	8,9749624	9,9980563	8,9769060	11,0230940	35
26	8,9762926	9,9980443	8,9782483	11,0217517	34
27	8,9776188	9,9980323	8,9795865	11,0204135	33
28	8,9789408	9,9980202	8,9809206	11,0190794	32
29	8,9802589	9,9980081	8,9822507	11,0177493	31
30	8,9815729	9,9979960	8,9835769	11,0164231	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	8,9815729	9,9979960	8,9835769	11,0164231	30
31	8,9828829	9,9979838	8,9848991	11,0151009	29
32	8,9841889	9,9979716	8,9862173	11,0137827	28
33	8,9854910	9,9979593	8,9875317	11,0124683	27
34	8,9867891	9,9979470	8,9888421	11,0111579	26
35	8,9880834	9,9979347	8,9901487	11,0098513	25
36	8,9893737	9,9979223	8,9914514	11,0085486	24
37	8,9906602	9,9979099	8,9927503	11,0072497	23
38	8,9919429	9,9978975	8,9940454	11,0059546	22
39	8,9932217	9,9978850	8,9953367	11,0046633	21
40	8,9944968	9,9978725	8,9966243	11,0033757	20
41	8,9957681	9,9978599	8,9979081	11,0020919	19
42	8,9970356	9,9978473	8,9991883	11,0008117	18
43	8,9982994	9,9978347	9,0004647	10,9995353	17
44	8,9995595	9,9978220	9,0017375	10,9982625	16
45	9,0008160	9,9978093	9,0030066	10,9969934	15
46	9,0020687	9,9977966	9,0042721	10,9957279	14
47	9,0033179	9,9977838	9,0055340	10,9944660	13
48	9,0045634	9,9977710	9,0067924	10,9932076	12
49	9,0058053	9,9977582	9,0080471	10,9919529	11
50	9,0070436	9,9977453	9,0092984	10,9907016	10
51	9,0082784	9,9977323	9,0105461	10,9894539	9
52	9,0095096	9,9977194	9,0117903	10,9882097	8
53	9,0107374	9,9977064	9,0130310	10,9869690	7
54	9,0119616	9,9976933	9,0142682	10,9857318	6
55	9,0131823	9,9976803	9,0155021	10,9844979	5
56	9,0143996	9,9976672	9,0167325	10,9832675	4
57	9,0156135	9,9976540	9,0179594	10,9820406	3
58	9,0168239	9,9976408	9,0191831	10,9808169	2
59	9,0180309	9,9976276	9,0204033	10,9795967	1
60	9,0192346	9,9976143	9,0216202	10,9783798	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

6 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,0192346	9,9976143	9,0216202	10,9783798	60
1	9,0204348	9,9976011	9,0228338	10,9771662	59
2	9,0216318	9,9975877	9,0240441	10,9759559	58
3	9,0228254	9,9975743	9,0252510	10,9747490	57
4	9,0240157	9,9975609	9,0264548	10,9735432	56
5	9,0252027	9,9975475	9,0276552	10,9723448	55
6	9,0263865	9,9975340	9,0288524	10,9711476	54
7	9,0275669	9,9975205	9,0300464	10,9699536	53
8	9,0287442	9,9975069	9,0312373	10,9687627	52
9	9,0299182	9,9974933	9,0324249	10,9675751	51
10	9,0310890	9,9974797	9,0336093	10,9663907	50
11	9,0322567	9,9974660	9,0347906	10,9652094	49
12	9,0334212	9,9974523	9,0359688	10,9640312	48
13	9,0345825	9,9974386	9,0371439	10,9628561	47
14	9,0357407	9,9974248	9,0383159	10,9616841	46
15	9,0368958	9,9974110	9,0394848	10,9605152	45
16	9,0380477	9,9973971	9,0406506	10,9593494	44
17	9,0391966	9,9973833	9,0418134	10,9581866	43
18	9,0403424	9,9973693	9,0429731	10,9570269	42
19	9,0414852	9,9973554	9,0441299	10,9558701	41
20	9,0426249	9,9973414	9,0452836	10,9547164	40
21	9,0437617	9,9973273	9,0464343	10,9535657	39
22	9,0448954	9,9973132	9,0475821	10,9524179	38
23	9,0460261	9,9972991	9,0487270	10,9512730	37
24	9,0471538	9,9972850	9,0498689	10,9501311	36
25	9,0482786	9,9972708	9,0510078	10,9489922	35
26	9,0494005	9,9972566	9,0521439	10,9478561	34
27	9,0505194	9,9972423	9,0532771	10,9467229	33
28	9,0516354	9,9972280	9,0544074	10,9455926	32
29	9,0527485	9,9972137	9,0555349	10,9444651	31
30	9,0538588	9,9971993	9,0566595	10,9433405	30
	Sine Com.	Sine.	Tan. Com.	Tangen.	Min.

83 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,0538588	9,9971993	9,0566595	10,9433405	30
31	9,0549661	9,9971849	9,0577813	10,9422187	29
32	9,0560706	9,9971704	9,0589002	10,9410998	28
33	9,0571723	9,9971559	9,0600164	10,9399836	27
34	9,0582711	9,9971414	9,0611297	10,9388703	26
35	9,0593672	9,9971268	9,0622403	10,9377597	25
36	9,0604604	9,9971122	9,0633482	10,9366518	24
37	9,0615509	9,9970976	9,0644533	10,9355467	23
38	9,0626386	9,9970829	9,0655556	10,9344444	22
39	9,0637235	9,9970682	9,0666553	10,9333447	21
40	9,0648057	9,9970535	9,0677522	10,9322478	20
41	9,0658852	9,9970387	9,0688465	10,9311535	19
42	9,0669619	9,9970239	9,0699381	10,9300619	18
43	9,0680360	9,9970090	9,0710270	10,9289730	17
44	9,0691074	9,9969941	9,0721133	10,9278867	16
45	9,0701761	9,9969792	9,0731969	10,9268031	15
46	9,0712421	9,9969642	9,0742779	10,9257221	14
47	9,0723055	9,9969492	9,0753563	10,9246437	13
48	9,0733663	9,9969342	9,0764321	10,9235679	12
49	9,0744244	9,9969191	9,0775053	10,9224947	11
50	9,0754799	9,9969040	9,0785760	10,9214240	10
51	9,0765329	9,9968888	9,0796441	10,9203559	9
52	9,0775832	9,9968736	9,0807096	10,9192904	8
53	9,0786310	9,9968584	9,0817726	10,9182274	7
54	9,0796762	9,9968431	9,0828331	10,9171669	6
55	9,0807189	9,9968278	9,0838911	10,9161089	5
56	9,0817590	9,9968125	9,0849466	10,9150534	4
57	9,0827966	9,9967971	9,0859996	10,9140004	3
58	9,0838317	9,9967817	9,0870501	10,9129499	2
59	9,0848643	9,9967662	9,0880981	10,9119019	1
60	9,0858945	9,9967507	9,0891438	10,9108562	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

7 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,0858945	9,9967507	9,0891438	10,9108562	60
1	9,0869221	9,9967352	9,0901869	10,9098131	59
2	9,0879473	9,9967196	9,0912277	10,9087723	58
3	9,0889700	9,9967040	9,0922660	10,9077340	57
4	9,0899903	9,9966884	9,0933020	10,9066980	56
5	9,0910082	9,9966727	9,0943355	10,9056645	55
6	9,0920237	9,9966570	9,0953667	10,9046333	54
7	9,0930367	9,9966412	9,0963955	10,9036045	53
8	9,0940474	9,9966254	9,0974219	10,9025781	52
9	9,0950556	9,9966096	9,0984460	10,9015540	51
10	9,0960615	9,9965937	9,0994678	10,9005322	50
11	9,0970651	9,9965778	9,1004872	10,8995128	49
12	9,0980662	9,9965619	9,1015044	10,8984956	48
13	9,0990651	9,9965459	9,1025192	10,8974808	47
14	9,1000616	9,9965299	9,1035317	10,8964683	46
15	9,1010558	9,9965138	9,1045420	10,8954580	45
16	9,1020477	9,9964977	9,1055500	10,8944500	44
17	9,1030373	9,9964816	9,1065557	10,8934443	43
18	9,1040246	9,9964655	9,1075591	10,8924409	42
19	9,1050096	9,9964493	9,1085004	10,8914396	41
20	9,1059924	9,9964330	9,1095594	10,8904406	40
21	9,1069729	9,9964167	9,1105562	10,8894438	39
22	9,1079512	9,9964004	9,1115508	10,8884492	38
23	9,1089272	9,9963841	9,1125431	10,8874569	37
24	9,1099010	9,9963677	9,1135333	10,8864667	36
25	9,1108726	9,9963513	9,1145213	10,8854787	35
26	9,1118420	9,9963348	9,1155072	10,8844928	34
27	9,1128092	9,9963183	9,1164909	10,8835091	33
28	9,1137742	9,9963018	9,1174724	10,8825276	32
29	9,1147370	9,9962852	9,1184518	10,8815482	31
30	9,1156977	9,9962686	9,1194291	10,8805709	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

82 Degrees.

7 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,1156977	9,9962686	9,1194291	10,8805709	30
31	9,1166562	9,9962519	9,1204043	10,8795957	29
32	9,1176125	9,9962352	9,1213773	10,8786227	28
33	9,1185667	9,9962185	9,1223482	10,8776518	27
34	9,1195188	9,9962017	9,1233171	10,8766829	26
35	9,1204688	9,9961849	9,1242839	10,8757161	25
36	9,1214167	9,9961681	9,1252486	10,8747514	24
37	9,1223624	9,9961512	9,1262112	10,8737888	23
38	9,1233061	9,9961343	9,1271718	10,8728282	22
39	9,1242477	9,9961174	9,1281303	10,8718697	21
40	9,1251872	9,9961004	9,1290868	10,8709132	20
41	9,1261246	9,9960834	9,1300413	10,8699587	19
42	9,1270600	9,9960663	9,1309937	10,8690063	18
43	9,1279934	9,9960492	9,1319442	10,8680558	17
44	9,1289247	9,9960321	9,1328926	10,8671074	16
45	9,1298539	9,9960149	9,1338391	10,8661609	15
46	9,1307812	9,9959977	9,1347835	10,8652165	14
47	9,1317064	9,9959804	9,1357260	10,8642740	13
48	9,1326297	9,9959631	9,1366665	10,8633335	12
49	9,1335509	9,9959458	9,1376051	10,8623949	11
50	9,1344702	9,9959284	9,1385417	10,8614583	10
51	9,1353875	9,9959111	9,1394764	10,8605236	9
52	9,1363028	9,9958936	9,1404092	10,8595908	8
53	9,1372161	9,9958761	9,1413400	10,8586600	7
54	9,1381275	9,9958586	9,1422689	10,8577311	6
55	9,1390370	9,9958411	9,1431959	10,8568041	5
56	9,1399445	9,9958235	9,1441210	10,8558790	4
57	9,1408501	9,9958059	9,1450442	10,8549558	3
58	9,1417537	9,9957882	9,1459655	10,8540345	2
59	9,1426555	9,9957705	9,1468849	10,8531151	1
60	9,1435553	9,9957528	9,1478025	10,8521975	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

82 Degrees,

8 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan.Comp.	
0	9,1435553	9,9957528	9,1478025	10,8521975	60
1	9,1444532	9,9957350	9,1487182	10,8512818	59
2	9,1453493	9,9957172	9,1496321	10,8503679	58
3	9,1462435	9,9956993	9,1505441	10,8494559	57
4	9,1471358	9,9996815	9,1514543	10,8485457	56
5	9,1480262	9,9956635	9,1523627	10,8476373	55
6	9,1489148	9,9956456	9,1532692	10,8467308	54
7	9,1498015	9,9956276	9,1541739	10,8458261	53
8	9,1506864	9,9996095	9,1550769	10,8449231	52
9	9,1515694	9,9955915	9,1559780	10,8440220	51
10	9,1524507	9,9955734	9,1568773	10,8431227	50
11	9,1533301	9,9955552	9,1577748	10,8422252	49
12	9,1542076	9,9955370	9,1586706	10,8413294	48
13	9,1550834	9,9955188	9,1595646	10,8404354	47
14	9,1559574	9,9955505	9,1604569	10,8395431	46
15	9,1568296	9,9954822	9,1613473	10,8386527	45
16	9,1577000	9,9954639	9,1622361	10,8377639	44
17	9,1585686	9,9954455	9,1631231	10,8368769	43
18	9,1594354	9,9954271	9,1640083	10,8359917	42
19	9,1603005	9,9954087	9,1648919	10,8351081	41
20	9,1611639	9,9953902	9,1657737	10,8342263	40
21	9,1620254	9,9953717	9,1666538	10,8333462	39
22	9,1628853	9,9953531	9,1675322	10,8324678	38
23	9,1637434	9,9953345	9,1684089	10,8315911	37
24	9,1645998	9,9953159	9,1692839	10,8307161	36
25	9,1654544	9,9952972	9,1701572	10,8298428	35
26	9,1663074	9,9952785	9,1710289	10,8289711	34
27	9,1671586	9,9952597	9,1718989	10,8281011	33
28	9,1680081	9,9952409	9,1727672	10,8272328	32
29	9,1688559	9,9952221	9,1736338	10,8263662	31
30	9,1697021	9,9952033	9,1744988	10,8255012	30
	Sine Com.	Sine.	Tan. Com.	Tang.	Min.

81 Degrees.

8 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,1697021	9,9952033	9,1744988	10,8255012	30
31	9,1705465	9,9951844	9,1753622	10,8246378	29
32	9,1713893	9,9951654	9,1762239	10,8237761	28
33	9,1722305	9,9951464	9,1770840	10,8229160	27
34	9,1730699	9,9951274	9,1779425	10,8229575	26
35	9,1739077	9,9951084	9,1787993	10,8212007	25
36	9,1747439	9,9950893	9,1796546	10,8203454	24
37	9,1755784	9,9950702	9,1805082	10,8194918	23
38	9,1764112	9,9950510	9,1813602	10,8186398	22
39	9,1772425	9,9950318	9,1822106	10,8177894	21
40	9,1780721	9,9950126	9,1830595	10,8169405	20
41	9,1789001	9,9949933	9,1839068	10,8160932	19
42	9,1797265	9,9949740	9,1847525	10,8152475	18
43	9,1805512	9,9949546	9,1855966	10,8144034	17
44	9,1813744	9,9949352	9,1864392	10,8135608	16
45	9,1821960	9,9949158	9,1872802	10,8127198	15
46	9,1830160	9,9948964	9,1881196	10,8118804	14
47	9,1838344	9,9948769	9,1889575	10,8110425	13
48	9,1846512	9,9948573	9,1897939	10,8102061	12
49	9,1854665	9,9948377	9,1906287	10,8093713	11
50	9,1862802	9,9948181	9,1914621	10,8085379	10
51	9,1870923	9,9947985	9,1922939	10,8077061	9
52	9,1879029	9,9947788	9,1931241	10,8068759	8
53	9,1887120	9,9947591	9,1939529	10,8060471	7
54	9,1895195	9,9947393	9,1947802	10,8052198	6
55	9,1903254	9,9947195	9,1956059	10,8043941	5
56	9,1911299	9,9946997	9,1964302	10,8035698	4
57	9,1919328	9,9946798	9,1972530	10,8027470	3
58	9,1927342	9,9946599	9,1980743	10,8019257	2
59	9,1935341	9,9946399	9,1988941	10,8011059	1
60	9,1943324	9,9946199	9,1997125	10,8002875	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

81 Degrees.

9 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,1943324	9,9946199	9,1997125	10,8002875	60
1	9,1951293	9,9945999	9,2005294	10,7994706	59
2	9,1959247	9,9945798	9,2013449	10,7986551	58
3	9,1967186	9,9945597	9,2021588	10,7978412	57
4	9,1975110	9,9945396	9,2029714	10,7970286	56
5	9,1983019	9,9945194	9,2037825	10,7962175	55
6	9,1990913	9,9944992	9,2045922	10,7954078	54
7	9,1998793	9,9944789	9,2054004	10,7945996	53
8	9,2006658	9,9944587	9,2062072	10,7937928	52
9	9,2014509	9,9944383	9,2070126	10,7929874	51
10	9,2022345	9,9944180	9,2078165	10,7921835	50
11	9,2030167	9,9943975	9,2086191	10,7913809	49
12	9,2037974	9,9943771	9,2094203	10,7905797	48
13	9,2045766	9,9943566	9,2102200	10,7897800	47
14	9,2053545	9,9943361	9,2110184	10,7889816	46
15	9,2061309	9,9943156	9,2118153	10,7881847	45
16	9,2069059	9,9942950	9,2126109	10,7873891	44
17	9,2076795	9,9942743	9,2134051	10,7865949	43
18	9,2084516	9,9942537	9,2141980	10,7858020	42
19	9,2092224	9,9942330	9,2149894	10,7850106	41
20	9,2099917	9,9942122	9,2157795	10,7842205	40
21	9,2107597	9,9941914	9,2165683	10,7834317	39
22	9,2115263	9,9941706	9,2173556	10,7826444	38
23	9,2122914	9,9941498	9,2181417	10,7818583	37
24	9,2130552	9,9941289	9,2189264	10,7810736	36
25	9,2138176	9,9941079	9,2197097	10,7802903	35
26	9,2145787	9,9940870	9,2204917	10,7795083	34
27	9,2153384	9,9940659	9,2212724	10,7787276	33
28	9,2160967	9,9940449	9,2220518	10,7779482	32
29	9,2168536	9,9940238	9,2228298	10,7771702	31
30	9,2176092	9,9940027	9,2236065	10,7763935	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

80 Degrees.

9 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,2176092	9,9940027	9,2236065	10,7763935	30
31	9,2183635	9,9939815	9,2243819	10,7756181	29
32	9,2191164	9,9939603	9,2251561	10,7748439	28
33	9,2198680	9,9939391	9,2259289	10,7740711	27
34	9,2206182	9,9939178	9,2267004	10,7732996	26
35	9,2213671	9,9938965	9,2274706	10,7725294	25
36	9,2221147	9,9938752	9,2282395	10,7717605	24
37	9,2228609	9,9938538	9,2290071	10,7709929	23
38	9,2236059	9,9938324	9,2297735	10,7702265	22
39	9,2243495	9,9938109	9,2305386	10,7694614	21
40	9,2250918	9,9937894	9,2313024	10,7686976	20
41	9,2258328	9,9937679	9,2320650	10,7679350	19
42	9,2265725	9,9937463	9,2328262	10,7671738	18
43	9,2273110	9,9937247	9,2335863	10,7664137	17
44	9,2280481	9,9937030	9,2343451	10,7656549	16
45	9,2287839	9,9936813	9,2351026	10,7648974	15
46	9,2295185	9,9936596	9,2358589	10,7641411	14
47	9,2302518	9,9936378	9,2366139	10,7633861	13
48	9,2309838	9,9936160	9,2373678	10,7626322	12
49	9,2317145	9,9935942	9,2381203	10,7618797	11
50	9,2324440	9,9935723	9,2388717	10,7611283	10
51	9,2331722	9,9935504	9,2396218	10,7603782	9
52	9,2338992	9,9935285	9,2403708	10,7596292	8
53	9,2346249	9,9935065	9,2411185	10,7588815	7
54	9,2353494	9,9934844	9,2418650	10,7581350	6
55	9,2360726	9,9934624	9,2426103	10,7573897	5
56	9,2367946	9,9934403	9,2433543	10,7566457	4
57	9,2375153	9,9934181	9,2440972	10,7559028	3
58	9,2382349	9,9933959	9,2448389	10,7551611	2
59	9,2389532	9,9933737	9,2455794	10,7544206	1
60	9,2396702	9,9933515	9,2463188	10,7536812	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

80 Degrees.

10 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,2396702	9,9933515	9,2463188	10,7536812	60
1	9,2403861	9,9933292	9,2470569	10,7529431	59
2	9,2411007	9,9933068	9,2477939	10,7522061	58
3	9,2418141	9,9932845	9,2485297	10,7514703	57
4	9,2425264	9,9932621	9,2492643	10,7507357	56
5	9,2432374	9,9932396	9,2499978	10,7500022	55
6	9,2439472	9,9932171	9,2507301	10,7492699	54
7	9,2446558	9,9931946	9,2514612	10,7485388	53
8	9,2453632	9,9931720	9,2521912	10,7478088	52
9	9,2460695	9,9931494	9,2529200	10,7470800	51
10	9,2467746	9,9931268	9,2536477	10,7463523	50
11	9,2474784	9,9931041	9,2543743	10,7456257	49
12	9,2481811	9,9930814	9,2550997	10,7449003	48
13	9,2488827	9,9930587	9,2558240	10,7441760	47
14	9,2495830	9,9930359	9,2565472	10,7434528	46
15	9,2502822	9,9930131	9,2572692	10,7427308	45
16	9,2509803	9,9929902	9,2579901	10,7420099	44
17	9,2516772	9,9929673	9,2587099	10,7412901	43
18	9,2523729	9,9929444	9,2594285	10,7405715	42
19	9,2530675	9,9929214	9,2601461	10,7398539	41
20	9,2537609	9,9928984	9,2608625	10,7391375	40
21	9,2544532	9,9928753	9,2615779	10,7384221	39
22	9,2551444	9,9928522	9,2622921	10,7377079	38
23	9,2558344	9,9928291	9,2630053	10,7369947	37
24	9,2565233	9,9928059	9,2637173	10,7362827	36
25	9,2572110	9,9927827	9,2644283	10,7355717	35
26	9,2578977	9,9927595	9,2651382	10,7348618	34
27	9,2585832	9,9927362	9,2658470	10,7341530	33
28	9,2592676	9,9927129	9,2665547	10,7334453	32
29	9,2599509	9,9926895	9,2672613	10,7327387	31
30	9,2606330	9,9926661	9,2679669	10,7320331	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

79 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,2606330	9,9926661	9,2679669	10,7320331	30
31	9,2613141	9,9926427	9,2686714	10,7313286	29
32	9,2619941	9,9926192	9,2693749	10,7306251	28
33	9,2626729	9,9925957	9,2700772	10,7299228	27
34	9,2633507	9,9925722	9,2707786	10,7292214	26
35	9,2640274	9,9925486	9,2714788	10,7285212	25
36	9,2647030	9,9925250	9,2721780	10,7278220	24
37	9,2653775	9,9925013	9,2728762	10,7271238	23
38	9,2660509	9,9924776	9,2735733	10,7264267	22
39	9,2667232	9,9924539	9,2742694	10,7257306	21
40	9,2673945	9,9924301	9,2749644	10,7250356	20
41	9,2680647	9,9924063	9,2756584	10,7243416	19
42	9,2687338	9,9923824	9,2763514	10,7236486	18
43	9,2694019	9,9923585	9,2770434	10,7229566	17
44	9,2700689	9,9923346	9,2777343	10,7222657	16
45	9,2707348	9,9923106	9,2784242	10,7215758	15
46	9,2713997	9,9922866	9,2791131	10,7208869	14
47	9,2720635	9,9922626	9,2798009	10,7201991	13
48	9,2727263	9,9922385	9,2804878	10,7195122	12
49	9,2733880	9,9922144	9,2811736	10,7188264	11
50	9,2740487	9,9921902	9,2818585	10,7181415	10
51	9,2747083	9,9921660	9,2825423	10,7174577	9
52	9,2753669	9,9921418	9,2832251	10,7167749	8
53	9,2760245	9,9921175	9,2839070	10,7160930	7
54	9,2766811	9,9920932	9,2845878	10,7154122	6
55	9,2773366	9,9920689	9,2852677	10,7147323	5
56	9,2779911	9,9920445	9,2859466	10,7140534	4
57	9,2786445	9,9920201	9,2866245	10,7133755	3
58	9,2792970	9,9919956	9,2873014	10,7126986	2
59	9,2799484	9,9919711	9,2879773	10,7120227	1
60	9,2805988	9,9919466	9,2886523	10,7113477	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

11 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,2805988	9,9919466	9,2886523	10,7113477	60
1	9,2812483	9,9919220	9,2893263	10,7106737	59
2	9,2818967	9,9918974	9,2899993	10,7100007	58
3	9,2825441	9,9918727	9,2906713	10,7093287	57
4	9,2831905	9,9918480	9,2913424	10,7086576	56
5	9,2838359	9,9918233	9,2920126	10,7079874	55
6	9,2844803	9,9917986	9,2926817	10,7073183	54
7	9,2851237	9,9917737	9,2933500	10,7066500	53
8	9,2857661	9,9917489	9,2940172	10,7059828	52
9	9,2864076	9,9917240	9,2946836	10,7053164	51
10	9,2870480	9,9916991	9,2953489	10,7046511	50
11	9,2876875	9,9916741	9,2960134	10,7039866	49
12	9,2883260	9,9916492	9,2966769	10,7033231	48
13	9,2889636	9,9916241	9,2973395	10,7026605	47
14	9,2896001	9,9915990	9,2980011	10,7019989	46
15	9,2902357	9,9915739	9,2986618	10,7013382	45
16	9,2908704	9,9915488	9,2993216	10,7006784	44
17	9,2915040	9,9915236	9,2999804	10,7000196	43
18	9,2921367	9,9914984	9,3006383	10,6993617	42
19	9,2927685	9,9914731	9,3012954	10,6987046	41
20	9,2933993	9,9914478	9,3019514	10,6980486	40
21	9,2940291	9,9914225	9,3026066	10,6973934	39
22	9,2946580	9,9913971	9,3032609	10,6967391	38
23	9,2952859	9,9913717	9,3039143	10,6960857	37
24	9,2959129	9,9913462	9,3045667	10,6954333	36
25	9,2965390	9,9913207	9,3052183	10,6947817	35
26	9,2971641	9,9912952	9,3058689	10,6941311	34
27	9,2977883	9,9912696	9,3065187	10,6934813	33
28	9,2984116	9,9912440	9,3071675	10,6928325	32
29	9,2990339	9,9912184	9,3078155	10,6921845	31
30	9,2996553	9,9911927	9,3084626	10,6915374	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

78 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,2996553	9,9911927	9,3084626	10,6915374	30
31	9,3002758	9,9911670	9,3091088	10,6908912	29
32	9,3008953	9,9911412	9,3097541	10,6902459	28
33	9,3015140	9,9911154	9,3103985	10,6896015	27
34	9,3021317	9,9910896	9,3110421	10,6889579	26
35	9,3027485	9,9910637	9,3116848	10,6883152	25
36	9,3033644	9,9910378	9,3123266	10,6876734	24
37	9,3039794	9,9910119	9,3129675	10,6870325	23
38	9,3045934	9,9909859	9,3136076	10,6863924	22
39	9,3052066	9,9909598	9,3142468	10,6857532	21
40	9,3058189	9,9909338	9,3148851	10,6851149	20
41	9,3064303	9,9909077	9,3155226	10,6844774	19
42	9,3070407	9,9908815	9,3161592	10,6838408	18
43	9,3076503	9,9908553	9,3167950	10,6832050	17
44	9,3082590	9,9908291	9,3174299	10,6825701	16
45	9,3088668	9,9908029	9,3180640	10,6819360	15
46	9,3094737	9,9907766	9,3186972	10,6813028	14
47	9,3100798	9,9907502	9,3193295	10,6806705	13
48	9,3106849	9,9907239	9,3199611	10,6800389	12
49	9,3112892	9,9906974	9,3205918	10,6794082	11
50	9,3118926	9,9906710	9,3212216	10,6787784	10
51	9,3124951	9,9906445	9,3218506	10,6781494	9
52	9,3130968	9,9906180	9,3224788	10,6775212	8
53	9,3136976	9,9905914	9,3231061	10,6768939	7
54	9,3142975	9,9905648	9,3237327	10,6762673	6
55	9,3148965	9,9905382	9,3243584	10,6756416	5
56	9,3154947	9,9905115	9,3249832	10,6750168	4
57	9,3160921	9,9904848	9,3256073	10,6743927	3
58	9,3166885	9,9904580	9,3262305	10,6737695	2
59	9,3172841	9,9904312	9,3268629	10,6731471	1
60	9,3178789	9,9904044	9,3274745	10,6725255	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

12 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan.Comp.	
0	9,3178789	9,9904044	9,3274745	10,6725255	60
1	9,3184728	9,9903775	9,3280953	10,6719047	59
2	9,3190659	9,9903506	9,3287153	10,6712847	58
3	9,3196581	9,9903237	9,3293345	10,6706655	57
4	9,3202495	9,9902967	9,3299528	10,6700472	56
5	9,3208406	9,9902697	9,3305704	10,6694296	55
6	9,3214297	9,9902426	9,3311872	10,6688128	54
7	9,3220186	9,9902155	9,3318031	10,6681969	53
8	9,3226066	9,9901883	9,3324183	10,6675817	52
9	9,3231938	9,9901612	9,3330327	10,6669673	51
10	9,3237802	9,9901339	9,3336463	10,6663537	50
11	9,3243657	9,9901067	9,3342591	10,6657409	49
12	9,3249505	9,9900794	9,3348711	10,6651289	48
13	9,3255344	9,9900521	9,3354823	10,6645177	47
14	9,3261174	9,9900247	9,3360927	10,6639073	46
15	9,3266997	9,9899973	9,3367024	10,6632976	45
16	9,3272811	9,9899698	9,3373113	10,6626887	44
17	9,3278617	9,9899423	9,3379194	10,6620806	43
18	9,3284416	9,9899148	9,3385267	10,6614733	42
19	9,3290206	9,9898873	9,3391333	10,6608667	41
20	9,3295988	9,9898597	9,3397391	10,6602609	40
21	9,3301761	9,9898320	9,3403441	10,6596559	39
22	9,3307527	9,9898043	9,3409484	10,6590516	38
23	9,3313285	9,9897766	9,3415519	10,6584481	37
24	9,3319035	9,9897489	9,3421546	10,6578454	36
25	9,3324777	9,9897211	9,3427566	10,6572434	35
26	9,3330511	9,9896932	9,3433578	10,6566422	34
27	9,3336237	9,9896654	9,3439583	10,6560417	33
28	9,3341955	9,9896374	9,3445580	10,6554420	32
29	9,3347665	9,9896095	9,3451570	10,6548430	31
30	9,3353368	9,9895815	9,3457552	10,6542448	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

77 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,3353368	9,9895815	9,3457552	10,6542448	30
31	9,3359062	9,9895535	9,3463527	10,6536473	29
32	9,3364749	9,9895254	9,3469494	10,6530506	28
33	9,3370428	9,9894973	9,3475454	10,6524546	27
34	9,3376099	9,9894692	9,3481407	10,6518593	26
35	9,3381762	9,9894410	9,3487352	10,6512648	25
36	9,3387418	9,9894128	9,3493290	10,6506710	24
37	9,3393056	9,9893845	9,3499220	10,6500780	23
38	9,3398706	9,9893562	9,3504143	10,6494857	22
39	9,3404338	9,9893279	9,3511059	10,6488941	21
40	9,3409963	9,9892995	9,3516968	10,6483032	20
41	9,3415580	9,9892711	9,3522869	10,6477131	19
42	9,3421190	9,9892427	9,3528763	10,6471237	18
43	9,3426792	9,9892142	9,3534650	10,6465350	17
44	9,3432386	9,9891856	9,3540530	10,6459470	16
45	9,3437973	9,9891571	9,3546402	10,6453598	15
46	9,3443552	9,9891285	9,3552267	10,6447733	14
47	9,3449124	9,9890998	9,3558126	10,6441874	13
48	9,3454688	9,9890711	9,3563977	10,6436023	12
49	9,3460245	9,9890424	9,3569821	10,6430179	11
50	9,3465794	9,9890137	9,3575658	10,6424342	10
51	9,3471336	9,9889849	9,3581487	10,6418513	9
52	9,3476870	9,9889560	9,3587310	10,6412690	8
53	9,3482397	9,9889271	9,3593126	10,6406874	7
54	9,3487917	9,9888982	9,3598935	10,6401065	6
55	9,3493429	9,9888693	9,3604736	10,6395264	5
56	9,3498934	9,9888403	9,3610531	10,6[illegible]89469	4
57	9,3504432	9,9888113	9,3616319	10,6383681	3
58	9,3509922	9,9887822	9,3622100	10,6377900	2
59	9,3515435	9,9887531	9,3627874	10,6372126	1
60	9,3520880	9,9887239	9,3633641	10,6366359	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,3520880	9,9887239	9,3633641	10,6366359	60
1	9,3526349	9,9886947	9,3639401	10,6360599	59
2	9,3531810	9,9886655	9,3645155	10,6354845	58
3	9,3537264	9,9886363	9,3650901	10,6349099	57
4	9,3542710	9,9886070	9,3656641	10,6343359	56
5	9,3548150	9,9885776	9,3662374	10,6337626	55
6	9,3553582	9,9885482	9,3668100	10,6331900	54
7	9,3559007	9,9885188	9,3673819	10,6326181	53
8	9,3564426	9,9884894	9,3679532	10,6320468	52
9	9,3569836	9,9884599	9,3685238	10,6314762	51
10	9,3575240	9,9884303	9,3690937	10,6309063	50
11	9,3580637	9,9884008	9,3696629	10,6303371	49
12	9,3586027	9,9883712	9,3702315	10,6297685	48
13	9,3591409	9,9883415	9,3707994	10,6292006	47
14	9,3596785	9,9883118	9,3713667	10,6286333	46
15	9,3602154	9,9882821	9,3719333	10,6280667	45
16	9,3607515	9,9882523	9,3724992	10,6275008	44
17	9,3612870	9,9882225	9,3730645	10,6269355	43
18	9,3618217	9,9881927	9,3736291	10,6263709	42
19	9,3623558	9,9881628	9,3741930	10,6258070	41
20	9,3628892	9,9881329	9,3747563	10,6252437	40
21	9,3634219	9,9881029	9,3753190	10,6246810	39
22	9,3639539	9,9880729	9,3758810	10,6241190	38
23	9,3644852	9,9880429	9,3764423	10,6235577	37
24	9,3650158	9,9880128	9,3770030	10,6229970	36
25	9,3655458	9,9879827	9,3775631	10,6224369	35
26	9,3660750	9,9879525	9,3781225	10,6218775	34
27	9,3666036	9,9879223	9,3786813	10,6213187	33
28	9,3671315	9,9878921	9,3792394	10,6207006	32
29	9,3676587	9,9878618	9,3797969	10,6202031	31
30	9,3681853	9,9878315	9,3803537	10,6196463	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

13 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,3681853	9,9878315	9,3803537	10,6196463	30
31	9,3687111	9,9878012	9,3809100	10,6190900	29
32	9,3692363	9,9877708	9,3814655	10,6185345	28
33	9,3697608	9,9877404	9,3820205	10,6179795	27
34	9,3702847	9,9877099	9,3825748	10,6174252	26
35	9.3708079	9,9876794	9,3831285	10,6168715	25
36	9,3713304	9,9876488	9,3836816	10,6163184	24
37	9,3718523	9,9876183	9,3842340	10,6157660	23
38	9,3723735	9,9875876	9,3847858	10,6152142	22
39	9,3728940	9,9875570	9,3853370	10,6146630	21
40	9,3734139	9,9875263	9,3858876	10,6141124	20
41	9,3739331	9,9874955	9,3864376	10,6135624	19
42	9,3744517	9,9874648	9,3869869	10,6130131	18
43	9,3749696	9,9874339	9,3875356	10,6124644	17
44	9,3754868	9,9874031	9,3880837	10,6119163	16
45	9,3760034	9,9873722	9,3886312	10,6113688	15
46	9,3765194	9,9873413	9,3891781	10,6108219	14
47	9,3770347	9,9873103	9,3897244	10,6102756	13
48	9,3775493	9,9872793	9,3902700	10,6097300	12
49	9,3780633	9,9872482	9,3908151	10,6091849	11
50	9,3785767	9,9872171	9,3913595	10,6086405	10
51	9,3790894	9,9871860	9,3919034	10,6080966	9
52	9,3796015	9,9871549	9,3924466	10,6075534	8
53	9,3801129	9,9871236	9,3929893	10,6070107	7
54	9,3806237	9,9870924	9,3935313	10,6064687	6
55	9,3811339	9,9870611	9,3940727	10,6059275	5
56	9,3816434	9 9870298	9,3946136	10,6055864	4
57	9,3821523	9,9869984	9,3951538	10,6048462	3
58	9,3826605	9,9869670	9,3956935	10,6043065	2
59	9,3831682	9,9869356	9,3962326	10,6037674	1
60	9,3836752	9,9869041	9,3967711	10,6032289	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

76 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,3836752	9,9869041	9,3967711	10,6032289	60
1	9,3841815	9,9868726	9,3973089	10,6026911	59
2	9,3846873	9,9868410	9,3978463	10,6021537	58
3	9,3851924	9,9868094	9,3983830	10,6016170	57
4	9,3856969	9,9867778	9,3989191	10,6010809	56
5	9,3862008	9,9867461	9,3994547	10,6005453	55
6	9,3867040	9,9867144	9,3999896	10,6000104	54
7	9,3872067	9,9866827	9,4005240	10,5994760	53
8	9,3877087	9,9866509	9,4010578	10,5989422	52
9	9,3882101	9,9866191	9,4015910	10,5984090	51
10	9,3887109	9,9865872	9,4021237	10,5978763	50
11	9,3892111	9,9865553	9,4026558	10,5973442	49
12	9,3897106	9,9865233	9,4031873	10,5968127	48
13	9,3902096	9,9864913	9,4037182	10,5962818	47
14	9,3907079	9,9864593	9,4042486	10,5957514	46
15	9,3912057	9,9864273	9,4047784	10,5952216	45
16	9,3917028	9,9863952	9,4053076	10,5946924	44
17	9,3921993	9,9863630	9,4058363	10,5941637	43
18	9,3926952	9,9863308	9,4063644	10,5936356	42
19	9,3931905	9,9862986	9,4068919	10,5931081	41
20	9,3936852	9,9862663	9,4074189	10,5925811	40
21	9,3941794	9,9862340	9,4079453	10,5920547	39
22	9,3946729	9,9862017	9,4084712	10,5915288	38
23	9,3951658	9,9861693	9,4089965	10,5910035	37
24	9,3956581	9,9861369	9,4095212	10,5904788	36
25	9,3961499	9,9861045	9,4100454	10,5899546	35
26	9,3966410	9,9860720	9,4105690	10,5894310	34
27	9,3971315	9,9860394	9,4110921	10,5889079	33
28	9,3976215	9,9860069	9,4116146	10,5883854	32
29	9,3981109	9,9859742	9,4121366	10,5878634	31
30	9,3985996	9,9859416	9,4126581	10,5873419	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,3985996	9,9859416	9,4126581	10,5873419	30
31	9,3990878	9,9859089	9,4131789	10,5868211	29
32	9,3995754	9,9858762	9,4136993	10,5863007	28
33	9,4000625	9,9858434	9,4142191	10,5857809	27
34	9,4005489	9,9858106	9,4147383	10,5852617	26
35	9,4010348	9,9857777	9,4152570	10,5847430	25
36	9,4015201	9,9857449	9,4157752	10,5842248	24
37	9,4020048	9,9857119	9,4162928	10,5837072	23
38	9,4024889	9,9856790	9,4168099	10,5831901	22
39	9,4029724	9,9856460	9,4173265	10,5826735	21
40	9,4034554	9,9856129	9,4178425	10,5821575	20
41	9,4039378	9,9855798	9,4183580	10,5816420	19
42	9,4044196	9,9855467	9,4188729	10,5811271	18
43	9,4049009	9,9855135	9,4193874	10,5806126	17
44	9,4053816	9,9854803	9,4199013	10,5800987	16
45	9,4058617	9,9854471	9,4204146	10,5795854	15
46	9,4063413	9,9854138	9,4209275	10,5790725	14
47	9,4068203	9,9853805	9,4214398	10,5785602	13
48	9,4072987	9,9853471	9,4219515	10,5780485	12
49	9,4077766	9,9853138	9,4224628	10,5775372	11
50	9,4082539	9,9852803	9,4229735	10,5770265	10
51	9,4087306	9,9852468	9,4234838	10,5765162	9
52	9,4092068	9,9852133	9,4239935	10,5760065	8
53	9,4096824	9,9851798	9,4245026	10,5754974	7
54	9,4101575	9,9851462	9,4250113	10,5749887	6
55	9,4106320	9,9851125	9,4255194	10,5744806	5
56	9,4111059	9,9850789	9,4260271	10,5739729	4
57	9,4115793	9,9850452	9,4265342	10,5734658	3
58	9,4120522	9,9850114	9,4270408	10,5729592	2
59	9,4125245	9,9849776	9,4275469	10,5724531	1
60	9,4129962	9,9849438	9,4280525	10,5719475	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,4129962	9,9849438	9,4280525	10,5719475	60
1	9,4134674	9,9849099	9,4285575	10,5714425	59
2	9,4139381	9,9848760	9,4290621	10,5709379	58
3	9,4144082	9,9848420	9,4295661	10,5704339	57
4	9,4148778	9,9848081	9,4300697	10,5699303	56
5	9,4153468	9,9847740	9,4305727	10,5694273	55
6	9,4158152	9,9847400	9,4310753	10,5689247	54
7	9,4162832	9,9847059	9,4315773	10,5684227	53
8	9,4167506	9,9846717	9,4320789	10,5679211	52
9	9,4172174	9,9846375	9,4325799	10,5674201	51
10	9,4176837	9,9846033	9,4330804	10,5669196	50
11	9,4181495	9,9845690	9,4335805	10,5664195	49
12	9,4186148	9,9845347	9,4340800	10,5659200	48
13	9,4190795	9,9845004	9,4345791	10,5654209	47
14	9,4195436	9,9844660	9,4350776	10,5649224	46
15	9,4200073	9,9844316	9,4355757	10,5644243	45
16	9,4204704	9,9843971	9,4360733	10,5639267	44
17	9,4209330	9,9843626	9,4365704	10,5634296	43
18	9,4213950	9,9843281	9,4370670	10,5629330	42
19	9,4218566	9,9842935	9,4375631	10,5624369	41
20	9,4223176	9,9842589	9,4380587	10,5619413	40
21	9,4227780	9,9842242	9,4385538	10,5614462	39
22	9,4232380	9,9841895	9,4390485	10,5609515	38
23	9,4236974	9,9841548	9,4395426	10,5604574	37
24	9,4241563	9,9841200	9,4400363	10,5599637	36
25	9,4246147	9,9840852	9,4405295	10,5594705	35
26	9,4250726	9,9840503	9,4410222	10,5589778	34
27	9,4255299	9,9840154	9,4415145	10,5584855	33
28	9,4259867	9,9839805	9,4420062	10,5579938	32
29	9,4264430	9,9839455	9,4424975	10,5575025	31
30	9,4268988	9,9839105	9,4429883	10,5570117	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

15 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,4268988	9,[illegible]0105	9,4429883	10,5570117	30
31	9,4273541	9,9[illegible]55	9,4434786	10,5565214	29
32	9,4278089	9,9838404	9,4439685	10,5560315	28
33	9,4282631	9,9838052	9,4444579	10,5555421	27
34	9,4287169	9,9837701	9,4449468	10,5550532	26
35	9,4291701	9,9837348	9,4454352	10,5545648	25
36	9,4296228	9,9836996	9,4459232	10,5540768	24
37	9,4300750	9,9836643	9,4464107	10,5535893	23
38	9,4305267	9,9836290	9,4468978	10,5531022	22
39	9,4309779	9,9835936	9,4473843	10,5526157	21
40	9,4314286	9,9835582	9,4478704	10,5521296	20
41	9,4318788	9,9835227	9,4483561	10,5516439	19
42	9,4323285	9,9834872	9,4488413	10,5511587	18
43	9,4327777	9,9834517	9,4493260	10,5506740	17
44	9,4332264	9,9834161	9,4498102	10,5501898	16
45	9,4336746	9,9833805	9,4502940	10,5497060	15
46	9,4341223	9,9833449	9,4507774	10,5492226	14
47	9,4345694	9,9833092	9,4512602	10,5487398	13
48	9,4350161	9,9832735	9,4517427	10,5482573	12
49	9,4354623	9,9832377	9,4522246	10,5477754	11
50	9,4359080	9,9832019	9,4527061	10,5472939	10
51	9,4363532	9,9831661	9,4531872	10,5468128	9
52	9,4367980	9,9831302	9,4536678	10,5463322	8
53	9,4372422	9,9830942	9,4541479	10,5458521	7
54	9,4376859	9,9830583	9,4546276	10,5453724	6
55	9,4381292	9,9830223	9,4551069	10,5448931	5
56	9,4385719	9,9829862	9,4555857	10,5444143	4
57	9,4390142	9,9829501	9,4560641	10,5439359	3
58	9,4394560	9,9829140	9,4565420	10,5434580	2
59	9,4398973	9,9828778	9,4570194	10,5429806	1
60	9,4403381	9,9828416	9,4574964	10,5425036	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

74 Degrees.

16 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,4403381	9,9828416	9,4574964	10,5425036	60
1	9,4407784	9,9828054	9,4579730	10,5420270	59
2	9,4412182	9,9827691	9,4584491	10,5415509	58
3	9,4416576	9,9827328	9,4589248	10,5410752	57
4	9,4420955	9,9826964	9,4594001	10,5405999	56
5	9,4425249	9,9826600	9,4598749	10,5401251	55
6	9,4429728	9,9826236	9,4603492	10,5396508	54
7	9,4434103	9,9825871	9,4608232	10,5391768	53
8	9,4438472	9,9825506	9,4612957	10,5387033	52
9	9,4442837	9,9825140	9,4617697	10,5382303	51
10	9,4447197	9,9824774	9,4622423	10,5377577	50
11	9,4451553	9,9824408	9,4627145	10,5372855	49
12	9,4455903	9,9824041	9,4631863	10,5368137	48
13	9,4460250	9,9823674	9,4636576	10,5363424	47
14	9,4464591	9,9823306	9,4641285	10,5358715	46
15	9,4468927	9,9822938	9,4645990	10,5354010	45
16	9,4473259	9,9822569	9,4650690	10,5349310	44
17	9,4477586	9,9822201	9,4655386	10,5344614	43
18	9,4481909	9,9821831	9,4660078	10,5339922	42
19	9,4486227	9,9821462	9,4664765	10,5335235	41
20	9,4490540	9,9821092	9,4669448	10,5330552	40
21	9,4494849	9,9820721	9,4674127	10,5325873	39
22	9,4499153	9,9820351	9,4678802	10,5321198	38
23	9,4503452	9,9819979	9,4683473	10,5316527	37
24	9,4507747	9,9819608	9,4688139	10,5311861	36
25	9,4512037	9,9819236	9,4692801	10,5307199	35
26	9,4516322	9,9818863	9,4697459	10,5302541	34
27	9,4520603	9,9818490	9,4702112	10,5297888	33
28	9,4524879	9,9818117	9,4706762	10,5293238	32
29	9,4529151	9,9817744	9,4711407	10,5288593	31
30	9,4533418	9,9817370	9,4716048	10,5283952	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

73 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,4533418	9,9817370	9,4716048	10,5283952	30
31	9,4537681	9,9816995	9,4720685	10,5279315	29
32	9,4541939	9,9816620	9,4725318	10,5274682	28
33	9,4546192	9,9816245	9,4729947	10,5270053	27
34	9,4550441	9,9815870	9,4734572	10,5265428	26
35	9,4554686	9,9815494	9,4739192	10,5260808	25
36	9,4558926	9,9815117	9,4743808	10,5256192	24
37	9,4563161	9,9814740	9,4748421	10,5251579	23
38	9,4567392	9,9814363	9,4753029	10,5246971	22
39	9,4571618	9,9813986	9,4757633	10,5242367	21
40	9,4575840	9,9813608	9,4762233	10,5237767	20
41	9,4580058	9,9813229	9,4766829	10,5233171	19
42	9,4584271	9,9812850	9,4771421	10,5228579	18
43	9,4588480	9,9812471	9,4776009	10,5223991	17
44	9,4592684	9,9812091	9,4780592	10,5219408	16
45	9,4596884	9,9811711	9,4785172	10,5214828	15
46	9,4601079	9,9811331	9,4789748	10,5210252	14
47	9,4605270	9,9810950	9,4794319	10,5205681	13
48	9,4609456	9,9810569	9,4798887	10,5201113	12
49	9,4613638	9,9810187	9,4803451	10,5196549	11
50	9,4617816	9,9809805	9,4808011	10,5191989	10
51	9,4621989	9,9809423	9,4812566	10,5187434	9
52	9,4626158	9,9809040	9,4817118	10,5182882	8
53	9,4630323	9,9808657	9,4821666	10,5178334	7
54	9,4634483	9,9808273	9,4826210	10,5173790	6
55	9,4638639	9,9807889	9,4830750	10,5169250	5
56	9,4642790	9,9807505	9,4835286	10,5164714	4
57	9,4646938	9,9807120	9,4839818	10,5160182	3
58	9,4651081	9,9806735	9,4844346	10,5155654	2
59	9,4655219	9,9806349	9,4848870	10,5151130	1
60	9,4659353	9,9805963	9,4853390	10,5146610	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,4659353	9,9805963	9,4853390	10,5146610	60
1	9,4663483	9,9805577	9,4857907	10,5142093	59
2	9,4667609	9,9805190	9,4862419	10,5137581	58
3	9,4671730	9,9804803	9,4866928	10,5133072	57
4	9,4675848	9,9804415	9,4871433	10,5128567	56
5	9,4679960	9,9804027	9,4875933	10,5124067	55
6	9,4684069	9,9803639	9,4880430	10,5119570	54
7	9,4688173	9,9803250	9,4884924	10,5115076	53
8	9,4692273	9,9802860	9,4889413	10,5110587	52
9	9,4696369	9,9802471	9,4893898	10,5106102	51
10	9,4700461	9,9802081	9,4898380	10,5101620	50
11	9,4704548	9,9801690	9,4902858	10,5097142	49
12	9,4708631	9,9801299	9,4907332	10,5092668	48
13	9,4712710	9,9800908	9,4911802	10,5088198	47
14	9,4716785	9,9800516	9,4916269	10,5083731	46
15	9,4720856	9,9800124	9,4920731	10,5079269	45
16	9,4724922	9,9799732	9,4925190	10,5074810	44
17	9,4728985	9,9799339	9,4929646	10,5070354	43
18	9,4733043	9,9798946	9,4934097	10,5065903	42
19	9,4737097	9,9798552	9,4938545	10,5061455	41
20	9,4741146	9,9798158	9,4942988	10,5057012	40
21	9,4745192	9,9797764	9,4947429	10,5052571	39
22	9,4749234	9,9797369	9,4951865	10,5048135	38
23	9,4753271	9,9796973	9,4956298	10,5043702	37
24	9,4757304	9,9796578	9,4960727	10,5039273	36
25	9,4761334	9,9796182	9,4965152	10,5034848	35
26	9,4765359	9,9795785	9,4969574	10,5030426	34
27	9,4769380	9,9795388	9,4973991	10,5026009	33
28	9,4773396	9,9794991	9,4978406	10,5021594	32
29	9,4777409	9,9794593	9,4982816	10,5017184	31
30	9,4781418	9,9794195	9,4987223	10,5012777	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

17 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,4781418	9,9794195	9,4987223	10,5012777	30
31	9,4785423	9,9793796	9,4991626	10,5008374	29
32	9,4789423	9,9793398	9,4996026	10,5003974	28
33	9,4793420	9,9792998	9,5000422	10,4999578	27
34	9,4797412	9,9792599	9,5004814	10,4995186	26
35	9,4801401	9,9792198	9,5009203	10,4990797	25
36	9,4805385	9,9791798	9,5013588	10,4986412	24
37	9,4809366	9,9791397	9,5017969	10,4982031	23
38	9,4813342	9,9790996	9,5022347	10,4977653	22
39	9,4817315	9,9790594	9,5026721	10,4973279	21
40	9,4821283	9,9790192	9,5031092	10,4968908	20
41	9,4825248	9,9789789	9,5035459	10,4964541	19
42	9,4829208	9,9789386	9,5039822	10,4960178	18
43	9,4833165	9,9788983	9,5044182	10,4955818	17
44	9,4837117	9,9788579	9,5048538	10,4951462	16
45	9,4841066	9,9788175	9,5052891	10,4947109	15
46	9,4845010	9,9787770	9,5057240	10,4942760	14
47	9,4848951	9,9787365	9,5061586	10,4938414	13
48	9,4852888	9,9786960	9,5065928	10,4934072	12
49	9,4856820	9,9786554	9,5070267	10,4929733	11
50	9,4860749	9,9786148	9,5074602	10,4925398	10
51	9,4864674	9,9785741	9,5078933	10,4921067	9
52	9,4868595	9,9785334	9,5083261	10,4916739	8
53	9,4872512	9,9784927	9,5087586	10,4912414	7
54	9,4876426	9,9784519	9,5091907	10,4908093	6
55	9,4880335	9,9784111	9,5096224	10,4903776	5
56	9,4884240	9,9783702	9,5100539	10,4899461	4
57	9,4888142	9,9783293	9,5104849	10,4895151	3
58	9,4892040	9,9782883	9,5109156	10,4890844	2
59	9,4895934	9,9782474	9,5113460	10,4886540	1
60	9,4899824	9,9782063	9,5117760	10,4882240	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

72 Degrees.

18 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,4899824	9,9782063	9,5117760	10,4882240	60
1	9,4903710	9,9781653	9,5122057	10,4877943	59
2	9,4907592	9,9781241	9,5126351	10,4873649	58
3	9,4911471	9,9780830	9,5130641	10,4869359	57
4	9,4915345	9,9780418	9,5134927	10,4865073	56
5	9,4919216	9,9780006	9,5139210	10,4860790	55
6	9,4923083	9,9779593	9,5143490	10,4856510	54
7	9,4926946	9,9779180	9,5147766	10,4852234	53
8	9,4930806	9,9778766	9,5152039	10,4847961	52
9	9,4934661	9,9778353	9,5156309	10,4843691	51
10	9,4938513	9,9777938	9,5160575	10,4839425	50
11	9,4942361	9,9777523	9,5164838	10,4835162	49
12	9,4946205	9,9777108	9,5169097	10,4830903	48
13	9,4950046	9,9776693	9,5173353	10,4826647	47
14	9,4953883	9,9776277	9,5177606	10,4822394	46
15	9,4957716	9,9775860	9,5181855	10,4818145	45
16	9,4961545	9,9775444	9,5186101	10,4813899	44
17	9,4965370	9,9775026	9,5190344	10,4809656	43
18	9,4969192	9,9774609	9,5194583	10,4805417	42
19	9,4973010	9,9774191	9,5198819	10,4801181	41
20	9,4976824	9,9773772	9,5203052	10,4796948	40
21	9,4980635	9,9773354	9,5207282	10,4792718	39
22	9,4984442	9,9772934	9,5211508	10,4788492	38
23	9,4988245	9,9772515	9,5215730	10,4784270	37
24	9,4992045	9,9772095	9,5219950	10,4780050	36
25	9,4995840	9,9771674	9,5224166	10,4775834	35
26	9,4999633	9,9771253	9,5228379	10,4771621	34
27	9,5003421	9,9770832	9,5232589	10,4767411	33
28	9,5007206	9,9770410	9,5236795	10,4763205	32
29	9,5010987	9,9769988	9,5240999	10,4759001	31
30	9,5014764	9,9769566	9,5245199	10,4754801	30
	Sine Com.	Sine.	Tan. Com	Tangent.	Min.

71 Degrees.

18 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,5014764	9,9769566	9,5245199	10,4754801	30
31	9,5018538	9,9769143	9,5249395	10,4750605	29
32	9,5022308	9,9768720	9,5253589	10,4746411	28
33	9,5026075	9,9768296	9,5257779	10,4742221	27
34	9,5029838	9,9767872	9,5261966	10,4738034	26
35	9,5033597	9,9767447	9,5266150	10,4733850	25
36	9,5037353	9,9767022	9,5270331	10,4729669	24
37	9,5041105	9,9766597	9,5274508	10,4725492	23
38	9,5044853	9,9766171	9,5278682	10,4721318	22
39	9,5048598	9,9765745	9,5282853	10,4717147	21
40	9,5052339	9,9765318	9,5287021	10,4712979	20
41	9,5056077	9,9764891	9,5291186	10,4708814	19
42	9,5059811	9,9764464	9,5295347	10,4704653	18
43	9,5063542	9,9764036	9,5299505	10,4700495	17
44	9,5067268	9,9763608	9,5303661	10,4696339	16
45	9,5070992	9,9763179	9,5307813	10,4692187	15
46	9,5074712	9,9762750	9,5311961	10,4688039	14
47	9,5078428	9,9762321	9,5316107	10,4683893	13
48	9,5082141	9,9761891	9,5320250	10,4679750	12
49	9,5085850	9,9761461	9,5324389	10,4675611	11
50	9,5089556	9,9761030	9,5328526	10,4671474	10
51	9,5093258	9,9760599	9,5332659	10,4667341	9
52	9,5096956	9,9760167	9,5336789	10,4663211	8
53	9,5100651	9,9759736	9,5340916	10,4659084	7
54	9,5104343	9,9759303	9,5345040	10,4654960	6
55	9,5108031	9,9758870	9,5349161	10,4650839	5
56	9,5111716	9,9758437	9,5353278	10,4646722	4
57	9,5115397	9,9758004	9,5357393	10,4642607	3
58	9,5119074	9,9757570	9,5361505	10,4638495	2
59	9,5122749	9,9757135	9,5365613	10,4634387	1
60	9,5126419	9,9756701	9,5369719	10,4630281	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

71 Degrees.

19 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,5126419	9,9756701	9,5369718	10,4630281	60
1	9,5130086	9,9756265	9,5373821	10,4626179	59
2	9,5133750	9,9755830	9,5377920	10,4622080	58
3	9,5137410	9,9755394	9,5382017	10,4617983	57
4	9,5141067	9,9754957	9,5386110	10,4613890	56
5	9,5144721	9,9754521	9,5390200	10,4609800	55
6	9,5148371	9,9754083	9,5394287	10,4605713	54
7	9,5152017	9,9753646	9,5398371	10,4601629	53
8	9,5155660	9,9753208	9,5402453	10,4597547	52
9	9,5159300	9,9752769	9,5406531	10,4593469	51
10	9,5162936	9,9752330	9,5410606	10,4589394	50
11	9,5166569	9,9751891	9,5414678	10,4585322	49
12	9,5170198	9,9751451	9,5418747	10,4581253	48
13	9,5173824	9,9751011	9,5422813	10,4577187	47
14	9,5177447	9,9750570	9,5426877	10,4573123	46
15	9,5181066	9,9750129	9,5430937	10,4569063	45
16	9,5184682	9,9749688	9,5434994	10,4565006	44
17	9,5188295	9,9749246	9,5439048	10,4560952	43
18	9,5191904	9,9748804	9,5443100	10,4556900	42
19	9,5195510	9,9748361	9,5447148	10,4552852	41
20	9,5199112	9,9747918	9,5451193	10,4548807	40
21	9,5202711	9,9747475	9,5455236	10,4544764	39
22	9,5206307	9,9747031	9,5459276	10,4540724	38
23	9,5209899	9,9746587	9,5463312	10,4536688	37
24	9,5213488	9,9746142	9,5467346	10,4532654	36
25	9,5217074	9,9745697	9,5471377	10,4528623	35
26	9,5220656	9,9745252	9,5475405	10,4524595	34
27	9,5224235	9,9744806	9,5479430	10,4520570	33
28	9,5227811	9,9744359	9,5483452	10,4516548	32
29	9,5231383	9,9743913	9,5487471	10,4512529	31
30	9,5234953	9,9743466	9,5491487	10,4508513	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

70 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,5234953	9,9743466	9,5491487	10,4508513	30
31	9,5238518	9,9743018	9,5495500	10,4504500	29
32	9,5242081	9,9742570	9,5499511	10,4500489	28
33	9,5245640	9,9742122	9,5593519	10,4496481	27
34	9,5249196	9,9741673	9,5507523	10,4492477	26
35	9,5252749	9,9741224	9,5511525	10,4488475	25
36	9,5256298	9,9740774	9,5515524	10,4484476	24
37	9,5259844	9,9740324	9,5519521	10,4480479	23
38	9,5263387	9,9739873	9,5523514	10,4476486	22
39	9,5266927	9,9739422	9,5527504	10,4472496	21
40	9,5270463	9,9738971	9,5531492	10,4468508	20
41	9,5273997	9,9738519	9,5535477	10,4464523	19
42	9,5277526	9,9738067	9,5539459	10,4460541	18
43	9,5281053	9,9737615	9,5543438	10,4456562	17
44	9,5284577	9,9737162	9,5547415	10,4452585	16
45	9,5288097	9,9736709	9,5551388	10,4448612	15
46	9,5291614	9,9736255	9,5555359	10,4444641	14
47	9,5295128	9,9735801	9,5559327	10,4440673	13
48	9,5298638	9,9735346	9,5563292	10,4436708	12
49	9,5302146	9,9734891	9,5567255	10,4432745	11
50	9,5305650	9,9734435	9,5571214	10,4428786	10
51	9,5309151	9,9733980	9,5575171	10,4424829	9
52	9,5312649	9,9733523	9,5579125	10,4420875	8
53	9,5316143	9,9733067	9,5583077	10,4416923	7
54	9,5319635	9,9732610	9,5587025	10,4412975	6
55	9,5323123	9,9732152	9,5590971	10,4409029	5
56	9,5326608	9,9731694	9,5594914	10,4405086	4
57	9,5330090	9,9731236	9,5598854	10,4401146	3
58	9,5333569	9,9730777	9,5602792	10,4397208	2
59	9,5337044	9,9730318	9,5606727	10,4393273	1
60	9,5340517	9,9729858	9,5610659	10,4389341	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

20 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,5340517	9,9729858	9,5610659	10,4389341	60
1	9,5343986	9,9729398	9,5614588	10,4385412	59
2	9,5347452	9,9728938	9,5618515	10,4381485	58
3	9,5350915	9,9728477	9,5622439	10,4377561	57
4	9,5354375	9,9728016	9,5626360	10,4373640	56
5	9,5357832	9,9727554	9,5630278	10,4369722	55
6	9,5361285	9,9727092	9,5634194	10,4365806	54
7	9,5364737	9,9726629	9,5638107	10,4361893	53
8	9,5368184	9,9726166	9,5642018	10,4357982	52
9	9,5371629	9,9725703	9,5645925	10,4354075	51
10	9,5375070	9,9725239	9,5649831	10,4350169	50
11	9,5378508	9 9724775	9,5653733	10,4346267	49
12	9,5381943	9,9724310	9,5657633	10,4342367	48
13	9,5385375	9,9723845	9,5661530	10,4338470	47
14	9,5388804	9,9723380	9,5665424	10,4334576	46
15	9,5392230	9,9722914	9,5669316	10,4330684	45
16	9,5395653	9,9722448	9,5673205	10,4326795	44
17	9,5399073	9,9721981	9,5677091	10,4322909	43
18	9,5402489	9,9721514	9,5680975	10,4319025	42
19	9,5405903	9,9721047	9,5684856	10,4315144	41
20	9,5409314	9,9720579	9,5688735	10,4311265	40
21	9,5412721	9,9720110	9,5692611	10,4307389	39
22	9,5416126	9,9719642	9,5696484	10,4303516	38
23	9,5419527	9,9719172	9,5700355	10,4299645	37
24	9,5422926	9,9718703	9,5704223	10,4295777	36
25	9,5426321	9,9718233	9,5708088	10,4291912	35
26	9,5429713	9,9717762	9,5711951	10,4288049	34
27	9,5433103	9,9717291	9,5715811	10,4284189	33
28	9,5436489	9,9716820	9,5719669	10,4280331	32
29	9,5439873	9,9716348	9,5723524	10,4276476	31
30	9,5443253	9,9715876	9,5727377	10,4272623	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

69 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,5443253	9,9715876	9,5727377	10,4272623	30
31	9,5446630	9,9715404	9,5731227	10,4268773	29
32	9,5450905	9,9714931	9,5735074	10,4264926	28
33	9,5453376	9,9714457	9,5738919	10,4261081	27
34	9,5456745	9,9713984	9,5742761	10,4257239	26
35	9,5460110	9,9713509	9,5746601	10,4253399	25
36	9,5463472	9,9713035	9,5750438	10,4249562	24
37	9,5466832	9,9712560	9,5754272	10,4245728	23
38	9,5470189	9,9712084	9,5758104	10,4241896	22
39	9,5473542	9,9711608	9,5761934	10,4238066	21
40	9,5476893	9,9711132	9,5765761	10,4234239	20
41	9,5480240	9,9710655	9,5769585	10,4230415	19
42	9,5483585	9,9710178	9,5773407	10,4226593	18
43	9,5486927	9,9709701	9,5777226	10,4222774	17
44	9,5490266	9,9709223	9,5781043	10,4218957	16
45	9,5493602	9,9708744	9,5784858	10,4215142	15
46	9,5496935	9,9708265	9,5788669	10,4211331	14
47	9,5500265	9,9707786	9,5792479	10,4207521	13
48	9,5503592	9,9707306	9,5796286	10,4203714	12
49	9,5506916	9,9706826	9,5800090	10,4199910	11
50	9,5510237	9,9706346	9,5803892	10,4196108	10
51	9,5513556	9,9705865	9,5807691	10,4192309	9
52	9,5516871	9,9705383	9,5811488	10,4188512	8
53	9,5520184	9,9704902	9,5815282	10,4184718	7
54	9,5523494	9,9704419	9,5819074	10,4180926	6
55	9,5526801	9,9703937	9,5822864	10,4177136	5
56	9,5530105	9,9703454	9,5826651	10,4173349	4
57	9,5533406	9,9702970	9,5830435	10,4169565	3
58	9,5536704	9,9702486	9,5834217	10,4165783	2
59	9,5539999	9,9702002	9,5837997	10,4162003	1
60	9,5543292	9,9701517	9,5841774	10,4158226	0
	Sine Com.	Sine,	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,5543292	9,9701517	9,5841774	10,4158226	60
1	9,5546581	9,9701032	9,5845549	10,4154451	59
2	9,5549868	9,9700547	9,5849321	10,4150679	58
3	9,5553152	9,9700061	9,5853091	10,4146909	57
4	9,5556433	9,9699574	9,5856859	10,4143141	56
5	9,5559711	9,9699087	9,5860624	10,4139376	55
6	9,5562987	9,9698600	9,5864386	10,4135614	54
7	9,5566259	9,9698112	9,5868147	10,4131853	53
8	9,5569529	9,9697624	9,5871904	10,4128096	52
9	9,5572796	9,9697136	9,5875660	10,4124340	51
10	9,5576060	9,9696647	9,5879413	10,4120587	50
11	9,5579321	9,9696158	9,5883163	10,4116837	49
12	9,5582579	9,9695668	9,5886912	10,4113088	48
13	9,5585835	9,9695177	9,5890657	10,4109343	47
14	9,5589088	9,9694687	9,5894401	10,4105599	46
15	9,5592338	9,9694196	9,5898142	10,4101858	45
16	9,5595585	9,9693704	9,5901881	10,4098119	44
17	9,5598829	9,9693212	9,5905617	10,4094383	43
18	9,5602071	9,9692720	9,5909351	10,4090649	42
19	9,5605310	9,9692227	9,5913082	10,4086918	41
20	9,5608546	9,9691734	9,5916812	10,4083188	40
21	9,5611779	9,9691241	9,5920539	10,4079461	39
22	9,5615010	9,9690746	9,5924263	10,4075737	38
23	9,5618237	9,9690252	9,5927985	10,4072015	37
24	9,5621462	9,9689757	9,5931705	10,4068295	36
25	9,5624685	9,9689262	9,5935423	10,4064577	35
26	9,5627904	9,9688760	9,5939138	10,4060862	34
27	9,5631121	9,9688270	9,5942851	10,4057149	33
28	9,5634335	9,9687773	9,5946561	10,4053439	32
29	9,5637546	9,9687276	9,5950269	10,4049731	31
30	9,5640754	9,9686779	9,5953975	10,4046025	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

21 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,5640754	9,9686779	9,5953975	10,4046025	30
31	9,5643960	9,9686281	9,5957679	10,4042321	29
32	9,5647163	9,9685783	9,5961380	10,4038620	28
33	9,5650363	9,9685284	9,5965079	10,4034921	27
34	9,5653561	9,9684785	9,5968776	10,4031224	26
35	9,5656756	9,9684286	9,5972470	10,4027530	25
36	9,5659948	9,9683786	9,5976162	10,4023838	24
37	9,5663137	9,9683285	9,5979852	10,4020148	23
38	9,5666324	9,9682784	9,5983540	10,4016460	22
39	9,5669508	9,9682283	9,5987225	10,4012775	21
40	9,5672689	9,9681781	9,5990908	10,4009092	20
41	9,5675868	9,9681279	9,5994588	10,4005412	19
42	9,5679044	9,9680777	9,5998267	10,4001733	18
43	9,5682217	9,9680274	9,6001943	10,3998057	17
44	9,5685387	9,9679771	9,6005617	10,3994383	16
45	9,5688555	9,9679267	9,6009289	10,3990711	15
46	9,5691721	9,9678763	9,6012958	10,3987042	14
47	9,5694883	9,9678258	9,6016625	10,3983375	13
48	9,5698043	9,9677753	9,6020290	10,3979710	12
49	9,5701200	9,9677247	9,6023953	10,3976047	11
50	9,5704355	9,9676741	9,6027613	10,3972387	10
51	9,5707506	9,9676235	9,6031271	10,3968729	9
52	9,5710656	9,9675728	9,6034927	10,3965073	8
53	9,5713802	9,9675221	9,6038581	10,3961419	7
54	9,5716946	9,9674713	9,6042233	10,3957767	6
55	9,5720087	9,9674205	9,6045882	10,3954118	5
56	9,5723226	9,9673697	9,6049529	10,3950471	4
57	9,5726362	9,9673188	9,6053174	10,3946826	3
58	9,5729495	9,9672679	9,6056817	10,3943183	2
59	9,5732626	9,9672169	9,6060457	10,3939543	1
60	9,5735754	9,9671659	9,6064096	10,3935904	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

68 Degrees.

22 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,5735754	9,9671659	9,6064096	10,3935904	60
1	9,5738880	9,9671148	9,6067732	10,3932268	59
2	9,5742003	9,9670637	9,6071366	10,3928634	58
3	9,5745123	9,9670125	9,6074997	10,3925003	57
4	9,5748240	9,9669614	9,6078627	10,3921373	56
5	9,5751356	9,9669101	9,6082254	10,3917746	55
6	9,5754468	9,9668588	9,6085880	10,3914120	54
7	9,5757578	9,9668075	9,6089503	10,3910497	53
8	9,5760685	9,9667562	9,6093124	10,3906876	52
9	9,5763790	9,9667048	9,6096742	10,3903258	51
10	9,5766892	9,9666533	9,6100359	10,3899641	50
11	9,5769991	9,9666018	9,6103973	10,3896027	49
12	9,5773088	9,9665503	9,6107586	10,3892414	48
13	9,5776183	9,9664986	9,6111196	10,3888804	47
14	9,5779275	9,9664471	9,6114804	10,3885196	46
15	9,5782364	9,9663954	9,6118409	10,3881591	45
16	9,5785450	9,9663437	9,6122013	10,3877987	44
17	9,5788535	9,9662920	9,6125615	10,3874385	43
18	9,5791616	9,9662402	9,6129214	10,3870786	42
19	9,5794695	9,9661884	9,6132812	10,3867188	41
20	9,5797772	9,9661365	9,6136407	10,3863593	40
21	9,5800845	9,9660846	9,6140000	10,3860000	39
22	9,5803917	9,9660326	9,6143591	10,3856409	38
23	9,5806986	9,9659806	9,6147180	10,3852820	37
24	9,5810052	9,9659285	9,6150766	10,3849234	36
25	9,5813116	9,9658764	9,6154351	10,3845649	35
26	9,5816177	9,9658243	9,6157934	10,3842066	34
27	9,5819236	9,9657721	9,6161514	10,3838486	33
28	9,5822292	9,9657199	9,6165093	10,3834907	32
29	9,5825345	9,9656677	9,6168669	10,3831331	31
30	9,5828397	9,9656153	9,6172243	10,3827757	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

67 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,5828397	9,9656153	9,6172243	10,3827757	30
31	9,5831445	9,9655630	9,6175815	10,3824185	29
32	9,5834491	9,9655106	9,6179385	10,3820615	28
33	9,5837535	9,9654582	9,6182953	10,3817047	27
34	9,5840576	9,9654057	9,6186519	10,3813481	26
35	9,5843615	9,9653532	9,6190083	10,3809917	25
36	9,5846651	9,9653006	9,6193645	10,3806355	24
37	9,5849685	9,9652480	9,6197205	10,3802795	23
38	9,5852716	9,9651953	9,6200762	10,3799238	22
39	9,5855745	9,9651426	9,6204318	10,3795682	21
40	9,5858771	9,9650899	9,6207872	10,3792128	20
41	9,5861795	9,9650371	9,6211423	10,3788577	19
42	9,5864816	9,9649843	9,6214973	10,3785027	18
43	9,5867835	9,9649314	9,6218520	10,3781480	17
44	9,5870851	9,9648785	9,6222066	10,3777934	16
45	9,5873865	9,9648256	9,6225609	10,3774391	15
46	9,5876876	9,9647726	9,6229150	10,3770850	14
47	9,5879885	9,9647195	9,6232690	10,3767310	13
48	9,5882892	9,9646665	9,6236227	10,3763773	12
49	9,5885896	9,9646133	9,6239763	10,3760237	11
50	9,5888897	9,9645602	9,6243296	10,3756704	10
51	9,5891897	9,9645069	9,6246827	10,3753173	9
52	9,5894893	9,9644537	9,6250356	10,3749644	8
53	9,5897888	9,9644004	9,6253884	10,3746116	7
54	9,5900880	9,9643470	9,6257409	10,3742591	6
55	9,5903869	9,9642937	9,6260932	10,3739068	5
56	9,5906856	9,9642402	9,6264454	10,3735546	4
57	9,5909841	9,9641868	9,6267973	10,3732027	3
58	9,5912823	9,9641332	9,6271491	10,3728509	2
59	9,5915803	9,9640797	9,6275006	10,3724994	1
60	9,5918780	9,9640261	9,6278519	10,3721481	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

23 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,5918780	9,9640261	9,6278519	10,3721481	60
1	9,5921755	9,9639724	9,6282031	10,3717969	59
2	9,5924728	9,9639187	9,6285540	10,3714460	58
3	9,5927698	9,9638650	9,6289048	10,3710952	57
4	9,5930666	9,9638112	9,6292553	10,3707447	56
5	9,5933631	9,9637574	9,6296057	10,3703943	55
6	9,5936594	9,9637036	9,6299558	10,3700442	54
7	9,5939555	9,9636496	9,6303058	10,3696942	53
8	9,5942513	9,9635957	9,6306556	10,3693444	52
9	9,5945469	9,9635417	9,6310052	10,3689948	51
10	9,5948422	9,9634877	9,6313545	10,3686455	50
11	9,5951373	9,9634336	9,6317037	10,3682963	49
12	9,5954322	9,9633795	9,6320527	10,3679473	48
13	9,5957268	9,9633253	9,6324015	10,3675985	47
14	9,5960212	9,9632711	9,6327501	10,3672499	46
15	9,5963154	9,9632168	9,6330985	10,3669015	45
16	9,5966093	9,9631625	9,6334468	10,3665532	44
17	9,5969030	9,9631082	9,6337948	10,3662052	43
18	9,5971965	9,9630538	9,6341426	10,3658574	42
19	9,5974897	9,9629994	9,6344903	10,3655097	41
20	9,5977827	9,9629449	9,6348378	10,3651622	40
21	9,5980754	9,9628904	9,6351850	10,3648150	39
22	9,5983679	9,9628358	9,6355321	10,3644679	38
23	9,5986602	9,9627812	9,6358790	10,3641210	37
24	9,5989523	9,9627266	9,6362257	10,3637743	36
25	9,5992441	9,9626719	9,6365722	10,3634278	35
26	9,5995357	9,9626172	9,6369185	10,3630815	34
27	9,5998273	9,9625624	9,6372646	10,3627354	33
28	9,6001181	9,9625076	9,6376106	10,3623894	32
29	9,6004090	9,9624527	9,6379563	10,3620437	31
30	9,6006997	9,9623978	9,6383019	10,3616981	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

66 Degrees.

23 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,6006997	9,9623978	9,6383019	10,3616981	30
31	9,6009901	9,9623428	9,6386473	10,3613527	29
32	9,6012803	9,9622878	9,6389925	10,3610075	28
33	9,6015793	9,9622328	9,6393375	10,3606625	27
34	9,6018600	9,9621777	9,6396823	10,3603177	26
35	9,6021495	9,9621226	9,6400269	10,3599731	25
36	9,6024388	9,9620674	9,6403714	10,3596286	24
37	9,6027278	9,9620122	9,6407156	10,3592844	23
38	9,6030166	9,9619569	9,6410597	10,3589403	22
39	9,6033052	9,9619016	9,6414036	10,3585964	21
40	9,6035936	9,9618463	9,6417473	10,3582527	20
41	9,6038817	9,9617909	9,6420908	10,3579092	19
42	9,6041696	9,9617355	9,6424342	10,3575658	18
43	9,6044573	9,9616800	9,6427773	10,3572227	17
44	9,6047448	9,9616245	9,6431203	10,3568797	16
45	9,6050320	9,9615689	9,6434631	10,3565369	15
46	9,6053190	9,9615133	9,6438057	10,3561943	14
47	9,6056057	9,9614576	9,6441481	10,3558519	13
48	9,6058923	9,9614020	9,6444903	10,3555097	12
49	9,6061786	9,9613462	9,6448324	10,3551676	11
50	9,6064647	9,9612904	9,6451743	10,3548257	10
51	9,6067506	9,9612346	9,6455160	10,3544840	9
52	9,6070362	9,9611787	9,6458575	10,3541425	8
53	9,6073216	9,9611228	9,6461988	10,3538012	7
54	9,6076068	9,9610668	9,6465400	10,3534600	6
55	9,6078918	9,9610108	9,6468810	10,3531190	5
56	9,6081765	9,9609548	9,6472217	10,3527783	4
57	9,6084611	9,9608987	9,6475624	10,3524376	3
58	9,6087454	9,9608426	9,6479028	10,3520972	2
59	9,6090294	9,9607864	9,6482431	10,3517569	1
60	9,6093133	9,9607302	9,6485831	10,3514169	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

66 Degrees.

24 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6093133	9,9607302	9,6485831	10,3514169	60
1	9,6095969	9,9606739	9,6489230	10,3510770	59
2	9,6098803	9,9606176	9,6492628	10,3507372	58
3	9,6101635	9,9605612	9,6496023	10,3503977	57
4	9,6104465	9,9605048	9,6499417	10,3500583	56
5	9,6107293	9,9604484	9,6502809	10,3497191	55
6	9,6110118	9,9603919	9,6506199	10,3493801	54
7	9,6112941	9,9603354	9,6509587	10,3490413	53
8	9,6115762	9,9602788	9,6512974	10,3487026	52
9	9,6118589	9,9602222	9,6516359	10,3483641	51
10	9,6121397	9,9601655	9,6519742	10,3480258	50
11	9,6124211	9,9601088	9,6523123	10,3476877	49
12	9,6127023	9,9600520	9,6526503	10,3473497	48
13	9,6129833	9,9599952	9,6529881	10,3470119	47
14	9,6132641	9,9599384	9,6533257	10,3466743	46
15	9,6135446	9,9598815	9,6536631	10,3463369	45
16	9,6138250	9,9598246	9,6540004	10,3459996	44
17	9,6141051	9,9597676	9,6543375	10,3456625	43
18	9,6143850	9,9597106	9,6546744	10,3453256	42
19	9,6146647	9,9596535	9,6550112	10,3449888	41
20	9,6149441	9,9595964	9,6553477	10,3446523	40
21	9,6152234	9,9595393	9,6556841	10,3443159	39
22	9,6155024	9,9594821	9,6560204	10,3439796	38
23	9,6157812	9,9594248	9,6563564	10,3436436	37
24	9,6160599	9,9593675	9,6566923	10,3433077	36
25	9,6163382	9,9593102	9,6570280	10,3429720	35
26	9,6166164	9,9592528	9,6573636	10,3426364	34
27	9,6168944	9,9591954	9,6576989	10,3423011	33
28	9,6171741	9,9591380	9,6580341	10,3419659	32
29	9,6174496	9,9590805	9,6583692	10,3416308	31
30	9,6177270	9,9590229	9,6587041	10,3412959	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

65 Degrees.

24 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,6177270	9,9590229	9,6587041	10,3412959	30
31	9,6180041	9,9589653	9,6590387	10,3409613	29
32	9,6182809	9,9589077	9,6593733	10,3406267	28
33	9,6185576	9,9588500	9,6597076	10,3402924	27
34	9,6188341	9,9587923	9,6600418	10,3399582	26
35	9,6191103	9,9587345	9,6603758	10,3396242	25
36	9,6193864	9,9586767	9,6607097	10,3392903	24
37	9,6196622	9,9586188	9,6610434	10,3389566	23
38	9,6199378	9,9585609	9,6613769	10,3386231	22
39	9,6202132	9,9585030	9,6617103	10,3382897	21
40	9,6204884	9,9584450	9,6620434	10,3379566	20
41	9,6207634	9,9583869	9,6623765	10,3376235	19
42	9,6210382	9,9583288	9,6627093	10,3372907	18
43	9,6213127	9,9582707	9,6630420	10,3369580	17
44	9,6215871	9,9582125	9,6633745	10,3366255	16
45	9,6218612	9,9581543	9,6637069	10,3362931	15
46	9,6221351	9,9580961	9,6640391	10,3359609	14
47	9,6224088	9,9580378	9,6643711	10,3356289	13
48	9,6226824	9,9579794	9,6647030	10,3352970	12
49	9,6229557	9,9579210	9,6650346	10,3349654	11
50	9,6232287	9,9578626	9,6653662	10,3346338	10
51	9,6235016	9,9578041	9,6656975	10,3343025	9
52	9,6237743	9,9577456	9,6660288	10,3339712	8
53	9,6240468	9,9576870	9,6663598	10,3336402	7
54	9,6243190	9,9576284	9,6666907	10,3333093	6
55	9,6245911	9,9575697	9,6670214	10,3329786	5
56	9,6248629	9,9575110	9,6673519	10,3326481	4
57	9,6251346	9,9574522	9,6676823	10,3323177	3
58	9,6254060	9,9573934	9,6680126	10,3319874	2
59	9,6256772	9,9573346	9,6683426	10,3316574	1
60	9,6259483	9,9572757	9,6686725	10,3313275	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

65 Degrees.

25 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6259483	9,9572757	9,6686725	10,3313275	60
1	9,6262191	9,9572168	9,6690023	10,3309977	59
2	9,6264897	9,9571578	9,6693319	10,3306681	58
3	9,6267601	9,9570988	9,6696613	10,3303387	57
4	9,6270303	9,9570397	9,6699906	10,3300094	56
5	9,6273003	9,9569806	9,6703197	10,3296803	55
6	9,6275701	9,9569215	9,6706486	10,3293514	54
7	9,6278397	9,9568623	9,6709774	10,3290226	53
8	9,6281092	9,9568030	9,6713060	10,3286940	52
9	9,6283782	9,9567437	9,6716345	10,3283655	51
10	9,6286472	9,9566844	9,6719628	10,3280372	50
11	9,6289160	9,9566250	9,6722910	10,3277090	49
12	9,6291845	9,9565656	9,6726190	10,3273810	48
13	9,6294529	9,9565061	9,6729468	10,3270532	47
14	9,6297211	9,9564466	9,6732745	10,3267255	46
15	9,6299890	9,9563870	9,6736020	10,3263980	45
16	9,6302568	9,9563274	9,6739294	10,3260706	44
17	9,6305243	9,9562678	9,6742566	10,3257434	43
18	9,6307917	9,9562081	9,6745836	10,3254164	42
19	9,6300589	9,9561483	9,6749105	10,3250895	41
20	9,6313258	9,9560886	9,6752372	10,3247628	40
21	9,6315926	9,9560287	9,6755638	10,3244362	39
22	9,6318591	9,9559689	9,6758903	10,3241097	38
23	9,6311255	9,9559089	9,6762165	10,3237835	37
24	9,6323916	9,9558490	9,6765426	10,3234574	36
25	9,6326576	9,9557890	9,6768686	10,3231314	35
26	9,6329233	9,9557289	9,6771944	10,3228056	34
27	9,6331889	9,9556688	9,6775201	10,3224799	33
28	9,6334542	9,9556087	9,6778456	10,3221544	32
29	9,6337194	9,9555485	9,6781709	10,3218291	31
30	9,6339844	9,9554882	9,6784961	10,3215039	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

64 Degrees.

25 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,6339844	9,9554882	9,6784961	10,3215039	30
31	9,6342491	9,9554280	9,6788211	10,3211789	29
32	9,6345137	9,9553676	9,6791460	10,3208540	28
33	9,6347780	9,9553073	9,6794708	10,3205292	27
34	9,6350422	9,9552469	9,6797953	10,3202047	26
35	9,6353062	9,9551864	9,6801198	10,3198802	25
36	9,6355699	9,9551259	9,6804440	10,3195560	24
37	9,6358335	9,9550653	9,6807682	10,3192318	23
38	9,6360969	9,9550047	9,6810921	10,3189079	22
39	9,6363601	9,9549441	9,6814160	10,3185840	21
40	9,6366231	9,9548834	9,6817396	10,3182604	20
41	9,6368859	9,9548227	9,6820632	10,3179368	19
42	9,6371484	9,9547619	9,6823865	10,3176135	18
43	9,6374108	9,9547011	9,6827098	10,3172902	17
44	9,6376731	9,9546402	9,6830328	10,3169672	16
45	9,6379351	9,9545793	9,6833557	10,3166443	15
46	9,6381969	9,9545184	9,6836785	10,3163215	14
47	9,6384585	9,9544574	9,6840011	10,3159989	13
48	9,6387199	9,9543963	9,6843236	10,3156764	12
49	9,6389812	9,9543352	9,6846459	10,3153541	11
50	9,6392422	9,9542741	9,6849681	10,3150319	10
51	9,6395030	9,9542129	9,6852901	10,3147099	9
52	9,6397637	9,9541517	9,6856120	10,3143880	8
53	9,6400241	9,9540904	9,6859338	10,3140662	7
54	9,6402844	9,9540291	9,6862553	10,3137447	6
55	9,6405445	9,9539677	9,6865768	10,3134232	5
56	9,6408044	9,9539063	9,6868981	10,3131019	4
57	9,6410640	9,9538448	9,6872192	10,3127808	3
58	9,6413235	9,9537833	9,6875402	10,3124598	2
59	9,6415828	9,9537218	9,6878611	10,3121389	1
60	9,6418420	9,9536602	9,6881818	10,3118182	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

64 Degrees.

26 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6418420	9,9536602	9,6881818	10,3118182	60
1	9,6421009	9,9535985	9,6885023	10,3114977	59
2	9,6423596	9,9535369	9,6888227	10,3111773	58
3	9,6426182	9,9534751	9,6891430	10,3108570	57
4	9,6428765	9,9534134	9,6894631	10,3105369	56
5	9,6431347	9,9533515	9,6897831	10,3102169	55
6	9,6433926	9,9532897	9,6901030	10,3098970	54
7	9,6436504	9,9532278	9,6904226	10,3095774	53
8	9,6439080	9,9531658	9,6907422	10,3092578	52
9	9,6441654	9,9531038	9,6910616	10,3089384	51
10	9,6444226	9,9530418	9,6913809	10,3086191	50
11	9,6446796	9,9529797	9,6917000	10,3083000	49
12	9,6449365	9,9529175	9,6920189	10,3079811	48
13	9,6451931	9,9528553	9,6923378	10,3076622	47
14	9,6454496	9,9527931	9,6926565	10,3073435	46
15	9,6457058	9,9527308	9,6939750	10,3070250	45
16	9,6459619	9,9526685	9,6932934	10,3067066	44
17	9,6462178	9,9526061	9,6936117	10,3063883	43
18	9,6464735	9,9525437	9,6939298	10,3060702	42
19	9,6467290	9,9524813	9,6942478	10,3057522	41
20	9,6469844	9,9524188	9,6945656	10,3054344	40
21	9,6472395	9,9523562	9,6948833	10,3051167	39
22	9,6474945	9,9522936	9,6952009	10,3047991	38
23	9,6477492	9,9522310	9,6955183	10,3044817	37
24	9,6480038	9,9521683	9,6958355	10,3041645	36
25	9,6482582	9,9521055	9,6961527	10,3038473	35
26	9,6485124	9,9520428	9,6964697	10,3035303	34
27	9,6487665	9,9519799	9,6967865	10,3032135	33
28	9,6490203	9,9519171	9,6971032	10,3028968	32
29	9,6492740	9,9518541	9,6974198	10,3025802	31
30	9,6495274	9,9517912	9,6977363	10,3022637	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

63 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,6495274	9,9517912	9,6977363	10,3022637	30
31	9,6497807	9,9517282	9,6980520	10,3019474	29
32	9,6500338	9,9516651	9,6983687	10,3016313	28
33	9,6502868	9,9516020	9,6986847	10,3013153	27
34	9,6505395	9,9515389	9,6990006	10,3009994	26
35	9,6507920	9,9514757	9,6993164	10,3006836	25
36	9,6510444	9,9514124	9,6996320	10,3003680	24
37	9,6512966	9,9513492	9,6999474	10,3000526	23
38	9,6515486	9,9512858	9,7002628	10,2997372	22
39	9,6518004	9,9512224	9,7005780	10,2994220	21
40	9,6520521	9,9511590	9,7008930	10,2991070	20
41	9,6523035	9,9510956	9,7012080	10,2987920	19
42	9,6525548	9,9510320	9,7015227	10,2984773	18
43	9,6528059	9,9509685	9,7018374	10,2981626	17
44	9,6530568	9,9509049	9,7021519	10,2978481	16
45	9,6533075	9,9508412	9,7024663	10,2975337	15
46	9,6535581	9,9507775	9,7027805	10,2972195	14
47	9,6538084	9,9507138	9,7030946	10,2969054	13
48	9,6540586	9,9506500	9,7034086	10,2965914	12
49	9,6543086	9,9505861	9,7037225	10,2962775	11
50	9,6545584	9,9505223	9,7040362	10,2959638	10
51	9,6548081	9,9504583	9,7043497	10,2956503	9
52	9,6550575	9,9503944	9,7046632	10,2953368	8
53	9,6553068	9,9503303	9,7049765	10,2950235	7
54	9,6555559	9,9502663	9,7052897	10,2947103	6
55	9,6558048	9,9502022	9,7056027	10,2943973	5
56	9,6560536	9,9501380	9,7059156	10,2940844	4
57	9,6563021	9,9500738	9,7062284	10,2937716	3
58	9,6565505	9,9500095	9,7065410	10,2934590	2
59	9,6567987	9,9499452	9,7068535	10,2931465	1
60	9,6570468	9,9498809	9,7071659	10,2928341	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

27 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6570468	9,9498809	9,7071659	10,2928341	60
1	9,6572946	9,9498165	9,7074781	10,2925219	59
2	9,6575423	9,9497521	9,7077902	10,2922098	58
3	9,6577898	9,9496876	9,7081022	10,2918978	57
4	9,6580371	9,9496230	9,7084141	10,2915859	56
5	9,6582842	9,9495585	9,7087258	10,2912742	55
6	9,6585312	9,9494938	9,7090374	10,2909626	54
7	9,6587780	9,9494292	9,7093488	10,2906512	53
8	9,6590246	9,9493645	9,7096601	10,2903399	52
9	9,6592710	9,9492997	9,7099713	10,2905287	51
10	9,6595173	9,9492349	9,7102824	10,2897176	50
11	9,6597634	9,9491700	9,7105933	10,2894067	49
12	9,6600093	9,9491051	9,7109041	10,2890959	48
13	9,6602530	9,9490402	9,7112148	10,2887852	47
14	9,6605005	9,9489752	9,7115254	10,2884746	46
15	9,6607459	9,9489101	9,7118358	10,2881642	45
16	9,6609911	9,9488450	9,7121461	10,2878539	44
17	9,6612361	9,9487799	9,7124562	10,2875438	43
18	9,6614810	9,9487147	9,7127662	10,2872338	42
19	9,6617257	9,9486495	9,7130761	10,2869239	41
20	9,6619702	9,9485842	9,7133859	10,2866141	40
21	9,6622145	9,9485189	9,7136956	10,2863044	39
22	9,6624586	9,9484535	9,7140051	10,2859949	38
23	9,6627026	9,9483881	9,7143145	10,2856855	37
24	9,6629464	9,9483227	9,7146237	10,2853763	36
25	9,6631900	9,9482572	9,7149329	10,2850671	35
26	9,6634335	9,9481916	9,7152419	10,2847581	34
27	9,6636768	9,9481260	9,7155508	10,2844492	33
28	9,6639199	9,9480604	9,7158595	10,2841405	32
29	9,6641628	9,9479947	9,7161682	10,2838318	31
30	9,6644056	9,9479289	9,7164767	10,2835233	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

62 Degrees.

27 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,6644056	9,9479287	9,7164767	10,2835233	30
31	9,6646482	9,9478631	9,7167851	10,2832149	29
32	9,6648906	9,9477973	9,7170933	10,2829067	28
33	9,6651329	9,9477314	9,7174014	10,2825986	27
34	9,6653749	9,9476655	9,7177095	10,2822906	26
35	9,6656168	9,9475995	9,7180173	10,2819827	25
36	9,6658586	9,9475335	9,7183251	10,2816749	24
37	9,6661001	9,9474674	9,7186327	10,2813673	23
38	9,6663415	9,9474013	9,7189402	10,2810598	22
39	9,6665828	9,9473352	9,7192476	10,2807524	21
40	9,6668238	9,9472689	9,7195549	10,2804451	20
41	9,6670647	9,9472027	9,7198620	10,2801380	19
42	9,6673054	9,9471364	9,7201690	10,2798310	18
43	9,6675459	9,9470700	9,7204759	10,2795241	17
44	9,6677863	9,9470036	9,7207827	10,2792173	16
45	9,6680265	9,9469372	9,7210893	10,2789107	15
46	9,6682665	9,9468707	9,7213958	10,2786042	14
47	9,6685064	9,9468042	9,7217022	10,2782978	13
48	9,6687461	9,9467376	9,7220085	10,2779915	12
49	9,6689856	9,9466710	9,7223147	10,2776853	11
50	9,6692252	9,9466043	9,7226207	10,2773793	10
51	9,6694642	9,9465376	9,7229266	10,2770734	9
52	9,6697032	9,9464708	9,7232324	10,2767676	8
53	9,6699420	9,9464040	9,7235381	10,2764619	7
54	9,6701807	9,9463371	9,7238436	10,2761564	6
55	9,6704192	9,9462702	9,7241490	10,2758510	5
56	9,6706576	9,9462032	9,7244543	10,2755457	4
57	9,6708958	9,9461362	9,7247595	10,2752405	3
58	9,6711338	9,9460692	9,7250646	10,2749354	2
59	9,6713716	9,9460021	9,7253695	10,2746305	1
60	9,6716093	9,9459349	9,7256774	10,2743256	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

62 Degrees.

28 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6716093	9,9459349	9,7256744	10,2743256	60
1	9,6718468	9,9458677	9,7259791	10,2740209	59
2	9,6720841	9,9458005	9,7262837	10,2737163	58
3	9,6723213	9,9457332	9,7265881	10,2734119	57
4	9,6725583	9,9456659	9,7268925	10,2731075	56
5	9,6727952	9,9455985	9,7271967	10,2728033	55
6	9,6730319	9,9455310	9,7275008	10,2724992	54
7	9,6732684	9,9454636	9,7278048	10,2721952	53
8	9,6735047	9,9453960	9,7281087	10,2718913	52
9	9,6737409	9,9453285	9,7284124	10,2715876	51
10	9,6739769	9,9452609	9,7287161	10,2712839	50
11	9,6742128	9,9451932	9,7290196	10,2709804	49
12	9,6744485	9,9451255	9,7293230	10,2706770	48
13	9,6746840	9,9450577	9,7296263	10,2703737	47
14	9,6749194	9,9449899	9,7299295	10,2700705	46
15	9,6751546	9,9449220	9,7302325	10,2697675	45
16	9,6753896	9,9448541	9,7305354	10,2694646	44
17	9,6756245	9,9447862	9,7308383	10,2691617	43
18	9,6758592	9,9447182	9,7311410	10,2688590	42
19	9,6760937	9,9446501	9,7314436	10,2685564	41
20	9,6763281	9,9445821	9,7317460	10,2682540	40
21	9,6765623	9,9445139	9,7320484	10,2679516	39
22	9,6767963	9,9444457	9,7323506	10,2676494	38
23	9,6770302	9,9443775	9,7326527	10,2673473	37
24	9,6772640	9,9443092	9,7329547	10,2670453	36
25	9,6774975	9,9442409	9,7332566	10,2667434	35
26	9,6777309	9,9441725	9,7335584	10,2664416	34
27	9,6779642	9,9441041	9,7338601	10,2661399	33
28	9,6781972	9,9440356	9,7341616	10,2658384	32
29	9,6784301	9,9439671	9,7344631	10,2655369	31
30	9,6786629	9,9438985	9,7347644	10,2652356	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

61 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,6786629	9,9438985	9,7347644	10,2652356	30
31	9,6788955	9,9438299	9,7350656	10,2649344	29
32	9,6791279	9,9437612	9,7353667	10,2646333	28
33	9,6793602	9,9436925	9,7356677	10,2643323	27
34	9,6795923	9,9436238	9,7359685	10,2640315	26
35	9,6798243	9,9435549	9,7362693	10,2637307	25
36	9,6800560	9,9434861	9,7365699	10,2634301	24
37	9,6802877	9,9434172	9,7368705	10,2631295	23
38	9,6805191	9,9433482	9,7371709	10,2628291	22
39	9,6807504	9,9432792	9,7374712	10,2625288	21
40	9,6809816	9,9432102	9,7377714	10,2622286	20
41	9,6812126	9,9431411	9,7380715	10,2619285	19
42	9,6814434	9,9430720	9,7383714	10,2616286	18
43	9,6816741	9,9430028	9,7386713	10,2613287	17
44	9,6819046	9,9429335	9,7389710	10,2610290	16
45	9,6821349	9,9428643	9,7392707	10,2607293	15
46	9,6823651	9,9427949	9,7395702	10,2604298	14
47	9,6825952	9,9427255	9,7398696	10,2601304	13
48	9,6828250	9,9426561	9,7401689	10,2598311	12
49	9,6830548	9,9425866	9,7404681	10,2595319	11
50	9,6832843	9,9425171	9,7407672	10,2592328	10
51	9,6835137	9,9424476	9,7410662	10,2589338	9
52	9,6837430	9,9423779	9,7413650	10,2586350	8
53	9,6839720	9,9423083	9,7416638	10,2583362	7
54	9,6842010	9,9422386	9,7419624	10,2580376	6
55	9,6844297	9,9421688	9,7422609	10,2577391	5
56	9,6846583	9,9420990	9,7425594	10,2574406	4
57	9,6848868	9,9420291	9,7428577	10,2571423	3
58	9,6851151	9,9419592	9,7431559	10,2568441	2
59	9,6853432	9,9418893	9,7434540	10,2565460	1
60	9,6855712	9,9418193	9,7437520	10,2562480	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

61 Degrees.

29 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6855712	9,9418193	9,7437520	10,2562480	60
1	9,6857991	9,9417492	9,7440499	10,2559501	59
2	9,6860267	9,9416791	9,7443476	10,2556524	58
3	9,6862542	9,9416090	9,7446453	10,2553547	57
4	9,6864816	9,9415388	9,7449428	10,2550572	56
5	9,6867088	9,9414685	9,7452403	10,2547597	55
6	9,6869359	9,9413982	9,7455376	10,2544624	54
7	9,6871628	9,9413279	9,7458349	10,2541651	53
8	9,6873895	9,9412575	9,7461320	10,2538680	52
9	9,6876161	9,9411871	9,7464290	10,2535710	51
10	9,6878425	9,9411166	9,7467259	10,2532741	50
11	9,6880688	9,9410461	9,7470227	10,2529773	49
12	9,6882949	9,9409755	9,7473194	10,2526806	48
13	9,6885209	9,9409048	9,7476160	10,2523840	47
14	9,6887467	9,9408342	9,7479125	10,2520875	46
15	9,6889723	9,9407634	9,7482089	10,2517911	45
16	9,6891978	9,9406927	9,7485052	10,2514948	44
17	9,6894232	9,9406219	9,7488013	10,2511987	43
18	9,6896484	9,9405510	9,7490974	10,2509026	42
19	9,6898734	9,9404801	9,7493934	10,2506066	41
20	9,6900983	9,9404091	9,7496892	10,2503108	40
21	9,6903231	9,9403381	9,7499850	10,2500150	39
22	9,6905476	9,9402670	9,7502806	10,2497194	38
23	9,6907721	9,9401959	9,7505762	10,2494238	37
24	9,6909964	9,9401248	9,7508716	10,2491284	36
25	9,6912205	9,9400535	9,7511669	10,2488331	35
26	9,6914445	9,9399823	9,7514622	10,2485378	34
27	9,6916683	9,9399110	9,7517573	10,2482427	33
28	9,6918919	9,9398396	9,7520523	10,2479477	32
29	9,6921155	9,9397682	9,7523472	10,2476528	31
30	9,6923388	9,9396968	9,7526420	10,2473580	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

60 Degrees.

29 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,6923388	9,9396968	9,7526420	10,2473580	30
31	9,6925620	9,9396253	9,7529368	10,2470632	29
32	9,6927851	9,9395537	9,7532314	10,2467686	28
33	9,6930080	9,9394821	9,7535259	10,2464741	27
34	9,6932308	9,9394105	9,7538203	10,2461797	26
35	9,6934534	9,9393388	9,7541146	10,2458854	25
36	9,6936758	9,9392671	9,7544088	10,2455912	24
37	9,6938981	9,9391953	9,7547029	10,2452971	23
38	9,6941203	9,9391234	9,7549969	10,2450031	22
39	9,6943423	9,9390515	9,7552908	10,2447092	21
40	9,6945642	9,9389796	9,7555846	10,2444154	20
41	9,6947859	9,9389076	9,7558783	10,2441217	19
42	9,6950074	9,9388356	9,7561718	10,2438282	18
43	9,6952288	9,9387635	9,7564653	10,2435347	17
44	9,6954501	9,9386914	9,7567587	10,2432413	16
45	9,6956712	9,9386192	9,7570520	10,2429480	15
46	9,6958922	9,9385470	9,7573452	10,2426548	14
47	9,6961130	9,9384747	9,7576383	10,2423617	13
48	9,6963336	9,9384024	9,7579313	10,2420689	12
49	9,6965541	9,9383300	9,7582242	10,2417758	11
50	9,6967745	9,9382576	9,7585170	10,2414830	10
51	9,6969947	9,9381851	9,7588096	10,2411904	9
52	9,6972148	9,9381126	9,7591022	10,2408978	8
53	9,6974347	9,9380400	9,7593947	10,2406053	7
54	9,6976545	9,9379674	9,7596871	10,2403129	6
55	9,6978741	9,9378947	9,7599794	10,2400206	5
56	9,6980936	9,9378220	9,7602716	10,2397284	4
57	9,6983129	9,9377492	9,7605637	10,2394363	3
58	9,6985321	9,9376764	9,7608557	10,2391443	2
59	9,6987511	9,9376035	9,7611476	10,2388524	1
60	9,6989700	9,9375306	9,7614394	10,2385606	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

60 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6989700	9,9375306	9,7614394	10,2385606	60
1	9,6991887	9,9374577	9,7617311	10,2382689	59
2	9,6994073	9,9373847	9,7620227	10,2379773	58
3	9,6996258	9,9373116	9,7623142	10,2376858	57
4	9,6998441	9,9372385	9,7626056	10,2373944	56
5	9,7000622	9,9371653	9,7628969	10,2371031	55
6	9,7002802	9,9370921	9,7631881	10,2368119	54
7	9,7004981	9,9370189	9,7634792	10,2365208	53
8	9,7007158	9,9369456	9,7637702	10,2362298	52
9	9,7009334	9,9368722	9,7640612	10,2359388	51
10	9,7011508	9,9367988	9,7643520	10,2356480	50
11	9,7013681	9,9367254	9,7646427	10,2353573	49
12	9,7015852	9,9366519	9,7649334	10,2350666	48
13	9,7018022	9,9365783	9,7652239	10,2347761	47
14	9,7020190	9,9365047	9,7655143	10,2344857	46
15	9,7022357	9,9364311	9,7658047	10,2341953	45
16	9,7024523	9,9363574	9,7660949	10,2339051	44
17	9,7026687	9,9362836	9,7663851	10,2336149	43
18	9,7028849	9,9362098	9,7666751	10,2333249	42
19	9,7031011	9,9361360	9,7669651	10,2330349	41
20	9,7033170	9,9360621	9,7672550	10,2327450	40
21	9,7035329	9,9359881	9,7675448	10,2324552	39
22	9,7037486	9,9359141	9,7678344	10,2321656	38
23	9,7039641	9,9358401	9,7681240	10,2318760	37
24	9,7041795	9,9357660	9,7684135	10,2315865	36
25	9,7043947	9,9356918	9,7687029	10,2312971	35
26	9,7046099	9,9356177	9,7689922	10,2310078	34
27	9,7048248	9,9355434	9,7692814	10,2307186	33
28	9,7050397	9,9354691	9,7695705	10,2304295	32
29	9,7052543	9,9353948	9,7698596	10,2301404	31
30	9,7054689	9,9353204	9,7701485	10,2298515	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,7054689	9,9353204	9,7701485	10,2298515	30
31	9,7056833	9,9352459	9,7704373	10,2295627	29
32	9,7058975	9,9351715	9,7707261	10,2292739	28
33	9,7061116	9,9350969	9,7710147	10,2289853	27
34	9,7063256	9,9350223	9,7713033	10,2286967	26
35	9,7065394	9,9349477	9,7715917	10,2284083	25
36	9,7067131	9,9348730	9,7718801	10,2281199	24
37	9,7069667	9,9347983	9,7721684	10,2278316	23
38	9,7071801	9,9347235	9,7724566	10,2275434	22
39	9,7073933	9,9346486	9,7727447	10,2272553	21
40	9,7076064	9,9345738	9,7730327	10,2269673	20
41	9,7078194	9,9344988	9,7733206	10,2266794	19
42	9,7080323	9,9344238	9,7736084	10,2263916	18
43	9,7082450	9,9343488	9,7738961	10,2261039	17
44	9,7084575	9,9342737	9,7741838	10,2258162	16
45	9,7086699	9,9341986	9,7744713	10,2255287	15
46	9,7088822	9,9341234	9,7747588	10,2252412	14
47	9,7090943	9,9340482	9,7750462	10,2249538	13
48	9,7093063	9,9339729	9,7753334	10,2246666	12
49	9,7095182	9,9338976	9,7756206	10,2243794	11
50	9,7097299	9,9338222	9,7759077	10,2240923	10
51	9,7099415	9,9337467	9,7761947	10,2238053	9
52	9,7101529	9,9336713	9,7764816	10,2235184	8
53	9,7103642	9,9335957	9,7767685	10,2232315	7
54	9,7105753	9,9335201	9,7770552	10,2229448	6
55	9,7107863	9,9334445	9,7773418	10,2226582	5
56	9,7109972	9,9333688	9,7776284	10,2223716	4
57	9,7112080	9,9332931	9,7779149	10,2220851	3
58	9,7114186	9,9332173	9,7782012	10,2217988	2
59	9,7116290	9,9331415	9,7784875	10,2215125	1
60	9,7118393	9,9330656	9,7787737	10,2212263	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

31 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7118393	9,9330656	9,7787737	10,2212263	60
1	9,7120495	9,9329897	9,7790599	10,2209401	59
2	9,7122596	9,9329137	9,7793459	10,2206541	58
3	9,7124695	9,9328376	9,7796318	10,2203682	57
4	9,7126792	9,9327616	9,7799177	10,2200823	56
5	9,7128889	9,9326854	9,7802034	10,2197966	55
6	9,7130983	9,9326092	9,7804891	10,2195109	54
7	9,7133077	9,9325330	9,7807747	10,2192253	53
8	9,7135169	9,9324567	9,7810602	10,2189398	52
9	9,7137260	9,9323804	9,7813456	10,2186544	51
10	9,7139349	9,9323040	9,7816309	10,2183691	50
11	9,7141437	9,9322276	9,7819162	10,2180838	49
12	9,7143524	9,9321511	9,7822013	10,2177987	48
13	9,7145609	9,9320746	9,7824864	10,2175136	47
14	9,7147693	9,9319980	9,7827713	10,2172287	46
15	9,7149776	9,9319213	9,7830562	10,2169430	45
16	9,7151857	9,9318447	9,7833410	10,2166598	44
17	9,7153937	9,9317679	9,7836258	10,2163742	43
18	9,7156015	9,9316911	9,7839104	10,2160896	42
19	9,7158092	9,9316143	9,7841949	10,2158051	41
20	9,7160168	9,9315374	9,7844794	10,2155206	40
21	9,7162243	9,9314605	9,7847638	10,2152362	39
22	9,7164316	9,9313835	9,7850481	10,2149519	38
23	9,7166387	9,9313065	9,7853323	10,2146677	37
24	9,7168458	9,9312294	9,7856164	10,2143836	36
25	9,7170526	9,9311522	9,7859004	10,2140996	35
26	9,7172594	9,9310750	9,7861844	10,2138156	34
27	9,7174660	9,9309978	9,7864682	10,2135318	33
28	9,7176725	9,9309205	9,7867520	10,2132480	32
29	9,7178789	9,9308432	9,7870357	10,2129643	31
30	9,7180851	9,9307658	9,7873193	10,2126807	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

58 Degrees.

31 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,7180851	9,9307658	9,7873193	10,2126807	30
31	9,7182912	9,9306883	9,7876028	10,2123972	29
32	9,7184971	9,9306109	9,7878863	10,2121137	28
33	9,7187030	9,9305333	9,7881696	10,2118304	27
34	9,7189086	9,9304557	9,7884529	10,2115471	26
35	9,7191142	9,9303781	9,7887361	10,2112639	25
36	9,7193196	9,9303004	9,7890192	10,2109808	24
37	9,7195249	9,9302226	9,7893023	10,2106977	23
38	9,7197300	9,9301448	9,7895852	10,2104148	22
39	9,7199350	9,9300670	9,7898681	10,2101319	21
40	9,7201399	9,9299891	9,7901508	10,2098492	20
41	9,7203447	9,9299112	9,7904335	10,2095665	19
42	9,7205493	9,9298332	9,7907161	10,2092839	18
43	9,7207538	9,9297551	9,7909987	10,2090013	17
44	9,7209581	9,9296770	9,7912811	10,2087189	16
45	9,7211623	9,9295989	9,7915635	10,2084365	15
46	9,7213664	9,9295207	9,7918458	10,2081542	14
47	9,7215704	9,9294424	9,7921280	10,2078720	13
48	9,7217742	9,9293641	9,7924101	10,2075899	12
49	9,7219779	9,9292857	9,7926921	10,2073079	11
50	9,7221814	9,9292073	9,7929741	10,2070259	10
51	9,7223848	9,9291289	9,7932560	10,2067440	9
52	9,7225881	9,9290504	9,7935378	10,2064622	8
53	9,7227913	9,9289718	9,7938195	10,2061805	7
54	9,7229943	9,9288932	9,7941011	10,2058989	6
55	9,7231972	9,9288145	9,7943827	10,2056173	5
56	9,7234000	9,9287358	9,7946641	10,2053359	4
57	9,7236026	9,9286571	9,7949455	10,2050545	3
58	9,7238051	9,9285783	9,7952268	10,2047732	2
59	9,7240075	9,9284994	9,7955081	10,2044919	1
60	9,7242097	9,9284205	9,7957892	10,2042108	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

58 Degrees.

32 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7242097	9,9284205	9,7957892	10,2042108	60
1	9,7244118	9,9283415	9,7960703	10,2039297	59
2	9,7246138	9,9282625	9,7963513	10,2036487	58
3	9,7248156	9,9281834	9,7966322	10,2033678	57
4	9,7250174	9,9281043	9,7969130	10,2030870	56
5	9,7252189	9,9280251	9,7971938	10,2028062	55
6	9,7254204	9,9279459	9,7974745	10,2025255	54
7	9,7256217	9,9278666	9,7977551	10,2022449	53
8	9,7258229	9,9277873	9,7980356	10,2019644	52
9	9,7260240	9,9277079	9,7983160	10,2016840	51
10	9,7262249	9,9276285	9,7985964	10,2014036	50
11	9,7264257	9,9275490	9,7988767	10,2011233	49
12	9,7266264	9,9274695	9,7991569	10,2008431	48
13	9,7268269	9,9273899	9,7994370	10,2005630	47
14	9,7270273	9,9273103	9,7997170	10,2002830	46
15	9,7272276	9,9272306	9,7999970	10,2000030	45
16	9,7274278	9,9271509	9,8002769	10,1997231	44
17	9,7276278	9,9270711	9,8005567	10,1994433	43
18	9,7278277	9,9269913	9,8008365	10,1991635	42
19	9,7280275	9,9269114	9,8011161	10,1988839	41
20	9,7282271	9,9268314	9,8013957	10,1986043	40
21	9,7284267	9,9267514	9,8016752	10,1983248	39
22	9,7286260	9,9266714	9,8019546	10,1980454	38
23	9,7288253	9,9265913	9,8022340	10,1977660	37
24	9,7290244	9,9265112	9,8025133	10,1974867	36
25	9,7292234	9,9264310	9,8027925	10,1972075	35
26	9,7294223	9,9263507	9,8030716	10,1969284	34
27	9,7296211	9,9262704	9,8033506	10,1966494	33
28	9,7298197	9,9261901	9,8036296	10,1963704	32
29	9,7300182	9,9261096	9,8039085	10,1960915	31
30	9,7302165	9,9260292	9,8041873	10,1958127	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

57 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,7302165	9,9260292	9,8041873	10,1958127	30
31	9,7304148	9,9259487	9,8044661	10,1955339	29
32	9,7306129	9,9258681	9,8047447	10,1952553	28
33	9,7308109	9,9257875	9,8050233	10,1949767	27
34	9,7310087	9,9257069	9,8053019	10,1946981	26
35	9,7312064	9,9256261	9,8055803	10,1944197	25
36	9,7314040	9,9255454	9,8058587	10,1941413	24
37	9,7316015	9,9254646	9,8061370	10,1938630	23
38	9,7317989	9,9253837	9,8064152	10,1935848	22
39	9,7319961	9,9253028	9,8066933	10,1933067	21
40	9,7321932	9,9252218	9,8069714	10,1930286	20
41	9,7323902	9,9251408	9,8072494	10,1927506	19
42	9,7325870	9,9250597	9,8075273	10,1924727	18
43	9,7327837	9,9249786	9,8078052	10,1921948	17
44	9,7329803	9,9248974	9,8080829	10,1919171	16
45	9,7331768	9,9248161	9,8083606	10,1916394	15
46	9,7333731	9,9247349	9,8086383	10,1913617	14
47	9,7335693	9,9246535	9,8089158	10,1910842	13
48	9,7337654	9,9245721	9,8091933	10,1908067	12
49	9,7339614	9,9244907	9,8094707	10,1905293	11
50	9,7341572	9,9244092	9,8097480	10,1902520	10
51	9,7343529	9,9243277	9,8100253	10,1899747	9
52	9,7345485	9,9242461	9,8103025	10,1896975	8
53	9,7347440	9,9241644	9,8105796	10,1894204	7
54	9,7349393	9,9240827	9,8108566	10,1891434	6
55	9,7351345	9,9240010	9,8111336	10,1888664	5
56	9,7353296	9,9239191	9,8114105	10,1885895	4
57	9,7355246	9,9238373	9,8116873	10,1883127	3
58	9,7357195	9,9237554	9,8119641	10,1880359	2
59	9,7359142	9,9236734	9,8122404	10,1877592	1
60	9,7361088	9,9235914	9,8125174	10,1874826	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7361088	9,9235914	9,8125174	10,1874826	60
1	9,7363032	9,9235093	9,8127939	10,1872061	59
2	9,7364976	9,9234272	9,8130704	10,1869296	58
3	9,7366918	9,9233450	9,8133468	10,1866532	57
4	9,7368859	9,9232628	9,8136231	10,1863769	56
5	9,7370799	9,9231805	9,8138993	10,1861007	55
6	9,7372737	9,9230982	9,8141755	10,1858245	54
7	9,7374675	9,9230158	9,8144516	10,1855484	53
8	9,7376611	9,9229334	9,8147277	10,1852723	52
9	9,7378546	9,9228509	9,8150036	10,1849964	51
10	9,7380479	9,9227684	9,8152795	10,1847205	50
11	9,7382412	9,9226858	9,8155554	10,1844446	49
12	9,7384343	9,9226032	9,8158311	10,1841689	48
13	9,7386273	9,9225205	9,8161068	10,1838932	47
14	9,7388201	9,9224377	9,8163824	10,1836176	46
15	9,7390129	9,9223549	9,8166580	10,1833420	45
16	9,7392055	9,9222721	9,8169335	10,1830665	44
17	9,7393980	9,9221891	9,8172089	10,1827911	43
18	9,7395904	9,9221062	9,8174842	10,1825158	42
19	9,7397827	9,9220232	9,8177595	10,1822405	41
20	9,7399748	9,9219401	9,8180347	10,1819653	40
21	9,7401668	9,9218570	9,8183098	10,1816902	39
22	9,7403587	9,9217738	9,8185849	10,1814151	38
23	9,7405505	9,9216906	9,8188599	10,1811401	37
24	9,7407421	9,9216073	9,8191348	10,1808652	36
25	9,7409337	9,9215240	9,8194096	10,1805904	35
26	9,7411251	9,9214406	9,8196844	10,1803156	34
27	9,7413164	9,9213572	9,8199592	10,1800408	33
28	9,7415075	9,9212737	9,8202338	10,1797662	32
29	9,7416986	9,9211902	9,8205084	10,1794916	31
30	9,7418895	9,9211066	9,8207829	10,1792171	30
	Sine Com.	Sine.	Tan. Com	Tangent.	Min.

33 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
30	9,7418895	9,9211066	9,8207829	10,1792171	30
31	9,7420803	9,9210229	9,8210574	10,1789426	29
32	9,7422710	9,9209393	9,8213317	10,1786683	28
33	9,7424616	9,9208555	9,8216060	10,1783940	27
34	9,7426520	9,9207717	9,8218803	10,1781197	26
35	9,7428423	9,9206878	9,8221545	10,1778455	25
36	9,7430325	9,9206039	9,8224286	10,1775714	24
37	9,7432226	9,9205200	9,8227026	10,1772974	23
38	9,7434126	9,9204360	9,8229766	10,1770234	22
39	9,7436024	9,9203519	9,8232505	10,1767495	21
40	9,7437921	9,9202678	9,8235244	10,1764756	20
41	9,7439817	9,9201836	9,8237981	10,1762019	19
42	9,7441712	9,9200994	9,8240719	10,1759281	18
43	9,7443606	9,9200151	9,8243455	10,1756545	17
44	9,7445498	9,9199308	9,8246191	10,1753809	16
45	9,7447390	9,9198464	9,8248926	10,1751074	15
46	9,7449280	9,9197619	9,8251660	10,1748340	14
47	9,7451169	9,9196775	9,8254394	10,1745606	13
48	9,7453056	9,9195929	9,8257127	10,1742873	12
49	9,7454943	9,9195083	9,8259860	10,1740140	11
50	9,7456828	9,9194237	9,8262592	10,1737408	10
51	9,7458712	9,9193390	9,8265323	10,1734677	9
52	9,7460595	9,9192542	9,8268053	10,1731947	8
53	9,7462477	9,9191694	9,8270783	10,1729217	7
54	9,7464358	9,9190845	9,8273513	10,1726487	6
55	9,7466237	9,9189996	9,8276241	10,1723759	5
56	9,7468115	9,9189146	9,8278969	10,1721031	4
57	9,7469992	9,9188296	9,8281696	10,1718304	3
58	9,7471868	9,9187445	9,8284423	10,1715577	2
59	9,7473743	9,9186594	9,8287149	10,1712851	1
60	9,7475617	9,9185742	9,8289874	10,1710126	0

56 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7475617	9,9185742	9,8289874	10,1710126	60
1	9,7477489	9,9184896	9,8292599	10,1707401	59
2	9,7479360	9,9184037	9,8295323	10,1704677	58
3	9,7481230	9,9183183	9,8298047	10,1701953	57
4	9,7483099	9,9182329	9,8300769	10,1699231	56
5	9,7484967	9,9181475	9,8303492	10,1696508	55
6	9,7486833	9,9180620	9,8306213	10,1693787	54
7	9,7488698	9,9179764	9,8308934	10,1691066	53
8	9,7490562	9,9178908	9,8311654	10,1688346	52
9	9,7492425	9,9178051	9,8314374	10,1685626	51
10	9,7494287	9,9177194	9,8317093	10,1682907	50
11	9,7496148	9,9176336	9,8319811	10,1680189	49
12	9,7498007	9,9175478	9,8322529	10,1677471	48
13	9,7499866	9,9174619	9,8325246	10,1674754	47
14	9,7501723	9,9173760	9,8327963	10,1672037	46
15	9,7503579	9,9172900	9,8330679	10,1669321	45
16	9,7505434	9,9172040	9,8333394	10,1666606	44
17	9,7507287	9,9171179	9,8336109	10,1663891	43
18	9,7509140	9,9170317	9,8338823	10,1661177	42
19	9,7510991	9,9169455	9,8341536	10,1658464	41
20	9,7512842	9,9168593	9,8344249	10,1655751	40
21	9,7514691	9,9167730	9,8346961	10,1653039	39
22	9,7516538	9,9166866	9,8349673	10,1650327	38
23	9,7518385	9,9166002	9,8352384	10,1647616	37
24	9,7520231	9,9165137	9,8355094	10,1644906	36
25	9,7522075	9,9164272	9,8357804	10,1642196	35
26	9,7523919	9,9163406	9,8360513	10,1639487	34
27	9,7525761	9,9162539	9,8363221	10,1636779	33
28	9,7527602	9,9161673	9,8365929	10,1634071	32
29	9,7529442	9,9160805	9,8368636	10,1631364	31
30	9,7531280	9,9159937	9,8371343	10,1628657	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,7531280	9,9159937	9,8371343	10,1628657	30
31	9,7533118	9,9159069	9,8374049	10,1625951	29
32	9,7534954	9,9158200	9,8376755	10,1623245	28
33	9,7536790	9,9157330	9,8379460	10,1620540	27
34	9,7538624	9,9156460	9,8382164	10,1617836	26
35	9,7540457	9,9155589	9,8384867	10,1615133	25
36	9,7542288	9,9154718	9,8387571	10,1612429	24
37	9,7544119	9,9153846	9,8390273	10,1609727	23
38	9,7545948	9,9152974	9,8392975	10,1607025	22
39	9,7547777	9,9152101	9,8395676	10,1604324	21
40	9,7549604	9,9151228	9,8398377	10,1601623	20
41	9,7551431	9,9150354	9,8401077	10,1598923	19
42	9,7553256	9,9149479	9,8403776	10,1596224	18
43	9,7555080	9,9148604	9,8406475	10,1593525	17
44	9,7556902	9,9147729	9,8409174	10,1590826	16
45	9,7558724	9,9146852	9,8411871	10,1588129	15
46	9,7560544	9,9145976	9,8414569	10,1585431	14
47	9,7562364	9,9145099	9,8417265	10,1582735	13
48	9,7564182	9,9144221	9,8419961	10,1580039	12
49	9,7565999	9,9143342	9,8422657	10,1577343	11
50	9,7567815	9,9142464	9,8425351	10,1574649	10
51	9,7569630	9,9141584	9,8428046	10,1571954	9
52	9,7571444	9,9140704	9,8430739	10,1569261	8
53	9,7573256	9,9139824	9,8433432	10,1566568	7
54	9,7575068	9,9138943	9,8436125	10,1563875	6
55	9,7576878	9,9138061	9,8438817	10,1561183	5
56	9,7578687	9,9137179	9,8441508	10,1558492	4
57	9,7580495	9,9136296	9,8444199	10,1555801	3
58	9,7582302	9,9135413	9,8446889	10,1553111	2
59	9,7584108	9,9134530	9,8449579	10,1550421	1
60	9,7585913	9,9133645	9,8452268	10,1547732	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

35 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7585913	9,9133645	9,8452268	10,1547732	60
1	9,7587717	9,9132760	9,8454956	10,1545044	59
2	9,7589519	9,9131875	9,8457644	10,1542356	58
3	9,7591321	9,9130989	9,8460332	10,1539668	57
4	9,7593121	9,9130102	9,8463018	10,1536982	56
5	9,7594920	9,9129215	9,8465705	10,1534295	55
6	9,7596718	9,9128328	9,8468390	10,1531610	54
7	9,7598515	9,9127440	9,8471075	10,1528925	53
8	9,7600311	9,9129551	9,8473760	10,1526240	52
9	9,7602106	9,9125662	9,8476444	10,1523556	51
10	9,7603899	9,9124772	9,8479127	10,1520873	50
11	9,7605692	9,9123882	9,8481810	10,1518190	49
12	9,7607483	9,9122991	9,8484492	10,1515508	48
13	9,7609274	9,9122099	9,8487174	10,1512826	47
14	9,7611063	9,9121207	9,8489855	10,1510145	46
15	9,7612851	9,9120315	9,8492536	10,1507464	45
16	9,7614638	9,9119422	9,8495216	10,1504784	44
17	9,7616424	9,9118528	9,8497896	10,1502104	43
18	9,7618208	9,9117634	9,8500575	10,1499425	42
19	9,7619992	9,9116739	9,8503253	10,1496747	41
20	9,7621775	9,9115844	9,8505931	10,1494069	40
21	9,7623556	9,9114948	9,8508608	10,1491392	39
22	9,7625337	9,9114051	9,8511285	10,1488715	38
23	9,7627116	9,9113155	9,8513961	10,1486039	37
24	9,7628894	9,9112257	9,8516637	10,1483363	36
25	9,7630671	9,9111359	9,8519312	10,1480688	35
26	9,7632447	9,9110460	9,8521987	10,1478013	34
27	9,7634222	9,9109561	9,8524661	10,1475339	33
28	9,7635996	9,9108661	9,8527335	10,1472665	32
29	9,7637769	9,9107761	9,8530008	10,1469992	31
30	9,7639540	9,9106860	9,8532680	10,1467320	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

54 Degrees.

35 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Com.	
30	9,7639540	9,9106860	9,8532680	10,1467320	30
31	9,7641311	9,9105959	9,8535352	10,1464648	29
32	9,7643080	9,9105057	9,8538023	10,1461977	28
33	9,7644849	9,9104155	9,8540694	10,1459306	27
34	9,7646616	9,9103251	9,8543365	10,1456635	26
35	9,7648382	9,9102348	9,8546034	10,1453966	25
36	9,7650147	9,9101444	9,8548704	10,1451296	24
37	9,7651911	9,9100539	9,8551372	10,1448628	23
38	9,7653674	9,9099634	9,8554041	10,1445959	22
39	9,7655436	9,9098728	9,8556708	10,1443292	21
40	9,7657197	9,9097821	9,8559376	10,1440624	20
41	9,7658957	9,9096915	9,8562042	10,1437958	19
42	9,7660715	9,9096007	9,8564708	10,1435292	18
43	9,7662473	9,9095090	9,8567374	10,1432626	17
44	9,7664229	9,9094190	9,8570039	10,1429961	16
45	9,7665985	9,9093281	9,8572704	10,1427296	15
46	9,7667739	9,9092371	9,8575368	10,1424632	14
47	9,7669492	9,9091461	9,8578031	10,1421969	13
48	9,7671244	9,9090550	9,8580694	10,1419306	12
49	9,7672996	9,9089639	9,8583357	10,1416643	11
50	9,7674746	9,9088727	9,8586019	10,1413981	10
51	9,7676494	9,9087814	9,8588680	10,1411320	9
52	9,7678242	9,9086901	9,8591341	10,1408659	8
53	9,7679989	9,9085988	9,8594002	10,1405998	7
54	9,7681735	9,9085073	9,8596661	10,1403339	6
55	9,7683480	9,9084159	9,8599321	10,1400679	5
56	9,7685223	9,9083243	9,8601980	10,1398020	4
57	9,7686966	9,9082327	9,8604638	10,1395362	3
58	9,7688707	9,9081411	9,8607296	10,1392704	2
59	9,7690448	9,9080494	9,8609954	10,1390046	1
60	9,7692187	9,9079576	9,8612610	10,1387390	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

54 Degrees.

36 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7692187	9,9079576	9,8612610	10,1387390	60
1	9,7693925	9,9078658	9,8615267	10,1384733	59
2	9,7695662	9,9077740	9,8617923	10,1382077	58
3	9,7697398	9,9076820	9,8620578	10,1379422	57
4	9,7699134	9,9075901	9,8623233	10,1376767	56
5	9,7700808	9,9074980	9,8625887	10,1374113	55
6	9,7702601	9,9074059	9,8628541	10,1371459	54
7	9,7704332	9,9073138	9,8631195	10,1368805	53
8	9,7706063	9,9072216	9,8633848	10,1366152	52
9	9,7707793	9,9071293	9,8636500	10,1363500	51
10	9,7709522	9,9070370	9,8639152	10,1360848	50
11	9,7711249	9,9069446	9,8641803	10,1358197	49
12	9,7712976	9,9068522	9,8644454	10,1355546	48
13	9,7714702	9,9067597	9,8647105	10,1352895	47
14	9,7716426	9,9066671	9,8649755	10,1350245	46
15	9,7718150	9,9065745	9,8652404	10,1347596	45
16	9,7719872	9,9064819	9,8655053	10,1344947	44
17	9,7721593	9,9063892	9,8657702	10,1342298	43
18	9,7723314	9,9062964	9,8660350	10,1339650	42
19	9,7725033	9,9062036	9,8662997	10,1337003	41
20	9,7726751	9,9061107	9,8665644	10,1334356	40
21	9,7728468	9,9060177	9,8668291	10,1331709	39
22	9,7730185	9,9059247	9,8670937	10,1329063	38
23	9,7731900	9,9058317	9,8673583	10,1326417	37
24	9,7733614	9,9057386	9,8676228	10,1323772	36
25	9,7735327	9,9056454	9,8678873	10,1321127	35
26	9,7737039	9,9055522	9,8681517	10,1318483	34
27	9,7738749	9,9054589	9,8684160	10,1315840	33
28	9,7740459	9,9053656	9,8686804	10,1313196	32
29	9,7742168	9,9052722	9,8689446	10,1310554	31
30	9,7743876	9,9051787	9,8692089	10,1307911	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

53 Degrees.

36 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,7743876	9,9051787	9,8692089	10,1307911	30
31	9,7745583	9,9050852	9,8694731	10,1305269	29
32	9,7747288	9,9049916	9,8697372	10,1302628	28
33	9,7748993	9,9048980	9,8700013	10,1299987	27
34	9,7750697	9,9048043	9,8702653	10,1297347	26
35	9,7752399	9,9047106	9,8705293	10,1294707	25
36	9,7754101	9,9046168	9,8707933	10,1292067	24
37	9,7755801	9,9045230	9,8710572	10,1289428	23
38	9,7757501	9,9044291	9,8713210	10,1286790	22
39	9,7759199	9,9043351	9,8715848	10,1284152	21
40	9,7760897	9,9042411	9,8718486	10,1281514	20
41	9,7762593	9,9041470	9,8721123	10,1278877	19
42	9,7764289	9,9040529	9,8723760	10,1276240	18
43	9,7765983	9,9039587	9,8726396	10,1273604	17
44	9,7767676	9,9038644	9,8729032	10,1270968	16
45	9,7769369	9,9037701	9,8731668	10,1268332	15
46	9,7771060	9,9036757	9,8734302	10,1265698	14
47	9,7772750	9,9035813	9,8736937	10,1263063	13
48	9,7774439	9,9034868	9,8739571	10,1260429	12
49	9,7776128	9,9033923	9,8742204	10,1257796	11
50	9,7777815	9,9032977	9,8744838	10,1255162	10
51	9,7779501	9,9032031	9,8747470	10,1252530	9
52	9,7781186	9,9031084	9,8750102	10,1249898	8
53	9,7782870	9,9030136	9,8752734	10,1247266	7
54	9,7784553	9,9029188	9,8755365	10,1244635	6
55	9,7786235	9,9028239	9,8757996	10,1242004	5
56	9,7787916	9,9027289	9,8760627	10,1239373	4
57	9,7789596	9,9026339	9,8763257	10,1236743	3
58	9,7791275	9,9025389	9,8765886	10,1234114	2
59	9,7792953	9,9024438	9,8768515	10,1231485	1
60	9,7794630	9,9023486	9,8771144	10,1228856	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

53 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7794630	9,9023486	9,8771144	10,1228856	60
1	9,7796306	9,9022534	9,8773772	10,1226228	59
2	9,7797981	9,9022581	9,8776400	10,1223600	58
3	9,7799655	9,9020628	9,8779027	10,1220973	57
4	9,7801328	9,9019674	9,8781654	10,1218346	56
5	9,7803000	9,9018719	9,8784281	10,1215719	55
6	9,7804671	9,9017764	9,8786907	10,1213093	54
7	9,7806341	9,9016808	9,8789533	10,1210467	53
8	9,7808010	9,9015852	9,8792158	10,1207842	52
9	9,7809677	9,9014895	9,8794782	10,1205218	51
10	9,7811344	9,9013938	9,8797407	10,1202593	50
11	9,7813010	9,9012980	9,8800031	10,1199969	49
12	9,7814675	9,9012021	9,8802654	10,1197346	48
13	9,7816339	9,9011062	9,8805277	10,1194723	47
14	9,7818002	9,9010102	9,8807900	10,1192100	46
15	9,7819664	9,9009142	9,8810522	10,1189478	45
16	9,7821324	9,9008181	9,8813144	10,1186856	44
17	9,7822984	9,9007219	9,8815765	10,1184235	43
18	9,7824643	9,9006257	9,8818386	10,1181614	42
19	9,7826301	9,9005294	9,8821007	10,1178993	41
20	9,7827958	9,9004331	9,8823627	10,1176373	40
21	9,7829614	9,9003367	9,8826246	10,1173754	39
22	9,7831268	9,9002403	9,8828866	10,1171134	38
23	9,7832922	9,9001438	9,8831484	10,1168516	37
24	9,7834575	9,9000472	9,8834103	10,1165897	36
25	9,7836227	9,8999506	9,8836721	10,1163279	35
26	9,7837878	9,8998539	9,8839338	10,1160662	34
27	9,7839528	9,8997572	9,8841956	10,1158044	33
28	9,7841177	9,8996604	9,8844572	10,1155428	32
29	9,7842824	9,8995636	9,8847189	10,1152811	31
30	9,7844471	9,8994667	9,8849805	10,1150195	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

37 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,7844471	9,8994667	9,8849805	10,1150195	30
31	9,7846117	9,8993697	9,8852420	10,1147580	29
32	9,7847762	9,8992727	9,8855035	10,1144965	28
33	9,7849406	9,8991756	9,8857650	10,1142350	27
34	9,7851049	9,8990784	9,8860264	10,1139736	26
35	9,7852691	9,8989812	9,8862878	10,1137122	25
36	9,7854332	9,8988840	9,8865492	10,1134508	24
37	9,7855972	9,8987867	9,8868105	10,1131895	23
38	9,7857611	9,8986893	9,8870718	10,1129282	22
39	9,7859249	9,8985919	9,8873330	10,1126670	21
40	9,7860886	9,8984944	9,8875942	10,1124058	20
41	9,7862522	9,8983968	9,8878554	10,1121446	19
42	9,7864157	9,8982992	9,8881165	10,1118835	18
43	9,7865791	9,8982015	9,8883775	10,1116225	17
44	9,7867424	9,8981038	9,8886386	10,1113614	16
45	9,7869056	9,8980060	9,8888996	10,1111004	15
46	9,7870687	9,8979082	9,8891605	10,1108395	14
47	9,7872317	9,8978103	9,8894214	10,1105786	13
48	9,7873946	9,8977123	9,8896823	10,1103177	12
49	9,7875574	9,8976143	9,8899432	10,1100568	11
50	9,7877202	9,8975162	9,8902040	10,1097960	10
51	9,7878828	9,8974181	9,8904647	10,1095353	9
52	9,7880453	9,8973199	9,8907254	10,1092746	8
53	9,7882077	9,8972216	9,8909861	10,1090139	7
54	9,7883701	9,8971233	9,8912468	10,1087532	6
55	9,7885323	9,8970249	9,8915074	10,1084926	5
56	9,7886944	9,8969265	9,8917679	10,1082321	4
57	9,7888565	9,8968280	9,8920285	10,1079715	3
58	9,7890184	9,8967294	9,8922890	10,1077110	2
59	9,7891802	9,8966308	9,8925494	10,1074506	1
60	9,7893420	9,8965321	9,8928098	10,1071902	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

53 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7893420	9,8965321	9,8928098	10,1071902	60
1	9,7895036	9,8964334	9,8930702	10,1069298	59
2	9,7896652	9,8963346	9,8933306	10,1066694	58
3	9,7898266	9,8962358	9,8935909	10,1064091	57
4	9,7899880	9,8961369	9,8938511	10,1061489	56
5	9,7901493	9,8960379	9,8941114	10,1058886	55
6	9,7903104	9,8959389	9,8943715	10,1056285	54
7	9,7904715	9,8958398	9,8946317	10,1053683	53
8	9,7906325	9,8957406	9,8948918	10,1051082	52
9	9,7907933	9,8956414	9,8951519	10,1048481	51
10	9,7909541	9,8955422	9,8954119	10,1045881	50
11	9,7911148	9,8954429	9,8956719	10,1043281	49
12	9,7912754	9,8953435	9,8959319	10,1040681	48
13	9,7914359	9,8952440	9,8961918	10,1038082	47
14	9,7915963	9,8951445	9,8964517	10,1035483	46
15	9,7917566	9,8950450	9,8967116	10,1032884	45
16	9,7919168	9,8949453	9,8969714	10,1030286	44
17	9,7920769	9,8948457	9,8972312	10,1027688	43
18	9,7922369	9,8947459	9,8974910	10,1025090	42
19	9,7923968	9,8946461	9,8977507	10,1022493	41
20	9,7925566	9,8945463	9,8980104	10,1019896	40
21	9,7927163	9,8944463	9,8982700	10,1017300	39
22	9,7928760	9,8943464	9,8985296	10,1014704	38
23	9,7930355	9,8942463	9,8987892	10,1012108	37
24	9,7931949	9,8941462	9,8990487	10,1009513	36
25	9,7933543	9,8940461	9,8993082	10,1006918	35
26	9,79[illegible]5135	9,8939458	9,8995677	10,1004323	34
27	9,7936727	9,8938456	9,8998271	10,1001729	33
28	9,7938317	9,8937452	9,9000865	10,0999135	32
29	9,7939907	9,8936448	9,9003459	10,0996541	31
30	9,7941496	9,8935444	9,9006052	10,0993948	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,7941496	9,8935444	9,9006052	10,0993948	30
31	9,7943083	9,8934439	9,9008645	10,0991355	29
32	9,7944670	9,8933433	9,9011237	10,0988763	28
33	9,7946256	9,8932426	9,9013830	10,0986170	27
34	9,7947841	9,8931419	9,9016422	10,0983578	26
35	9,7949425	9,8930412	9,9019013	10,0980987	25
36	9,7951008	9,8929404	9,9021604	10,0978396	24
37	9,7952590	9,8928395	9,9024195	10,0975805	23
38	9,7954171	9,8927385	9,9026786	10,0973214	22
39	9,7955751	9,8926375	9,9029376	10,0970624	21
40	9,7957330	9,8925305	9,9031966	10,0968034	20
41	9,7958909	9,8924354	9,9034555	10,0965445	19
42	9,7960486	9,8923342	9,9037144	10,0962856	18
43	9,7962062	9,8922329	9,9039733	10,0960267	17
44	9,7963638	9,8921316	9,9042321	10,0957679	16
45	9,7965212	9,8920303	9,9044910	10,0955090	15
46	9,7966786	9,8919289	9,9047497	10,0952503	14
47	9,7968359	9,8918274	9,9050085	10,0949915	13
48	9,7969930	9,8917258	9,9052672	10,0947328	12
49	9,7971501	9,8916242	9,9055259	10,0944741	11
50	9,7973071	9,8915226	9,9057845	10,0942155	10
51	9,7974640	9,8914208	9,9060431	10,0939569	9
52	9,7976208	9,8913191	9,9063017	10,0936983	8
53	9,7977775	9,8912172	9,9065603	10,0934397	7
54	9,7979341	9,8911153	9,9068188	10,0931812	6
55	9,7980906	9,8910133	9,9070773	10,0929227	5
56	9,7982470	9,8909113	9,9073357	10,0926643	4
57	9,7984034	9,8908092	9,9075941	10,0924059	3
58	9,7985596	9,8907071	9,9078525	10,0921475	2
59	9,7987158	9,8906049	9,9081109	10,0918891	1
60	9,7988718	9,8905026	9,9083692	10,0916308	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7988718	9,8905026	9,9083692	10,0916308	60
1	9,7990278	9,8904003	9,9086275	10,0913725	59
2	9,7991836	9,8902979	9,9088858	10,0911142	58
3	9,7993394	9,8901954	9,9091440	10,0908560	57
4	9,7994951	9,8900929	9,9094022	10,0905978	56
5	9,7996507	9,8899903	9,9096603	10,0903397	55
6	9,7998062	9,8898877	9,9099185	10,0900815	54
7	9,7999616	9,8897850	9,9101766	10,0898234	53
8	9,8001169	9,8896822	9,9104347	10,0895653	52
9	9,8002721	9,8895794	9,9106927	10,0893073	51
10	9,8004272	9,8894765	9,9109507	10,0890493	50
11	9,8005823	9,8893736	9,9112087	10,0887913	49
12	9,8007372	9,8892706	9,9114666	10,0885334	48
13	9,8008921	9,8891675	9,9117245	10,0882755	47
14	9,8010468	9,8890644	9,9119824	10,0880176	46
15	9,8012015	9,8889612	9,9122403	10,0877597	45
16	9,8013561	9,8888580	9,9124981	10,0875019	44
17	9,8015106	9,8887547	9,9127559	10,0872441	43
18	9,8016649	9,8886513	9,9130137	10,0869863	42
19	9,8018192	9,8885479	9,9132714	10,0867286	41
20	9,8019735	9,8884444	9,9135291	10,0864709	40
21	9,8021276	9,8883408	9,9137868	10,0862132	39
22	9,8022816	9,8882372	9,9140444	10,0859556	38
23	9,8024355	9,8881335	9,9143020	10,0856980	37
24	9,8025894	9,8880298	9,9145596	10,0854404	36
25	9,8027431	9,8879260	9,9148171	10,0851829	35
26	9,8028968	9,8878221	9,9150747	10,0849253	34
27	9,8030504	9,8877182	9,9153322	10,0846678	33
28	9,8032038	9,8876142	9,9155896	10,0844104	32
29	9,8033572	9,8875102	9,9158471	10,0841529	31
30	9,8035105	9,8874061	9,9161045	10,0838955	30
	Sine Com.	S ne.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp	
30	9,8035105	9,8874061	9,9161045	10,0838955	30
31	9,8036637	9,8873019	9,9163618	10,0836382	29
32	9,8038168	9,8871977	9,9166192	10,0833808	28
33	9,8039699	9,8870934	9,9168765	10,0831235	27
34	9,8041228	9,8869890	9,9171338	10,0828662	26
35	9,8042757	9,8868846	9,9173911	10,0826089	25
36	9,8044284	9,8867801	9,9176483	10,0823517	24
37	9,8045811	9,8866756	9,9179055	10,0820945	23
38	9,8047336	9,8865710	9,9181627	10,0818373	22
39	9,8048861	9,8864663	9,9184198	10,0815802	21
40	9,8050385	9,8863616	9,9186769	10,0813231	20
41	9,8051908	9,8862568	9,9189340	10,0810660	19
42	9,8053430	9,8861519	9,9191911	10,0808089	18
43	9,8054951	9,8860470	9,9194481	10,0805519	17
44	9,8056472	9,8859420	9,9197051	10,0802949	16
45	9,8057991	9,8858370	9,9199621	10,0800379	15
46	9,8059510	9,8857319	9,9202191	10,0797809	14
47	9,8061027	9,8856267	9,9204760	10,0795240	13
48	9,8062544	9,8855215	9,9207329	10,0792671	12
49	9,8064060	9,8854162	9,9209898	10,0790102	11
50	9,8065575	9,8853109	9,9212466	10,0787534	10
51	9,8067089	9,8852055	9,9215034	10,0784966	9
52	9,8068602	9,8851000	9,9217602	10,0782398	8
53	9,8070114	9,8849945	9,9220170	10,0779830	7
54	9,8071626	9,8848889	9,9222737	10,0777263	6
55	9,8073136	9,8847832	9,9225304	10,0774696	5
56	9,8074646	9,8846775	9,9227871	10,0772129	4
57	9,8076154	9,8845717	9,9230437	10,0769563	3
58	9,8077662	9,8844659	9,9233004	10,0766996	2
59	9,8079169	9,8843599	9,9235570	10,0764430	1
60	9,8080675	9,8842540	9,9238135	10,0761865	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

40 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,8080675	9,8842540	9,9238135	10,0761865	60
1	9,8082180	9,8841479	9,9240701	10,0759299	59
2	9,8083684	9,8840418	9,9243266	10,0756734	58
3	9,8085188	9,8839357	9,9245831	10,0754169	57
4	9,8086690	9,8838294	9,9248396	10,0751604	56
5	9,8088192	9,8837232	9,9250960	10,0749040	55
6	9,8089692	9,8836168	9,9253524	10,0746476	54
7	9,8091192	9,8835104	9,9256088	10,0743912	53
8	9,8092691	9,8834039	9,9258652	10,0741348	52
9	9,8094189	9,8832974	9,9261215	10,0738785	51
10	9,8095686	9,8831908	9,9263778	10,0736222	50
11	9,8097182	9,8830841	9,9266341	10,0733659	49
12	9,8098678	9,8829774	9,9268904	10,0731096	48
13	9,8100172	9,8828706	9,9271466	10,0728534	47
14	9,8101666	9,8827638	9,9274028	10,0725972	46
15	9,8103159	9,8826568	9,9276590	10,0723410	45
16	9,8104650	9,8825499	9,9279152	10,0720848	44
17	9,8106141	9,8824428	9,9281713	10,0718287	43
18	9,8107631	9,8823357	9,9284274	10,0715726	42
19	9,8109121	9,8822285	9,9286835	10,0713165	41
20	9,8110609	9,8821213	9,9289396	10,0710604	40
21	9,8112096	9,8820140	9,9291956	10,0708044	39
22	9,8113583	9,8819067	9,9294516	10,0705484	38
23	9,8115069	9,8817992	9,9297076	10,0702924	37
24	9,8116554	9,8816918	9,9299636	10,0700364	36
25	9,8118038	9,8815842	9,9302195	10,0697805	35
26	9,8119521	9,8814766	9,9304755	10,0695245	34
27	9,8121003	9,8813689	9,9307314	10,0692686	33
28	9,8122484	9,8812612	9,9309872	10,0690128	32
29	9,8123965	9,8811534	9,9312431	10,0687569	31
30	9,8125444	9,8810455	9,9314989	10,0685011	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

49 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,8125444	9,8810455	9,9314989	10,0685011	30
31	9,8126923	9,8809376	9,9317547	10,0682453	29
32	9,8128401	9,8808296	9,9320105	10,0679895	28
33	9,8129878	9,8807215	9,9322662	10,0677338	27
34	9,8131354	9,8806134	9,9325220	10,0674780	26
35	9,8132829	9,8805052	9,9327777	10,0672223	25
36	9,8134303	9,8803970	9,9330334	10,0669666	24
37	9,8135777	9,8802887	9,9332890	10,0667110	23
38	9,8137250	9,8801803	9,9335446	10,0664554	22
39	9,8138721	9,8800719	9,9338003	10,0661997	21
40	9,8140192	9,8799634	9,9340559	10,0659441	20
41	9,8141662	9,8798548	9,9343114	10,0656886	19
42	9,8143131	9,8797462	9,9345670	10,0654330	18
43	9,8144600	9,8796375	9,9348225	10,0651775	17
44	9,8146067	9,8795287	9,9350780	10,0649220	16
45	9,8147534	9,8794199	9,9353335	10,0646665	15
46	9,8148999	9,8793110	9,9355889	10,0644111	14
47	9,8150464	9,8792021	9,9358444	10,0641556	13
48	9,8151928	9,8790930	9,9360998	10,0639002	12
49	9,8153391	9,8789840	9,9363552	10,0636448	11
50	9,8154854	9,8788748	9,9366105	10,0633895	10
51	9,8156315	9,8787656	9,9368659	10,0631341	9
52	9,8157776	9,8786563	9,9371212	10,0628788	8
53	9,8159235	9,8785470	9,9373765	10,0626235	7
54	9,8160694	9,8784376	9,9376318	10,0623682	6
55	9,8162152	9,8783281	9,9378871	10,0621129	5
56	9,8163609	9,8782186	9,9381423	10,0618577	4
57	9,8165066	9,8781090	9,9383975	10,0616025	3
58	9,8166521	9,8779994	9,9386527	10,0613473	2
59	9,8167975	9,8778896	9,9389079	10,0610921	1
60	9,8169429	9,8777799	9,9391631	10,0608369	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,8169429	9,8777799	9,9391631	10,0608369	60
1	9,8170882	9,8776700	9,9394182	10,0605818	59
2	9,8172334	9,8775601	9,9396733	10,0603267	58
3	9,8173785	9,8774501	9,9399284	10,0600716	57
4	9,8175235	9,8773401	9,9401835	10,0598165	56
5	9,8176685	9,8772300	9,9404385	10,0595615	55
6	9,8178133	9,8771198	9,9406936	10,0593064	54
7	9,8179581	9,8770096	9,9409486	10,0590514	53
8	9,8181028	9,8768993	9,9412036	10,0587964	52
9	9,8182474	9,8767889	9,9414585	10,0585415	51
10	9,8183919	9,8766785	9,9417135	10,0582865	50
11	9,8185364	9,8765680	9,9419684	10,0580316	49
12	9,8186807	9,8764574	9,9422233	10,0577767	48
13	9,8188250	9,8763468	9,9424782	10,0575218	47
14	9,8189692	9,8762361	9,9427331	10,0572669	46
15	9,8191133	9,8761253	9,9429879	10,0570121	45
16	9,8192573	9,8760145	9,9432428	10,0567572	44
17	9,8194012	9,8759036	9,9434976	10,0565024	43
18	9,8195450	9,8757927	9,9437524	10,0562476	42
19	9,8196888	9,8756816	9,9440072	10,0559928	41
20	9,8198325	9,8755706	9,9442619	10,0557381	40
21	9,8199761	9,8754594	9,9445166	10,0554834	39
22	9,8201196	9,8753482	9,9447714	10,0552286	38
23	9,8202630	9,8752369	9,9450261	10,0549739	37
24	9,8204063	9,8751256	9,9452807	10,0547193	36
25	9,8205496	9,8750142	9,9455354	10,0544646	35
26	9,8206927	9,8749027	9,9457900	10,0542100	34
27	9,8208358	9,8747912	9,9460447	10,0539553	33
28	9,8209788	9,8746795	9,9462993	10,0537007	32
29	9,8211217	9,8745679	9,9465539	10,0534461	31
30	9,8212646	9,8744561	9,9468084	10,0531916	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,8212646	9,8744561	9,9468084	10,0531916	30
31	9,8214073	9,8743443	9,9470630	10,0529370	29
32	9,8215500	9,8742325	9,9473175	10,0526825	28
33	9,8216926	9,8741205	9,947572	10,0524280	27
34	9,8218351	9,8740085	9,9478265	10,0521735	26
35	9,8219775	9,8738965	9,9480810	10,0519190	25
36	9,8221198	9,8737844	9,9483355	10,0516645	24
37	9,8222621	9,8736722	9,9485899	10,0514101	23
38	9,8224042	9,8735599	9,9488443	10,0511557	22
39	9,8225463	9,8734476	9,9490987	10,0509013	21
40	9,8226883	9,8733352	9,9493531	10,0506469	20
41	9,8228302	9,8732227	9,9496075	10,0503925	19
42	9,8229721	9,8731102	9,9498619	10,0501381	18
43	9,8231138	9,8729976	9,9501162	10,0498838	17
44	9,8232555	9,8728849	9,9503705	10,0496295	16
45	9,8233971	9,8727722	9,9506248	10,0493752	15
46	9,8235386	9,8726594	9,9508791	10,0491209	14
47	9,8236800	9,8725466	9,9511334	10,0488666	13
48	9,8238213	9,8724337	9,9513876	10,0486124	12
49	9,8239626	9,8723207	9,9516419	10,0483581	11
50	9,8241037	9,8722076	9,9518961	10,0481039	10
51	9,8242448	9,8720945	9,9521503	10,0478497	9
52	9,8243858	9,8719813	9,9524045	10,0475955	8
53	9,8245267	9,8718681	9,9526587	10,0473413	7
54	9,8246676	9,8717548	9,9529128	10,0470872	6
55	9,8248083	9,8716414	9,9531670	10,0468330	5
56	9,8249490	9,8715279	9,9534211	10,0465789	4
57	9,8250896	9,8714144	9,9536752	10,0463248	3
58	9,8252301	9,8713008	9,9539293	10,0460707	2
59	9,8253795	9,8711872	9,9541834	10,0458166	1
60	9,8255109	9,8710735	9,9544374	10,0455626	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,8255109	9,8710735	9,9544374	10,0455626	60
1	9,8256512	9,8709597	9,9546915	10,0453085	59
2	9,8257913	9,8708458	9,9549455	10,0450545	58
3	9,8259314	9,8707319	9,9551995	10,0448005	57
4	9,8260715	9,8706179	9,9554535	10,0445465	56
5	9,8262114	9,8705039	9,9557075	10,0442925	55
6	9,8263512	9,8703898	9,9559615	10,0440385	54
7	9,8264910	9,8702756	9,9562154	10,0437846	53
8	9,8266307	9,8701613	9,9564694	10,0435306	52
9	9,8267703	9,8700470	9,9567233	10,0432767	51
10	9,8269098	9,8699326	9,9569772	10,0430228	50
11	9,8270493	9,8698182	9,9572311	10,0427689	49
12	9,8271887	9,8697037	9,9574850	10,0425150	48
13	9,8273279	9,8695891	9,9577389	10,0422611	47
14	9,8274671	9,8694744	9,9579927	10,0420073	46
15	9,8276063	9,8693597	9,9582465	10,0417535	45
16	9,8277453	9,8692449	9,9585004	10,0414996	44
17	9,8278843	9,8691301	9,9587542	10,0412458	43
18	9,8280231	9,8690152	9,9590080	10,0409920	42
19	9,8281619	9,8689002	9,9592618	10,0407382	41
20	9,8283006	9,8687851	9,9595155	10,0404845	40
21	9,8284393	9,8686700	9,9597693	10,0402307	39
22	9,8285778	9,8685548	9,9600230	10,0399770	38
23	9,8287163	9,8684396	9,9602767	10,0397233	37
24	9,8288547	9,8683242	9,9605305	10,0394695	36
25	9,8289930	9,8682088	9,9607842	10,0392158	35
26	9,8291312	9,8680934	9,9610378	10,0389622	34
27	9,8292694	9,8679779	9,9612915	10,0387085	33
28	9,8294075	9,8678623	9,9615452	10,0384548	32
29	9,8295454	9,8677466	9,9617988	10,0382012	31
30	9,8296833	9,8676309	9,9620525	10,0379475	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,8296833	9,8676309	9,9620525	10,0379475	30
31	9,8298212	9,8675151	9,9623061	10,0376939	29
32	9,8299589	9,8673992	9,9625597	10,0374403	28
33	9,8300966	9,8672833	9,9628133	10,0371867	27
34	9,8302342	9,8671673	9,9630669	10,0369331	26
35	9,8303717	9,8670512	9,9633204	10,0366796	25
36	9,8305091	9,8669351	9,9635740	10,0364260	24
37	9,8306464	9,8668189	9,9638275	10,0361725	23
38	9,8307837	9,8667026	9,9640811	10,0359186	22
39	9,8309209	9,8665863	9,9643346	10,0356654	21
40	9,8310580	9,8664699	9,9645881	10,0354119	20
41	9,8311950	9,8663534	9,9648416	10,0351584	19
42	9,8313320	9,8662369	9,9650951	10,0349049	18
43	9,8314688	9,8661203	9,9653486	10,0346514	17
44	9,8316056	9,8660036	9,9656020	10,0343980	16
45	9,8317423	9,8658868	9,9658555	10,0341445	15
46	9,8318789	9,8657700	9,9661089	10,0338911	14
47	9,8320155	9,8656531	9,9663623	10,0336377	13
48	9,8321519	9,8655362	9,9666157	10,0333843	12
49	9,8322883	9,8654192	9,9668692	10,0331308	11
50	9,8324246	9,8653021	9,9671225	10,0328775	10
51	9,8325609	9,8651849	9,9673759	10,0326241	9
52	9,8326970	9,8650677	9,9676293	10,0323707	8
53	9,8328331	9,8649504	9,9678827	10,0321173	7
54	9,8329691	9,8648331	9,9681360	10,0318640	6
55	9,8331050	9,8647156	9,9683893	10,0316107	5
56	9,8332408	9,8645981	9,9686427	10,0313573	4
57	9,8333766	9,8644806	9,9688960	10,0311040	3
58	9,8335122	9,8643629	9,9691493	10,0308507	2
59	9,8336478	9,8642452	9,9694026	10,0305974	1
60	9,8337833	9,8641275	9,9696559	10,0303441	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

43 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,8337833	9,8641275	9,9696559	10,0303441	60
1	9,8339188	9,8640096	9,9699091	10,0300909	59
2	9,8340541	9,8638917	9,9701624	10,0298376	58
3	9,8341894	9,8637737	9,9704157	10,0295843	57
4	9,8343246	9,8636557	9,9706689	10,0293311	56
5	9,8344597	9,8635376	9,9709221	10,0290779	55
6	9,8345948	9,8634194	9,9711754	10,0288246	54
7	9,8347297	9,8633011	9,9714286	10,0285714	53
8	9,8348646	9,8631828	9,9716818	10,0283182	52
9	9,8349994	9,8630644	9,9719350	10,0280650	51
10	9,8351341	9,8629460	9,9721882	10,0278118	50
11	9,8352688	9,8628274	9,9724413	10,0275587	49
12	9,8354033	9,8627088	9,9726945	10,0273055	48
13	9,8355378	9,8625902	9,9729477	10,0270523	47
14	9,8356722	9,8624714	9,9732008	10,0267992	46
15	9,8358066	9,8623526	9,9734539	10,0265461	45
16	9,8359408	9,8622338	9,9737071	10,0262929	44
17	9,8360750	9,8621148	9,9739602	10,0260398	43
18	9,8362091	9,8619958	9,9742133	10,0257867	42
19	9,8363431	9,8618767	9,9744664	10,0255336	41
20	9,8364771	9,8617576	9,9747195	10,0252805	40
21	9,8366109	9,8616383	9,9749726	10,0250274	39
22	9,8367447	9,8615190	9,9752257	10,0247743	38
23	9,8368784	9,8613997	9,9754787	10,0245213	37
24	9,8370121	9,8612803	9,9757318	10,0242682	36
25	9,8371456	9,8611608	9,9759849	10,0240151	35
26	9,8372791	9,8610412	9,9762379	10,0237621	34
27	9,8374125	9,8609215	9,9764909	10,0235091	33
28	9,8375458	9,8608018	9,9767440	10,0232560	32
29	9,8376790	9,8606821	9,9769970	10,0230030	31
30	9,8378122	9,8605622	9,9772500	10,0227500	30
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

46 Degrees.

43 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,8378122	9,8605622	9,9772500	10,0227500	30
31	9,8379453	9,8604423	9,9775030	10,0224970	29
32	9,8380783	9,8603223	9,9777560	10,0222440	28
33	9,8382112	9,8602022	9,9780090	10,0219910	27
34	9,8383441	9,8600821	9,9782620	10,0217380	26
35	9,8384769	9,8599619	9,9785149	10,0214851	25
36	9,8386096	9,8598416	9,9787679	10,0212321	24
37	9,8387422	9,8597213	9,9790209	10,0209791	23
38	9,8388747	9,8596009	9,9792738	10,0207262	22
39	9,8390072	9,8594804	9,9795268	10,0204732	21
40	9,8391396	9,8593599	9,9797797	10,0202203	20
41	9,8392719	9,8592393	9,9800326	10,0199674	19
42	9,8394041	9,8591186	9,9802856	10,0197144	18
43	9,8395363	9,8589978	9,9805385	10,0194615	17
44	9,8396684	9,8588770	9,9807914	10,0192086	16
45	9,8398004	9,8587561	9,9810443	10,0189557	15
46	9,8399323	9,8586351	9,9812972	10,0187028	14
47	9,8400642	9,8585141	9,9815501	10,0184499	13
48	9,8401959	9,8583929	9,9818030	10,0181970	12
49	9,8403276	9,8582718	9,9820559	10,0179441	11
50	9,8404593	9,8581505	9,9823087	10,0176913	10
51	9,8405908	9,8580202	9,9825616	10,0174384	9
52	9,8407223	9,8579078	9,9828145	10,0171855	8
53	9,8408537	9,8577863	9,9830673	10,0169327	7
54	9,8409850	9,8576648	9,9833202	10,0166798	6
55	9,8411162	9,8575432	9,9835730	10,0164270	5
56	9,8412474	9,8574215	9,9838259	10,0161741	4
57	9,8413785	9,8572998	9,9840787	10,0159213	3
58	9,8415095	9,8571779	9,9843315	10,0156685	2
59	9,8416404	9,8570561	9,9845844	10,0154156	1
60	9,8417713	9,8569341	9,9848372	10,0151628	0
	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.

46 Degrees.

44 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,8417713	9,8569341	9,9848372	10,0151628	60
1	9,8419021	9,8568121	9,9850900	10,0149100	59
2	9,8420328	9,8566900	9,9853428	10,0146572	58
3	9,8424634	9,8565678	9,9855956	10,0144044	57
4	9,8422939	9,8564455	9,9858484	10,0141516	56
5	9,8424244	9,8563232	9,9861012	10,0138988	55
6	9,8425548	9,8562008	9,9863540	10,0136460	54
7	9,8426851	9,8560784	9,9866068	10,0133932	53
8	9,8428154	9,8559558	9,9868596	10,0131404	52
9	9,8429456	9,8558332	9,9871123	10,0128877	51
10	9,8430757	9,8557106	9,9873651	10,0126349	50
11	9,8432057	9,8555878	9,9876179	10,0123821	49
12	9,8433356	9,8554650	9,9878706	10,0121294	48
13	9,8434655	9,8553421	9,9881234	10,0118766	47
14	9,8435953	9,8552192	9,9883761	10,0116239	46
15	9,8437250	9,8550961	9,9886289	10,0113711	45
16	9,8438547	9,8549730	9,9888816	10,0111184	44
17	9,8439842	9,8548499	9,9891344	10,0108656	43
18	9,8441137	9,8547266	9,9893871	10,0106129	42
19	9,8442432	9,8546033	9,9896399	10,0103601	41
20	9,8443725	9,8544799	9,9898926	10,0101074	40
21	9,8445018	9,8543564	9,9901453	10,0098547	39
22	9,8446310	9,8542329	9,9903981	10,0096019	38
23	9,8447601	9,8541093	9,9906508	10,0093492	37
24	9,8448891	9,8539856	9,9909035	10,0090965	36
25	9,8450181	9,8538619	9,9911562	10,0088438	35
26	9,8451470	9,8537381	9,9914089	10,0085911	34
27	9,8452758	9,8536142	9,9916616	10,0083384	33
28	9,8454045	9,8534902	9,9919143	10,0080857	32
29	9,8455332	9,8533662	9,9921670	10,0078330	31
30	9,8456618	9,8532421	9,9924197	10,0075803	30
	Tan. Com.	Sine.	Sine Com.	Tangent.	Min.

45 Degrees.

Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,8456618	9,8532421	9,9924197	10,0075803	30
31	9,8457903	9,8531179	9,9926724	10,0073276	29
32	9,8459188	9,8529936	9,9929251	10,0070749	28
33	9,8460471	9,8528693	9,9931778	10,0068222	27
34	9,8461754	9,8527449	9,9934305	10,0065695	26
35	9,8463036	9,8526204	9,9936832	10,0063168	25
36	9,8464318	9,8524959	9,9939359	10,0060641	24
37	9,8465599	9,8523713	9,9941880	10,0058114	23
38	9,8466879	9,8522466	9,9944413	10,0055587	22
39	9,8468158	9,8521218	9,9946940	10,0053060	21
40	9,8469436	9,8519970	9,9949466	10,0050534	20
41	9,8470714	9,8518721	9,9951993	10,0048007	19
42	9,8471991	9,8517471	9,9954520	10,0045480	18
43	9,8473267	9,8516220	9,9957047	10,0042953	17
44	9,8474543	9,8514969	9,9959573	10,0040427	16
45	9,8475817	9,8513717	9,9962100	10,0037900	15
46	9,8477091	9,8512465	9,9964627	10,0035373	14
47	9,8478365	9,8511211	9,9967154	10,0032846	13
48	9,8479637	9,8509957	9,9969680	10,0030320	12
49	9,8480909	9,8508702	9,9972207	10,0027793	11
50	9,8482180	9,8507446	9,9974734	10,0025206	10
51	9,8483450	9,8506190	9,9977260	10,0022740	9
52	9,8484720	9,8504933	9,9979787	10,0020213	8
53	9,8485989	9,8503675	9,9982314	10,0017686	7
54	9,8487257	9,8502417	9,9984840	10,0015160	6
55	9,8488524	9,8501157	9,9987367	10,0012633	5
56	9,8489791	9,8499897	9,9989893	10,0010107	4
57	9,8491057	9,8498637	9,9992420	10,0007580	3
58	9,8492322	9,8497375	9,9994947	10,0005053	2
59	9,8493586	9,8496113	9,9997473	10,0002527	1
60	9,8494850	9,8494850	10,0000000	10,0000000	0
	Tan. Com.	Sine.	Sine Com.	Tangent.	Min.

A TABLE OF DIFFERENCE OF LATITUDE AND DEPARTURE

To every DEGREE and QUARTER POINT of the COMPASS.

Cour.		Dist. 1.		Dist. 2.		Dist. 3.		Cour.	
Pts	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts
	1	0,9998	0,0175	1,9997	0,0349	2,9995	0,0524	89	
	2	0,9994	0,0349	1,9988	0,0698	2,9982	0,1047	88	
0.¼		0,9988	0,0491	1,9976	0,0981	2,9964	0,1472		7.¾
	3	0,9986	0,0523	1,9973	0,1047	2,9959	0,1570	87	
	4	0,9976	0,0698	1,9951	0,1395	2,9927	0,2093	86	
	5	0,9962	0,0872	1,9924	0,1743	2,9886	0,2615	85	
0.½		0,9952	0,0980	1,9904	0,1960	2,9856	0,2940		7.½
	6	0,9945	0,1045	1,9890	0,2091	2,9836	0,3136	84	
	7	0,9925	0,1219	1,9851	0,2437	2,9776	0,3656	83	
	8	0,9903	0,1392	1,9805	0,2783	2,9708	0,4175	82	
0.¾		0,9892	0,1467	1,9784	0,2935	2,9675	0,4402		7.¼
	9	0,9877	0,1564	1,9754	0,3129	2,9631	0,4693	81	
	10	0,9848	0,1736	1,9696	0,3473	2,9544	0,5209	80	
	11	0,9816	0,1908	1,9633	0,3816	2,9449	0,5724	79	
1.		0,9808	0,1951	1,9616	0,3902	2,9424	0,5853		7.
	12	0,9781	0,2079	1,9563	0,4158	2,9344	0,6237	78	
	13	0,9744	0,2250	1,9487	0,4499	2,9231	0,6749	77	
	14	0,9703	0,2419	1,9406	0,4838	2,9108	0,7258	76	
1.¼		0,9700	0,2430	1,9401	0,4860	2,9101	0,7289		6.¾
	15	0,9659	0,2588	1,9319	0,5176	2,8978	0,7765	75	
	16	0,9613	0,2756	1,9225	0,5513	2,8838	0,8269	74	
1.½		0,9569	0,2903	1,9139	0,5806	2,8708	0,8709		6.½
	17	0,9563	0,2924	1,9126	0,5847	2,8689	0,8771	73	
	18	0,9511	0,3090	1,9021	0,6180	2,8532	0,9271	72	
	19	0,9455	0,3256	1,8910	0,6511	2,8366	0,9767	71	
1.¾		0,9415	0,3369	1,8831	0,6738	2,8246	1,0107		6.¼
	20	0,9397	0,3420	1,8794	0,6840	2,8191	1,0261	70	
	21	0,9336	0,3584	1,8672	0,7167	2,8007	1,0751	69	
	22	0,9272	0,3746	1,8544	0,7492	2,7816	1,1238	68	
2.		0,9239	0,3827	1,8478	0,7654	2,7716	1,1480		6.
Pts	D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.	Pts
		Dist. 1.		Dist. 2.		Dist 3.			

Cour.		Dist. 4.		Dist. 5.		Dist. 6.		Cour.	
Pts	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts
	1	3,9994	0,0698	4,9992	0,0873	5,9991	0,1047	89	
	2	3,9976	0,1396	4,9970	0,1745	5,9963	0,2094	88	
0.$\frac{1}{4}$		3,9952	0,1963	4,9940	0,2453	5,9928	0,2944		7.$\frac{3}{4}$
	3	3,9945	0,2093	4,9931	0,2617	5,9918	0,3140	87	
	4	3,9903	0,2790	4,9878	0,3488	5,9854	0,4185	86	
	5	3,9848	0,3486	4,9810	0,4358	5,9772	0,5229	85	
0.$\frac{1}{2}$		3,9807	0,3921	4,9759	0,4901	5,9711	0,5881		7.$\frac{1}{2}$
	6	3,9781	0,4181	4,9726	0,5226	5,9671	0,6272	84	
	7	3,9702	0,4875	4,9627	0,6093	5,9553	0,7312	83	
	8	3,9611	0,5567	4,9513	0,6959	5,9416	0,8350	82	
0.$\frac{3}{4}$		3,9567	0,5869	4,9459	0,7337	5,9351	0,8804		7.$\frac{1}{4}$
	9	3,9508	0,6257	4,9384	0,7822	5,9261	0,9386	81	
	10	3,9392	0,6946	4,9240	0,8682	5,9088	1,0419	80	
	11	3,9265	0,7632	4,9081	0,9540	5,8898	1,1449	79	
1.		3,9231	0,7804	4,9039	0,9754	1,8847	1,1705		7.
	12	3,9126	0,8316	4,8907	1,0396	5,8689	1,2475	78	
	13	3,8975	0,8998	4,8718	1,1248	5,8462	1,3497	77	
	14	3,8812	0,9677	4,8515	1,2096	5,8218	1,4515	76	
1.$\frac{1}{4}$		3,8801	0,9719	4,8502	1,2149	5,8202	1,4579		6.$\frac{3}{4}$
	15	3,8637	1,0353	4,8296	1,2941	5,7956	1,5529	75	
	16	3,8450	1,1025	4,8063	1,3782	9,7676	1,6538	74	
1.$\frac{1}{2}$		3,8278	1,1611	4,7847	1,4514	5,7416	1,7417		6.$\frac{1}{2}$
	17	3,8252	1,1695	4,7815	1,4619	5,7378	1,7542	73	
	18	3,8042	1,2361	4,7553	1,5451	5,7063	1,8541	72	
	19	3,7821	1,3023	4,7276	1,6278	5,6731	1,9534	71	
1.$\frac{3}{4}$		3,7662	1,3476	4,7077	1,6844	5,6493	2,0213		6.$\frac{1}{4}$
	20	3,7588	1,3681	4,6985	1,7101	5,6382	2,0521	70	
	21	3,7343	1,4335	4,6679	1,7918	5,6015	2,1502	69	
	22	3,7087	1,4984	4,6359	1,8730	5,5631	2,2476	68	
2.		3,6955	1,5307	4,6194	1,9134	5,5433	2,2961		6.
Pts	D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.	Pts
		Dist. 4.		Dist. 5.		Dist. 6.			

Cour.		Dist. 7.		Dist. 8.		Dist. 9.			Cour.
Pts	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts
	1	6,9989	0,1222	7,9988	0,1396	8,9986	0,1571	89	
	2	6,9957	0,2443	7,9951	0,2792	8,9945	0,3141	88	
0.¼		6,9916	0,3435	7,9904	0,3925	8,9892	0,4416		7.¾
	3	6,9904	0,3664	7,9890	0,4187	8,9877	0,4710	87	
	4	6,9829	0,4883	7,9805	0,5580	8,9781	0,6278	86	
	5	6,9734	0,6101	7,9696	0,6972	8,9658	0,7844	85	
0.½		6,9663	0,6861	7,9615	0,7841	8,9567	0,8822		7.½
	6	6,9617	0,7317	7,9562	0,8362	8,9507	0,9408	84	
	7	6,9478	0,8531	7,9404	0,9750	8,9329	1,0968	83	
	8	6,9319	0,9742	7,9221	1,1134	8,9124	1,2526	82	
0.¾		6,9242	1,0271	7,9134	1,1738	8,9026	1,3206		7.¼
	9	6,9138	1,0950	7,9015	1,2515	8,8892	1,4079	81	
	10	6,8937	1,2155	7,8785	1,3892	8,8633	1,5628	80	
	11	6,8714	1,3357	7,8530	1,5265	8,8346	1,7173	79	
1.		6,8655	1,3656	7,8463	1,5607	8,8271	1,7558		7.
	12	6,8479	1,4554	7,8252	1,6633	8,8033	1,8712	78	
	13	6,8206	1,5746	7,7950	1,7996	8,7693	2,0246	77	
	14	6,7921	1,6935	7,7624	1,9354	8,7327	2,1773	76	
1.¼		6,7902	1,7009	7,7602	1,9438	8,7303	2,1868		6.¾
	15	6,7615	1,8117	7,7274	2,0706	8,6933	2,3294	75	
	16	6,7288	1,9295	7,6901	2,2051	8.6513	2,4807	74	
1.½		6,6986	2,0320	7,6555	2,3[illegible]23	8,6125	2,6126		6.½
	17	,6941	2,0466	7,6504	2,3390	8,6067	2,6313	73	
	18	6,6574	2,1631	7,6084	2,4721	8,5595	2,7812	72	
	19	6,6186	2,2790	7,5642	2,6045	8,5097	2,9301	71	
1.¾		9,5908	2,3582	7,5324	2,6951	8,4739	3,0320		6.¼
	20	6,5779	2,3941	7,5175	2,7362	8,4572	3,0782	70	
	21	6,5351	2,5086	7,4686	2,8669	8,4022	3,2253	69	
	22	6,4903	2,6222	7,4175	2,9969	8,3447	3,3715	68	
2.		6,4672	2,6788	7,3910	3,0615	8,3149	3,4441		6.
Pts	D.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	D.	Pts
		Dist. 7.		Dist. 8.		Dist. 9.			

Cour.		Dist. 1.		Dist. 2.		Dist. 3.		Cour.	
Pts	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts
	23	0,9205	0,3907	1,8410	0,7815	2,7615	1,1722	67	
	24	0,9135	0,4067	1,8270	0,8135	2,7406	1,2202	66	
	25	0,9063	0,4226	1,8126	0,8452	2,7189	1,2679	65	
2.¼		0,9040	0,4276	1,8080	0,8551	2,7120	1,2827		5.¾
	26	0,8988	0,4384	1,7976	0,8767	2, 964	1,3151	64	
	27	0,8910	0,4540	1,7820	0,9080	2,6730	1,3620	63	
	28	0,8829	0,4695	1,7659	0,9389	2,6488	1,4084	62	
2.½		0,8819	0,4714	1,7638	0,9428	2,6458	1,4142		5.½
	29	0,8746	0,4848	1,7492	0,9696	2,6239	1,4544	61	
	30	0,8660	0,5000	1,7320	1,0000	2,5981	1,5000	60	
2.¾		0,8577	0,5141	1,7155	1,0282	2,5732	1,5423		5.¼
	31	0,8572	0,5150	1,7143	1,0301	2,5715	1,5451	59	
	32	0,8480	0,5299	1,6961	1,0598	2,5441	1,5896	58	
	33	0,8387	0,5446	1,6773	1,0893	2,5160	1,6339	57	
3.		0,8315	0,5556	1,6629	1,1111	2,4944	1,6667		5
	34	0,8290	0,5592	1,6581	1,1184	2,4871	1,6776	56	
	35	0,8192	0,5736	,6383	1,1472	2,4575	1,7207	55	
	36	0,8090	0,5878	1,6180	1,1756	2,4271	1,7634	54	
3.¼		0,8032	0,5957	1,6064	1,1914	2,4096	1,7871		5.¼
	37	0,7986	0,6018	1,5973	1,2036	2,3959	1,8054	53	
	38	0,7880	0,6157	1,5760	1,2313	2,3640	1,8470	52	
	39	0,7771	0,6293	1,5543	1,2586	2,3314	1,8880	51	
3.½		0,7730	0,6344	1,5460	1,2688	2,3190	1,9032		4.½
	40	0,7660	0,6428	1,5321	1,2856	2,2981	1,9284	50	
	41	0,7547	0,6561	1,5094	1,3121	2,2641	1,9682	49	
	42	0,7431	0,6691	1,4863	1,3383	2,2294	2,0074	48	
3.¾		0,7410	0,6716	1,4819	1,3431	2,2229	2,0147		4.¼
	43	0,7314	0,6820	1,4628	1,3640	2,1941	2,0460	47	
	44	0,7193	0,6947	1,4387	1,3894	2,1580	2,0840	46	
4.	45	0,7071	0,7071	1,4142	1,4142	2,1213	2,1213	45	4.
Pts	D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.	Pts
		Dist. 1.		Dist. 2.		Dist. 3.			

Cour.		Dist. 4.		Dist. 5.		Dist. 6.		Cour.	
Pts	D.	Lat.	Dep.	Lat.	Dep.	Lat	Dep.	D.	Pts
	23	3,6820	1,5629	4,6025	1,9537	5,5230	2,3444	67	
	24	3,6542	1,6269	4,5677	2,0337	5,4813	2,4404	66	
	25	3,6252	1,6905	4,5315	2,1131	5,4378	2,5357	65	
2·1/4		3,6160	1,7102	4,5199	2,1378	5,4239	2,5653		5·3/4
	26	3,5952	1,7535	4,4940	2,1919	5,3928	2,6302	64	
	27	3,5640	1,8160	4,4550	2,2699	5,3460	2,7239	63	
	28	3,5318	1,8779	4,4147	2,3474	5,2977	2,8168	62	
2·1/2		3,5277	1,8856	4,4096	2,3570	5,2915	2,8284		5·1/2
	29	3,4985	1,9392	4,3731	2,4240	5,2477	2,9089	61	
	30	3,4641	2,0000	4,3301	2,5000	5,1961	3,0000	60	
2·3/4		3,4309	2,0564	4,2886	2,5705	5,1464	3,0846		5·1/4
	31	3,4287	2,0602	4,2858	2,5752	5,1430	3,0902	59	
	32	3,3922	2,1197	4,2402	2,6496	5,0883	3,1795	58	
	33	3,3547	2,1786	4,1934	2,7232	5,0320	3,2678	57	
3·		3,3259	2,2223	4,1573	2,7778	4,9888	3,3334		5·
	34	3,3162	2,2368	4,1452	2,7960	4,9742	3,3552	56	
	35	3,2766	2,2943	4,0958	2,8679	4,9149	3,4415	55	
	36	3,2361	2,3511	4,0451	2,9389	4,8541	3,5267	54	
3·1/4		3,2128	2,3828	4,0160	2,9785	4,8192	3,5742		4·3/4
	37	3,1945	2,4073	3,9932	3,0091	4,7918	3,6109	53	
	38	3,1520	2,4626	3,9401	3,0783	4,7281	3,6940	52	
	39	3,1086	2,5173	3,8857	3,1466	4,6629	3,7759	51	
3·1/2		3,0920	2,5376	3,8650	3,1720	4,6381	3,8064		4·1/2
	40	3,0642	2,5712	3,8302	3,2139	4,5963	3,8567	50	
	41	3,0188	2,6242	3,7736	3,2803	4,5283	3,9363	49	
	42	2,9726	2,6765	3,7157	3,3457	4,4589	4,0148	48	
3·3/4		2,9638	2,6862	3,7048	3,3578	4,4457	4,0294		4·1/4
	43	2,9254	2,7280	3,6568	3,4100	4,3881	4,0920	47	
	44	2,8774	2,7786	3,5967	3,4733	4,3160	4,1679	46	
4·	45	2,8284	2,8284	3,5355	3,5355	4,2426	4.2426	45	4·
Pts	D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.	Pts
		Dist. 4.		Dist. 5.		Dist. 6.			

Cour.		Dist. 7.		Dist. 8.		Dist. 9.			Cour.
Pts	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts
	23	6,4435	2,7351	7,3640	3,1258	8,2845	3,5166	67	
	24	6,3948	2,8472	7,3084	3,1539	8,2219	3,6606	66	
	25	6,3442	2,9583	7,2505	3,3809	8,1568	3,8036	65	
2.¼		6,3279	2,9929	7,1319	3,4204	8,1359	3,848[illegible]		5.¾
	26	6,2916	3,0686	7,1904	3,5070	8,0891	3,9453	64	
	27	6,2370	3,1779	7,1280	3,6319	8,0191	4,0859	63	
	28	6,1806	3,2863	7,0636	3,7558	7,9465	4,2252	62	
2.½		6,1734	3,2998	7,0554	3,7712	7,9373	4,2426		5.½
	29	6,1223	3,3937	6,9972	3,8785	7,8716	4,3632	61	
	30	6,0622	3,5000	6,9282	4,0000	7,7942	4,5000	60	
2.¾		6,0041	3,5987	6,8618	4,1128	7,7196	4,6269		5.¼
	31	6,0002	3,6052	6,8573	4,1203	7,7145	4,6353	59	
	32	5,9363	3,7094	6,7843	4,2394	7,6324	4,7693	58	
	33	5,8707	3,8124	6,7094	4,3571	7,5480	4,9018	57	
3.		5,8203	3,8890	6,6518	4,4446	7,4832	5,0001		5.
	34	5,8033	3,9144	6,6323	4,4735	7,4613	5,0327	56	
	35	5,7341	4,0150	6,5532	4,5886	7,3724	5,1622	55	
	36	5,6631	4,1145	6,4721	4,7023	7,2812	5,2901	54	
3.¼		5,6224	4,1699	6,4257	4,7656	7,2289	5,3613		4.¾
	37	5,5904	4,2127	6,3891	4,8145	7,1877	5,4163	53	
	38	5,5161	4,3096	6,3041	4,9253	7,0921	5,5409	52	
	39	5,4400	4,4052	6,1272	5,0346	6,9943	5,6639	51	
3.½		5,4111	4,4408	6,1841	5,0751	6,9571	5,7095		4.½
	40	5,3623	4,4995	6,1284	5,1423	6,8944	5,7851	50	
	41	5,2830	4,5924	6,0377	5,2485	6,7924	5,9045	49	
	42	5,2020	4,6839	5,9452	5,3530	6,6883	6,0222	48	
3.¾		5,1867	4,7009	5,9276	5,3725	6,6686	6,0440		4.¼
	43	5,1195	4,7740	5,8508	5,4560	6,5822	6,1380	47	
	44	5,0354	4,8626	5,7547	5,5573	6,4741	6,2519	46	
4.	45	4,9497	4,9497	5,6569	5,6569	6,3640	6,3640	45	4.
Pts	D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.	Pts
		Dist. 7.		Dist. 8.		Dist. 9.			